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APPLICATION OF THE STOCHASTIC TRANSPORT THEORY TO REACTIVITY MEASUREMENTS IN A SUBCRITICAL ASSEMBLY DRIVEN BY A PULSED SOURCE

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ABSTRACT

Using the stochastic transport theory formalism developed by Muñoz-Cobo for a radioactive source and for a spallation source, we derive an expression for the Cross Power Spectral Density (CPSD) for a subcritical system driven by an external pulsed neutron source. The derivation of this expression has been done in a rigorous and general way including the energy and spatial dependency. The CPSD shows some peaks at the source frequency together with its harmonics. The final expression containing some approximations (fundamental mode analysis; Dirac delta pulses for the source) is compared with calculations using a Monte Carlo code. In the future, comparisons with measurements in the MUSE project are planned.

Key Words: Noise, Stochastic, Pulse, Subcritical

1. INTRODUCTION

The control of Accelerator Driven Systems (ADS) requires the development of methods to monitor the reactivity of the reactor while not interfering with its normal operation. Because the ADS presents many challenges for fluctuation based measurements, the MUSE project was proposed to study these methods, Gandini and Salvatores [1]. In order to do so, a deuterium accelerator (GENEPI) that can operate with either a deuterium or tritium target has been coupled to a subcritical reactor core containing MOX fuel (MASURCA). In this paper, a deuterium target producing neutrons with energy of 2.45 MeV at a maximum frequency of 5KHz has been considered.

Several authors, Pazsit and Ceder [3]; Kuang and Pazsit [4]; Degweker [2], and Rugama et al [5] proposed the use of random noise techniques in ADS, which differ from the classical noise techniques because of the non-Poissonian neutron source. In this paper, a theoretical framework to compute the covariance between two neutron detectors has been derived. The method is based on the stochastic neutron transport theory and determines various kinetics parameters through the analysis of neutron detector signals. From these parameters the value of the K_{eff} can be extracted. Although at the end, the fundamental mode approximation has been considered, the derivation includes the spatial and energy dependency in order to have a general expression.

In this work, we first analyse the stochastic transport theory for subcritical reactivity measurements and give its main limitations. The results is an analytical expression for the Cross Power Spectral Density, CPSD, based on previous developments of Pal [6], Bell [7], and Muñoz-Cobo et al [8] and [9]. Monte Carlo simulations have been performed with the MCNP-DSP code [13] to compare the analytical expression with the calculation results for the CPSD between pairs of detectors located in the reflector region of MASURCA. Finally, we give some recommendations for further applications.

In the future, measurements will be performed within the framework of the EC-supported MUSE project in order to check the applicability of this technique together with other noise techniques like Feynman-α and Rossi-α methods.

2. THE CROSS CORRELATION FUNCTION WITH A TIME DEPENDENT SOURCE

Usually, zero power noise is described by the master equations, which define the probability distribution that can be used to calculate the correlation between two neutron detectors located in the reactor region. The analytical derivation contains two steps. First, we derive the distribution of the number of neutrons or its generating function due to one single source neutron. Secondly, we derive a formula that connects the generating function of the single-particle-induced distribution with that induced by an external source. Contributions of delayed neutrons are neglected .

The source considered in this paper corresponds to a pulsed neutron source at $\vec{r}_0$ with characteristics similar to those of the source used in the MUSE project and with statistical properties defined by Pazsit and Ceder [3]:

$$S(1,t) = \frac{S_o}{T_w}\delta(\vec{r}-\vec{r}_0)\, f_s(v,\Omega)\sum_{n=0}^{N} H(t-nT_p) - H(t-(nT_p+T_w)) \tag{1}$$

The source probability generating function can be described by:

$$G_s^{D2D3}(z_1,z_2) = \sum_{Nc=0}^{\infty}\sum_{Mc=0}^{\infty} z_1^{Nc} z_2^{Mc} P_{NcMc}(D_2D_3) \tag{2}$$

Where $P_{NcMc}(D_2D_3)$ is the joint probability to have Nc counts in detector D_2 when Mc counts are obtained in detector D_3 after a neutron source pulse at time t_o. The expression of this probability can be written as:

$$\begin{aligned}P_{N_cM_c}(D_2,D_3) = \exp\left\{-\int_{to}^{tf} S(1,t')dt'\right\}\Big[\delta_{N_c0}\delta_{M_c0} + \int_{to}^{tf} S(1,t_1)dt_1\int d\Omega_1\int dv_1 f_s(v_1,\Omega_1) K_{n_1,m_1}(\vec{r}_o,v_1,\Omega_1,t_1\,|\,d_2,d_3)\\ +\int_{to}^{tf} S(1,t_1)dt_1\int_{t1}^{tf} S(1,t_2)dt_2\int d\Omega_1\int dv_1 f_s(v_1,\Omega_1)\int d\Omega_2\int dv_2 f_s(v_2,\Omega_2)\\ K_{n_1,m_1}(\vec{r}_o,v_1,\Omega_1,t_1\,|\,d_2,d_3) K_{n_2,m_2}(\vec{r}_o,v_2,\Omega_2,t_2\,|\,d_2,d_3)+....\Big]\end{aligned} \tag{3}$$

After some mathematical manipulations, the source probability generating function (2) obtained from the probability described by equation (3) writes:

$$G_s^{D2D3}(z_1,z_2)=\exp\left\{-\int_{to}^{tf} S(1,t')dt'\right\}\left\{1+\int_{to}^{tf} S(1,t_1)dt_1 T_k^s G_k^{D_2D_3}+\frac{1}{2}\left(\int_{to}^{tf} S(1,t_1)dt_1 T_k^s G_k^{D_2D_3}\right)^2+...\right\}= \\ =\exp\left\{-\int_{to}^{tf} dtS(1,t)\left(T_k^s G_k^{D_2D_3}-1\right)\right\} \tag{4}$$

We define $G_k^{D_2D_3}\left(\vec{r},v,\Omega,t\right)$ as the kernel probability generating function from the definition of $K_{n_c,m_c}\left(\vec{r},v,\Omega,t\,|\,d_2,d_3\right)$, the probability having n_c counts in detector D_2 and m_c counts in D_3 after a neutron has been injected at the phase space point $\left(\vec{r},v,\Omega,t\right)$:

$$G_k^{D_2D_3}\left(\vec{r},v,\Omega,t\right)=\sum_{n_c=0}^{\infty}\sum_{m_c=0}^{\infty} z_1^{n_c} z_2^{m_c} K_{n_c,m_c}\left(\vec{r},v,\Omega,t\,|\,d_2,d_3\right) \tag{5}$$

The product $T_k^s G_k^{D_2D_3}$ writes:

$$T_k^s G_k^{D_2D_3}=\int dv\int d\Omega f_s\left(v,\Omega\right) G_k^{D_2D_3}\left(\vec{r},v,\Omega,t\right) \tag{6}$$

From the first and second moment of the source probability generating function described in equation (4), the following expressions can be obtained:

$$\bar{N}_C=\frac{\partial}{\partial z_1}G_s^{D_2D_3}(z_1,z_2)=\left[\int_{to}^{tf} dtS(1,t)T_k^s\frac{\partial}{\partial z_1}G_K^{D_2D_3}\right]G_s^{D_2D_3}(z_1,z_2)=\left\langle S(1,t)\middle|\bar{n}_C(1,t\,|\,d_2\right\rangle \tag{7}$$

$$\bar{M}_C=\frac{\partial}{\partial z_2}G_s^{D_2D_3}(z_1,z_2)=\left[\int_{to}^{tf} dtS(1,t)T_k^s\frac{\partial}{\partial z_2}G_K^{D_2D_3}\right]G_s^{D_2D_3}(z_1,z_2)=\left\langle S(1,t)\middle|\bar{m}_C(1,t\,|\,d_3\right\rangle \tag{8}$$

$$\overline{N_C M_C}=\frac{\partial^2}{\partial z_1\partial z_2}G_s^{D_2D_3}(z_1,z_2)=\left[\int_{to}^{tf} dtS(1,t)T_k^s\frac{\partial^2}{\partial z_1\partial z_2}G_K^{D_2D_3}\right]G_s^{D_2D_3}(z_1,z_2)+\bar{N}_C\bar{M}_C \tag{9}$$

Using the definition for the covariance ($Cov(D_2,D_3)=\overline{N_C M_C}-\bar{N}_C\bar{M}_C$) and applying $z_1=z_2=1$ to equations (7), (8) and (9), the expression we get for the covariance of a time-dependent source reads:

$$Cov(D_2,D_3)=\left\langle S(1,t)\middle|\overline{n_C m}_C(1,t\,|\,d_2,d_3\right\rangle=\int_{to}^{tf} dtS(1,t)\int dv\int d\Omega f_s(v,\Omega)\overline{n_c m_c}\left(\vec{r}_0,v,\Omega,t\,|\,d_2,d_3\right) \tag{10}$$

Here $\overline{n_c m_c}\left(\vec{r}_0,v,\Omega,t\,|\,d_2,d_3\right)$ is the second moment of the kernel probability generating function, which equals the average of the number of simultaneous counts in detectors D_2 and D_3 after injection of one neutron at the phase space point $(\vec{r}_0,v,\Omega,t)$.

Note that $\overline{n_c m_c}\left(\vec{r}_0,v,\Omega,t\,|\,d_2,d_3\right)$ obeys the equation gives by Muñoz-Cobo et al [8]:

$$\left\{-\frac{1}{v}\frac{\partial}{\partial t}+L^{+}\right\}\overline{n_C m_C}(1,t\,|\,d_2,d_3)=\overline{\nu(\nu-1)}\Sigma_f(\vec{r},v,t)I^{D_2}(\vec{r},v,t\,|\,d_2)I^{D_3}(\vec{r},v,t\,|\,d_3) \qquad (11)$$

The importance $I^{D_i}(\vec{r},v,t\,|\,d_i)$ of detector D_i is defined as the spectral averaged number of counts produced by a neutron source at point $\vec{r}$.

$$\begin{aligned} I^{D_2}(\vec{r},t,d_2) &= \int dv\int\frac{d\Omega}{4\pi}\chi(v)\overline{n}_C(\vec{r},v,\Omega,t\,|\,d_2) \\ I^{D_3}(\vec{r},t,d_3) &= \int dv\int\frac{d\Omega}{4\pi}\chi(v)\overline{m}_C(\vec{r},v,\Omega,t\,|\,d_3) \end{aligned} \qquad (12)$$

The neutron flux transport equation including an external time-dependent source writes:

$$\frac{1}{v}\frac{\partial}{\partial t}\Phi(1,t)+L\Phi(1,t)=S(1,t) \qquad (13)$$

where L is the transport operator defined by:

$$L=\Omega\vec{\nabla}_r+\Sigma\left(\vec{r},v,t\right)-(S_c+F) \qquad (14)$$

Using the definitions (11) and (13), the covariance equation described in (10) can be recasted into a function of the importances:

$$\begin{aligned} Cov(D_2,D_3) &= \left\langle \Phi(1,t)\middle|\overline{\nu(\nu-1)}\Sigma_f(\vec{r},v)I^{D_2}(\vec{r},t,d_2)I^{D_3}(\vec{r},t,d_3)\right\rangle \\ &= \overline{\nu(\nu-1)}\int_{to}^{tf}dt\int d^3r F(\vec{r},t)I^{D_2}(\vec{r},t,d_2)I^{D_3}(\vec{r},t,d_3) \end{aligned} \qquad (15)$$

The fission rate density at time t, $F(\vec{r},t)$, is defined by:

$$F(\vec{r},t)=\int dv\int d\Omega\Sigma_f(\vec{r},v)\Phi(\vec{r},v,\Omega,t) \qquad (16)$$

The descriptor used will be the cross-correlation function $\Psi_{23}(tf_2,tf_3)$, which is defined in terms of the instantaneous count rates as the following limit of the covariance function for infinitely small gate lengths At_{c2} and At_{c3}:

$$\Psi_{23}(tf_2,tf_3)=\lim_{\substack{At_{c2}\to 0\\ At_{c3}\to 0}}\frac{Cov(D_2,D_3)}{\Delta t_{c2}\Delta t_{c3}}=\overline{\nu(\nu-1)}\int_{to}^{tf}dt\int d^3r F(\vec{r},t)I^{D_2}(\vec{r},tf_2-t)I^{D_3}(\vec{r},tf_3-t) \qquad (17)$$

The limits of equation (17) give the importance functions and write:

$$I^{D_2}(\vec{r},tf_2-t)=\lim_{At_{c2}\to 0}\frac{I^{D_2}(\vec{r},t,d_2)}{\Delta t_{c2}}=\int dv\int\frac{d\Omega}{4\pi}\chi(v)\overline{n}_C(\vec{r},v,\Omega,tf_2-t) \qquad (18)$$

$$I^{D_3}(\vec{r},tf_3-t)=\lim_{At_{c3}\to 0}\frac{I^{D_3}(\vec{r},t,d_3)}{\Delta t_{c3}}=\int dv\int\frac{d\Omega}{4\pi}\chi(v)\overline{m}_C(\vec{r},v,\Omega,tf_3-t) \qquad (19)$$

$\overline{n}_C(\vec{r},v,\Omega,tf_2-t)$ and $\overline{m}_C(\vec{r},v,\Omega,tf_3-t)$ are the count rates at tf_2 and tf_3 produced by a single neutron injected at (1, t) . They are described as a displacement kernel.

3. CROSS POWER SPECTRAL DENSITY USING A PULSED NEUTRON SOURCE

The external time-dependent source produces a time-dependent neutron flux $\Phi(1,t)$. To obtain the CPSD, we operate $\int d\tau \exp(-iw\tau)$ on both sides of the equation (17). Introducing $tf_3 = tf_2 + \tau$, we get:

$$CPSD_{23}(w) = \overline{v(v-1)}\frac{1}{2\pi}\int dw_2 \int d^3r \exp\{i(w_2+w)tf_2\} F(\vec{r},w_2+w) I^{D_2}(\vec{r},w_2) I^{D_3}(\vec{r},w) \qquad (20)$$

In equation (20) the upper limit can be considered as $+\infty$ because $\begin{cases}\overline{n}_C(1,tf_2-t)=0 & t>tf_2\\ \overline{m}_C(1,tf_3-t)=0 & t>tf_3\end{cases}$ while the lower limit can be extended to $-\infty$ when we assume that the neutron pulse generator (GENEPI) was turned on in the remote past.

To simplify the calculations in equation (20) we assume that the periodic source, considered at the beginning of section 2, has the form of a series of Dirac delta pulses:

$$S(1,t) = S_o\delta(\vec{r}-\vec{r}_o) f_s(v,\Omega)\sum_{n=0}^{N}\delta(t-nT_p) \qquad (21)$$

Introducing the angular source frequency $\Theta = {2\pi}/{T_p}$ with T_p the source period and S_o the number of neutrons per pulse, the Fourier transform of the source described in equation (21) yields the source spectrum:

$$S(1,w) = S_o\delta(\vec{r}-\vec{r}_o) f_s(v,\Omega)\Theta\sum_{n=0}^{N}\delta(w-n\Theta) = S_o\delta(\vec{r}-\vec{r}_o) f_s(v,\Omega)\sum_{n}\exp(inwT_p) \qquad (22)$$

The Fourier transform of the transport equation defined in equation (13) yields:

$$(-\frac{iw}{v}+L)\Phi(1,w) = S(1,w) \qquad (23)$$

Considering that the $\alpha - eigenvalue$ equation is given by:

$$L\Phi_n(1) = -\frac{\alpha_n}{v}\Phi_n(1) \qquad (24)$$

Inserting the expansion of the flux in $\alpha-\text{modes}$ as described by Bell and Glasstone [11], $\Phi(\vec{r},v,\Omega,t) = \sum_n T_n(t)\times\Phi_n(\vec{r},v,\Omega)$, in the Fourier transform of the transport equation (23), we obtain an expression for the transport equation in the frequency domain as a function of the $\alpha - eigenvalue$:

$$\sum_n \left(\frac{-iw - \alpha_n}{v} \right) T_n(w)\Phi_n(1) = S(1,w) \tag{25}$$

The frequency-dependent function $T_n(w)$ can be obtained from the scalar multiplication by the adjoint eigenfunction $\Phi_n^+(1)$:

$$T_n(w) = \frac{(\Phi_n^+(1), S(1,w))}{N_n(-iw-\alpha_n)} = \frac{S_n(w)}{N_n(-iw-\alpha_n)} \tag{26}$$

It has been considered that the adjoint and direct eigenfunctions satisfy the biorthogonal relationship:

$$\left(\frac{1}{v}\Phi_m^+, \Phi_n \right) = N_n \delta_{mn} \tag{27}$$

The expansion of the kernel displacement, $n_C(1,w) = \sum_n T_n^+(w)\Phi_n^+(1)$, in $\alpha - \text{modes}$ is done in a similar way. During this derivation we use the expression for the adjoint source operator given by Muñoz-Cobo et al [8]. From both considerations we arrive at the expansion of the adjoint transport equation in $\alpha - \text{modes}$ described by the eigenvalues and the detector location and its characteristics:

$$\sum_n \left(\frac{iw - \alpha_n}{v} \right) T_n^+(w)\Phi_n^+(1) = \eta_n \Sigma_{D\,j} \tag{28}$$

Therefore, the Fourier transform of the spectrally and directionally weighted importance for detector D_2 and D_3 can be rewritten as:

$$I^{D_j}(\vec{r}, w) = \int dv \int \frac{d\Omega}{4\pi} \chi(v) \sum_n \frac{(\eta\Sigma_{D_j}, \Phi_n(1))}{(iw-\alpha_n)} \Phi_n^+(1) \qquad j = 2,3 \tag{29}$$

Once that the source and the flux has been expanded in alpha modes, we can rewrite the expression of the CPSD given in equation (20) as a function of the source frequency and the $\alpha - eigenvalues$ and $\alpha - eigenfunctions$:

$$CPSD_{23}(w) = \overline{v(v-1)} \frac{1}{2\pi} \int dw_2 \int d^3r \exp\{i(w_2+w)tf_2\} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left\{ \int dv \int d\Omega \Sigma_f \Phi_k(1) \chi^2(v) \Phi_n^+(1) \Phi_m^+(1) \right\}$$
$$\frac{\mathrm{S_k}(w+w_2)}{(-i(w+w_2)-\alpha_k)N_k} \frac{(\eta\Sigma_{D_2}\Phi_n)}{(iw_2-\alpha_n)N_n} \frac{(\eta\Sigma_{D_3}\Phi_m)}{(iw-\alpha_m)N_m} \tag{30}$$

Limiting us to the fundamental mode, the formula for the CPSD given in equation (30) simplifies considerably, and will still be valid for slightly subcritical systems. In case of highly subcritical systems the higher harmonics should be included as has been done by

Rugama et al [5]. A study to the applicability of the fundamental mode approximation should be done for each particular system. The result of the fundamental-mode approximation writes:

$$CPSD_{23}(w) = \overline{\nu(\nu-1)}\frac{1}{2\pi}\int d^3r\int d\nu\int d\Omega\Sigma_f\Phi_k(1)\chi^2(\nu)\Phi_n^+(1)\Phi_m^+(1)\sum_{n=0}^{N}\exp\left\{i(w)(tf_2-nT_p)\right\}$$
$$\frac{S_0 I_0(\vec{r}_0)}{N_0^3(-i(w+(tf_2-nT_p))-\alpha_0)}\frac{(\eta\Sigma_{D_2},\Phi_0)}{(i(tf_2-nT_p)-\alpha_0)}\frac{(\eta\Sigma_{D_3},\Phi_0)}{(iw-\alpha_0)} \qquad (31)$$

The fundamental-mode approximation will be denoted with the subindex 0. To simplify the calculation, the pulsed source has been considered as a point source at $\vec{r}_o$. The expression for the source importance used in the equation above follows the scalar product $I_0(\vec{r}_o) = \left(\Phi_0^+(1), f_s(\nu,\Omega)\delta(\vec{r}-\vec{r}_o)\right)$. From the expression (31) we can conclude that the $CPSD_{23}$ depends on the source frequency ($\Theta = 2\pi/T_p$) and the reactor frequency (α_0 for the fundamental mode approximation).

Figure 1 illustrates expression (31) for a source frequency of 500 Hz and a reactivity of –400 pcm, which corresponds to a reactor frequency of about 2050 Hz. Peaks corresponding to the source frequency of 500 Hz and its harmonics are observed. From the shape of this CPSD we can derive the reactor transfer function as is being done for a critical reactor.

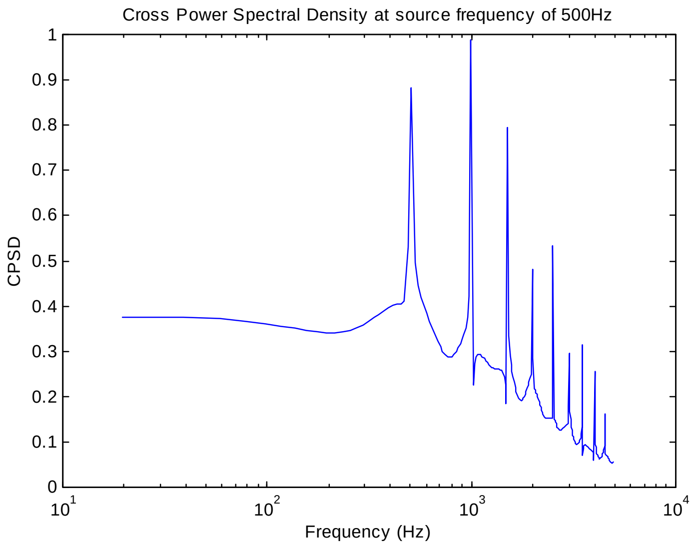

Figure 1 $CPSD_{23}$ from expression (31)

4. CPSD CALCULATED WITH MCNPDS-DSP

The Monte Carlo code MCNP-DSP (where DSP stand for digital signal processing) [13], has been used to simulate the CPSD. MCNP-DSP is a modified version of MCNP-4A that, besides other parameters used in noise techniques, can calculate the auto and cross power spectral density functions. The calculations are performed in an analogue way. This code provides a tool to design and study the experimental methods that can be applied in subcritical driven systems. All calculations have been done for a subcritical configuration using a simplified model of the MASURCA reactor and fission chambers (U-235) with high efficiency.

In order to check the expression obtained by the stochastic transport theory for the CPSD, equation (31), we performed MCNP-DSP calculations at the same conditions presented in figure 1. A pulsed deuterium source (D-D reaction) was assumed operating at a frequency of 500 Hz coupled to the MASURCA reactor in a slightly subcritical state (-400 pcm, Rugama et al [10]). The U-235 fission chambers modelled in the calculations were located in the reflector where the reaction rates are highest. In the calculations, the starting time for a pulse is randomly sampled between t=0 and t=T_p, with T_p being the source period. This is equivalent to considering the source as quasi-Markovian.

Figure 2 shows the comparison between the calculated and the theoretical curves. Clearly, a dependency on the source frequency predicted by equation (31) is observed in the Monte Carlo calculations as well as in the theoretical expression. The agreement between both curves gives us some confidence in the equation derived. Comparisons with future measurements in MASURCA will be performed to confirm the results from figures 1 and 2.

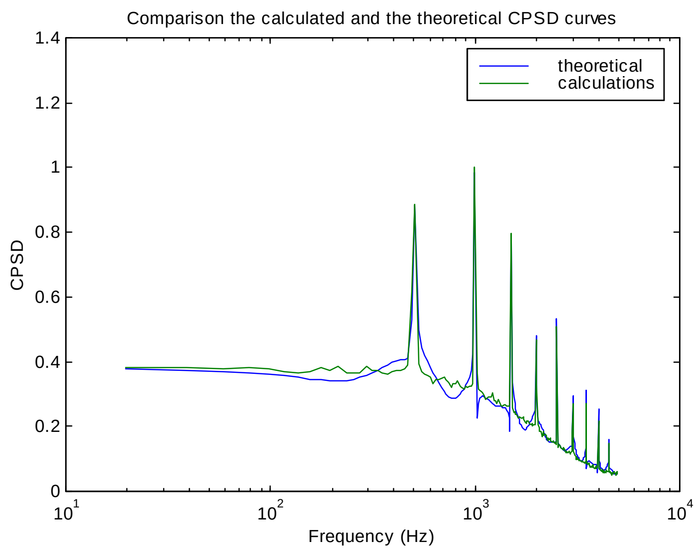

Figure 2 Comparison Cross Power Spectral Density from expression (31) and for calculations.

5. CONCLUSIONS

A general expression has been derived for the Cross Power Spectral Density, $CPSD_{23}(w)$, between two detectors D_2 and D_3 in a pulsed source-driven system. As expected, the results depends on the frequency of the source. The general expression includes a full space, energy and time dependency, which has been simplified using only the fundamental mode. Because of the operating characteristics of the source driven systems, the influence of the higher harmonics will be studied in the near future.

A method to obtain the reactivity in a subcritical reactor driven by a time-dependent source is presented. At high source frequencies, the fundamental mode eigenvalue can be easily obtained, while at low frequencies a primary tendency should be calculated. However, the frequency range to which the method can be applied is wide enough, say from 100 Hz in the present case. At source frequencies higher than the fundamental mode the transfer function can be treated as independent of the source frequency.

The comparisons with MCNP-DSP calculations give confidence in the expression developed. MCNP-DSP has shown to be a useful tool to define noise techniques in pulsed source-driven systems. In the future, a comparison with measurements will be made.

6. NOMENCLATURE

$1=\left(\vec{r},v,\hat{\Omega}\right)$ Phase space co-ordinates

$d_i =$ Counting interval of detector D_i i.e. $\left(tf_i-\Delta tc_i, tf_i\right)$

$F(\vec{r})$= Fission rate per unit volume at point $\vec{r}$, of the subcritical system $\left(cm^{-3}s^{-1}\right)$

$f_S(v,\hat{\Omega})$ = Direction and velocity source neutron distribution function

$Keff$ =Multiplication constant of the system

$L=\Omega\nabla_r+\Sigma\left(\vec{r},v,t\right)-(S+F))$ Neutron transport operator

$L^+=-\Omega\nabla_r+\Sigma\left(\vec{r},v,t\right)-(S^+ +F^+))$ Adjoint neutron transport operator

$\overline{n}_C\left(1,t\,|\,d_i\right)$ =Average number of counts in detector D_i upon injection of one single neutron at (1,t)

$\overline{n}_C\left(1,t,tf_i\right)$ =Detector counting rate at time tf_i upon injection of one single neutron at $\left(1,t\right)$

$H(t-nT_p)$ = Heaviside function

S_c, S_c^{+} = Direct and Adjoint scattering operators respectively

$S(t)$ = Time dependent neutron source $\left(s^{-1}\right)$

S_o = Number of neutrons per pulse

T_p = Period of the neutron source (s)

T_w= pulse width (s)

v= neutron speed $\left(cm.s^{-1}\right)$

tf_2 , tf_3 = Final counting time of detectors D_2 and D_3 respectively

α_n =n-th α-eigenvalue of the neutron transport equation

$\Psi_{ij}(tf_1, tf_2)$ = Cross correlation between detectors D_i and D_j

$\Phi_m(\vec{r}, v, \Omega)$ =mth α-eigenfunction of the neutron transport equation

$\Phi_m^{+}(\vec{r}, v, \Omega)$ =mth α-eigenfunction of the adjoint neutron transport equation

Σ_f = Fission cross section $\left(cm^{-1}\right)$

$\chi(v)$ = Normalized spectrum of the fission neutrons

$\hat{\Omega}$ =Unitary vector indicating the direction

η =Neutron detector efficiency

Σ_{D_i} = Neutron detector cross section $\left(cm^{-1}\right)$

$(\, , \,) = \int dv \int d\Omega \int d^3 r$

$\langle \; | \; \rangle = \int_{to}^{tf} dt \int dv \int d\Omega \int d^3 r$

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