An Approximate Procedure for Solving Base-Isolated Structures

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Dynamic analysis of several shear-type structures with base isolation indicates that the response of these structures follows their fundamental mode shape. Based on this observation, this paper uses an approximate procedure for computing the response of base-isolated structures. The procedure consists of modeling the structure and its base by a two-degree of freedom system, one representing the base and the other the structure. The response from the two-degree of freedom model and mode shapes of the structure are used to compute the response of the structure to earthquake excitation. The approximate procedure is simple, requires substantially less computational time than other methods, and gives results that are in excellent agreement with those from direct integration. Nonlinear properties and nonproportional damping are easily included in the model. Savings of approximately 54–77 percent in computational time result by using the approximate model.

Introduction

Base isolation design is used to minimize damage to structures and equipment due to earthquakes. The design is based on the premise that it is possible to confine the motion from earthquakes to the base, and thereby reduce the displacements and forces in both the structure and attached equipment. According to Kelly (1986), base isolation is effective for short and stiff structures (five stories or less in buildings) where the fundamental frequency of the structure without base isolation is close to predominant frequencies of earthquakes.

In addition to minimizing the damage to structures, base isolation reduces the vibratory motion induced in nonstructural components such as equipment, sensitive instruments, piping, etc. Even though a structure may not experience any damage, the vibratory motion from earthquakes may cause malfunction or failure of sensitive equipment. In some cases, such as telecommunication facilities, the equipment housed in the structure may be worth substantially more than the structure itself. Moreover, in many instances the facility must be functional immediately after an earthquake.

Severe earthquakes cause nonlinear deformation in the structure and equipment. In analyzing base-isolated structures, the small stiffness and large damping of the isolator compared to structure result in an inelastic nonproportional damping behavior. Methods such as direct integration, complex mode, or modified undamped mode superposition (Karasudshi et al., 1975; Chang and Mohraz, 1990, 1991; Mohraz et al., 1991, 1992) may be used to compute the response. These methods, however, require significant computational effort and time.

Because base isolation confines the earthquake motion to the base, in most cases the response of the structure remains in the elastic range. Comparisons of maximum displacement response and the number of times the columns in a typical five-story shear-type structure experience yielding are shown in Table 1. The structure is subjected to the S00E component of El Centro, the Imperial Valley earthquake of May 18, 1940. The properties of the structure are given in the Appendix. The yield displacement for the columns is selected so that the columns undergo substantial yielding when no base isolation is used. The results in Table 1 show that by introducing base isolation, not only the displacement response is reduced significantly, but also the yielding is confined to the base. Results similar to Table 1 were also depicted for the same frame and for two other frames (a three and four-story) subjected to the N65E component of Cholame, Shandon Array No. 2, the Parkfield earthquake of June 27, 1966 and the S16E component of Pacoima Dam, the San Fernando earthquake of February 9, 1971. In all cases, the results indicate that the presence of base isolation not only reduces the displacement response of the structure significantly by confining the earthquake motion to the base, but is also prevents yielding of columns.

Table 1 Comparison of maximum displacement response and number of times columns experience yielding for the five-story structure subjected to the S00E component of El Centro, 1940

story	Without base isolation		With base isolation	
	Yield No.	Max. Disp.*(in.)*	Max. Disp.*(in.)*	Yield No.
Top		1.785	0.222	
	0			0
2		1.792	0.195	
	10			0
3		1.876	0.154	
	212			0
4		3.459	0.106	
	1520			0
5		0.802	0.055	
	136			0
base			3.393	
				119

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Table 2 Comparison of maximum displacement response and mode shape for a five-story base-isolated structure

story	computed response			normalized response			mode shape
	El Centro	Parkfield	Pacoima	El Centro	Parkfield	Pacoima	
Top	0.222	0.379	0.479	1.00	1.00	1.00	1.00
2	0.195	0.332	0.418	0.88	0.88	0.87	0.89
3	0.154	0.263	0.332	0.69	0.69	0.69	0.70
4	0.106	0.181	0.228	0.48	0.48	0.48	0.49
5	0.055	0.091	0.115	0.25	0.24	0.24	0.25
base	3.393	8.960	13.886	15.28	23.64	28.99	12.50

El Centro – S00E Component of El Centro, 1940.
Parkfield – N65E Component of Parkfield, 1966.
Pacoima – S16E Component of Pacoima Dam, 1971.

Table 2 shows the displacement response of the same five-story structure with base isolation to the motion of three different earthquakes. The fundamental mode shape is also shown in the table. Comparisons of the response ratios (normalized to the top story) and the mode shape indicate that even though the base displacements are different, the response of the structure follows its fundamental mode shape very closely. This observation leads to the possibility of predicting the response of a base-isolated structure from its mode shape once the response of the base is known.

This paper presents an approximate procedure for computing the response of base-isolated structures. The procedure consists of modeling the structure and its base by two masses or stories, one representing the base and the other the structure, and two resisting elements, the first representing the base isolation and the second the structure, Fig. 1. The mass of the base and the stiffness and damping properties of the isolator in the model are identical to those in the structure under consideration. The mass m_s in the model represents the total mass of the structure above the base, whereas the stiffness and damping properties, k_s and c_s, of the model are the same as the stiffness and damping properties, k_N and c_N of the columns directly above the base.

The response of the model to earthquake ground motion together with the vibration mode shapes of the structure are used to compute the response of the structure. Any number of modes can be included in computing the response. The solutions are compared with those from direct integration.

Formulation

Referring to Fig. 1(*b*), the equations of motion for the model may be written as

$$m_s\ddot{z}_b(t) + m_s\ddot{z}_s(t) + c_s\dot{z}_s(t) + k_s z_s(t) = -m_s\ddot{x}_g(t) - r_{ns}(x,t) \quad (1)$$

$$m_s\ddot{z}_s(t) + (m_b + m_s)\ddot{z}_b(t) + c_b\dot{z}_b(t) + k_b z_b(t) = -(m_b + m_s)\ddot{x}_g(t) - r_{nb}(x,t) \quad (2)$$

where k_b and k_s are the linear stiffnesses of the base isolation and the structure, r_{ns} and r_{nb} are the nonlinear contributions to the restoring forces in the isolator and in the structure, c_b and c_s are the damping coefficients of the isolator and the structure, z_b and z_s are the displacements of the base and the structure, m_b is the mass of the base, and m_s is the mass of the structure above the base.

The solution of Eqs. (1) and (2) gives the displacements z_b and z_s of the model. The displacements in the structure may be computed from the displacements of the model and the vibration mode shapes of the structure by proportioning the mode shapes such that the displacement of the story directly above the base (story N) is equal to the displacement z_s of the model. The number of modes to be used depends on the degree of accuracy desired. Usually, the first mode shape will provide a reasonable estimate. The procedure for computing the displacement response of the structure is as follows: Assuming

$$\{\phi_1\}^T = \{\nu_{1,1}\ \nu_{1,2} \cdots \nu_{1,3} \cdots \nu_{1,N}\ \nu_{1,N+1}\}^T \quad (3)$$

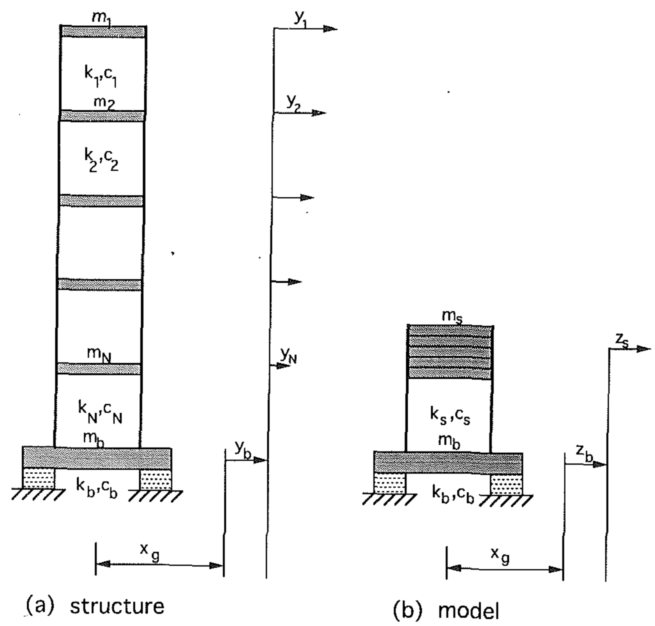

Fig. 1 Base-isolated frame and its approximate model

is the first mode shape, and

$$\{y\}^T = \{y_1\ y_2 \cdots y_i \cdots y_N\}^T \quad (4)$$

denotes the displacements of the structure to be computed, then the displacement of the ith story is obtained by proportioning the mode shape such that $y_N = z_s$. Thus,

$$y_i = (\nu_{1,i}/\nu_{1,N})z_s \quad (5)$$

If other modes are to be considered, participation factors are used to include their contributions. Assuming the modal participations are

$$\{p\}^T = \{p_1\ p_2 \cdots p_i \cdots p_N\ p_{N+1}\}^T \quad (6)$$

the contribution of mode 1 is

$$(y_i)_1 = (\nu_{1,i}/\nu_{1,N})\left(p_1 / \sum_{m=1}^{M} p_m\right) z_s$$

where Σp_m is the sum of the participation factors of the first M modes considered in the computation. Similarly, the contribution of mode 2 is

$$(y_i)_2 = (\nu_{2,i}/\nu_{2,N})\left(p_2 / \sum_{m=1}^{M} p_m\right) z_s \quad (7)$$

or, in general, the contribution of mode j is

$$(y_i)_j = (\nu_{j,i}/\nu_{j,N})\left(p_j / \sum_{m=1}^{M} p_m\right) z_s \quad (8)$$

The final displacement response is obtained by adding the displacements proportioned from different modes. Thus,

$$y_i = \sum_{j=1}^{M}\left[(\nu_{j,i}/\nu_{j,N})\left(p_j / \sum_{m=1}^{M} p_m\right)\right] z_s \quad (9)$$

Results

Three shear-type plane frames—a three, a four, and a five-story with base isolation—are used to demonstrate the applicability of the approximate procedure. A damping ratio of five percent of critical is assumed for the structure and a ratio of 20 percent for the isolation bearings. The mass, stiffness, and yield properties for the three frames are given in the Appendix. The fundamental frequencies of the frames without base isolation range from 2.50 to 3.45 Hz. Novak and Hen-

Table 3 Ratio of isolation bearing yield load to total weight of the structure

	3 - story	4 - story	5 - story
total weight (*kip*)	73.511	89.843	106.174
yield load (*kip*)	5.149	5.650	6.323
ratio	0.071	0.063	0.060

Table 4 Comparison of maximum displacements (in.) for the three-story frame

	approximate method		direct integration
story	1st mode	all modes	
Top	0.110	0.112	0.113
2	0.085	0.086	0.086
3	0.046	0.046	0.046
Base	3.323	3.323	3.325
CPU ratio	0.45	0.46	1.00

derson (1989) have shown that by shifting the fundamental frequency close to 0.5 Hz, serious damage to structures can be avoided. For the frames with base isolation, the isolation bearing stiffnesses were selected such that the fundamental frequencies remained close to 0.5 Hz.

Experimental and analytical studies (Meggett, 1978; Lee and Medland, 1979; Blakeley et al., 1980; Filiatrault and Cherry, 1988) have shown that the isolation bearings can be assumed to have elastic-plastic properties and that a reasonable yield load for the bearings is about 5 to 10 percent of the total weight of the structure. For the frames considered, the isolation bearings were selected so that the yield loads ranged from 6.0 to 7.1 percent of the total weight (Table 3). The frames are subjected to the S00E component of the accelerogram from the El Centro, Imperial Valley earthquake of May 18, 1940. Wilson-θ method (Wilson et al., 1973) with an integration time step of 0.01 s was used in the direct integration to compute the maximum displacements.

Tables 4–6 show a comparison of the maximum displacement for a three, a four, and five-story frame from the direct integration and the approximate procedure. The results from the approximate procedure are given for two cases—one when only the first mode is used and the other when all modes are included in computing the displacement response. The tables show that the maximum displacements computed from the approximate procedure are in excellent agreement with those from the direct integration. The CPU times (using IBM 3081D) indicate that substantial savings in computational time results when using this approximate procedure. The CPU times for the approximate procedure not only are significantly less than those for the direct integration, but as one would expect, remain nearly the same whether the first mode or all modes are included in computing the response.

Conclusions

The presence of base isolation in structures not only reduces the vibratory motion in the structure, but also confines the inelastic action to the isolation bearings. The response of several shear-type structures with base isolation shows that the structures remain elastic and their response follows their fundamental mode shapes.

Using this observation, the response of a structure with base isolation may be computed from the response of a two-degree-of-freedom model—one representing the base and the second the structure—and the vibration mode shapes of the structure. The model is easy to use, requires substantially less computational time than direct integration, and gives results that are in excellent agreement with those from direct integration. Nonlinear properties and nonproportional damping are easily included in the model. Savings in computational time of approximately 54–77 percent are observed for the problems considered.

Table 5 Comparison of maximum displacements (in.) for the four-story frame

	approximate method		direct integration
story	1st mode	all modes	
Top	0.153	0.155	0.157
2	0.128	0.130	0.131
3	0.091	0.092	0.093
4	0.048	0.048	0.049
Base	3.300	3.300	3.302
CPU ratio	0.31	0.32	1.00

Table 6 Comparison of maximum displacements (in.) for the five-story frame

	approximate method		direct integration
story	1st mode	all modes	
Top	0.215	0.220	0.222
2	0.190	0.193	0.195
3	0.151	0.153	0.154
4	0.104	0.105	0.106
5	0.054	0.054	0.056
Base	3.390	3.390	3.393
CPU ratio	0.23	0.24	1.00

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APPENDIX

The properties of the three, four and five-story frames used in the example problems are given as follows:

THREE-STORY FRAME

story	mass ($k\text{-}sec^2/in.$)	stiffness ($k/in.$)	yield displ. ($in.$)
Top	0.0423		
		69.9	0.01
2	0.0423		
		93.2	0.01
3	0.0423		
		116.5	0.01
base	0.0635		
bearings		2.8397	2.00

FOUR-STORY FRAME

story	mass ($k\text{-}sec^2/in.$)	stiffness ($k/in.$)	yield displ. ($in.$)
Top	0.0423		
		69.9	0.01
2	0.0423		
		93.2	0.01
3	0.0423		
		116.5	0.01
4	0.0423		
		139.8	0.01
base	0.0635		
bearings		3.383	1.67

FIVE-STORY FRAME

story	mass ($k\text{-}sec^2/in.$)	stiffness ($k/in.$)	yield displ. ($in.$)
Top	0.0423		
		62.0	0.01
2	0.0423		
		64.0	0.01
3	0.0423		
		66.0	0.01
4	0.0423		
		139.8	0.01
5	0.0423		
		163.1	0.01
base	0.0635		
bearings		4.215	1.50