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Research Article

Robust Finite-Time Control for Spacecraft with Coupled Translation and Attitude Dynamics

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Robust finite-time control for spacecraft with coupled translation and attitude dynamics is investigated in the paper. An error-based spacecraft motion model in six-degree-of-freedom is firstly developed. Then a finite-time controller based on nonsingular terminal sliding mode control technique is proposed to achieve translation and attitude maneuvers in the presence of model uncertainties and environmental perturbations. A finite-time observer is designed and a modified controller is then proposed to deal with uncertainties and perturbations and alleviate chattering. Numerical simulations are finally provided to illustrate the performance of the proposed controllers.

1. Introduction

Recent years have witnessed plentiful research in space missions such as spacecraft rendezvous and docking, assembly of large space structural systems and formation flying [1, 2]. Such space missions require spacecraft to perform large angle slews or complex complicated translational maneuvers. Current research mainly separates attitude motion from translation motion. In practice, the attitude and translation motions are coupled and highly nonlinear. Furthermore, the model parameters of the spacecraft cannot be acquired exactly, and the spacecraft is always subject to environmental perturbations. All of above issues make it difficult to achieve ideal control performance for spacecraft coupled translation and attitude maneuvers. It is worth pointing out that some previous work has proposed various approaches to solve this problem. The sliding mode control and the state-dependent Riccati equation method were respectively proposed to achieve six-degree-of-freedom position and attitude maneuvers in the absence of uncertainties and perturbations [3, 4]. A high-pass filter and an output feedback control law were designed to solve tracking control problem for a spacecraft with coupled translation and attitude motion in the absence of translational and angular velocity measurements. Although the abovementioned control algorithms have shown adequate reliability, they only guaranteed asymptotic stability and convergence. This implies that the control objective can be completed in infinite time [5]. The capability of fast maneuver is, however, highly desirable in many space missions.

To advance the research in the control of spacecraft translation and attitude maneuvers, the finite-time control for spacecraft with coupled translation and attitude dynamics is investigated in this paper. The error-based motion model with coupled translation and attitude dynamics is firstly developed, and then a finite time controller is designed to make the spacecraft achieve the desired position and attitude in finite-time. A finite-time observer and a modified finite-time controller are further proposed to address the robustness of the closed-loop system in the presence of model uncertainties and environmental perturbations. Numerical simulations are finally presented to demonstrate the performance of proposed controllers.

2. Problem Definition

2.1. Spacecraft Dynamics and Kinematics. The dynamic equations of a six-degree-of-freedom (6-DOF) spacecraft, which performs translational and rotational motion, are governed by the following differential equations [3]:

$$m\dot{v} + m\omega^{\times}v = u_1, \quad (1)$$

$$J\dot{\omega} + \omega^{\times}(J\omega) = \rho^{\times}u_1 + u_2, \quad (2)$$

where m and J are the mass and the inertia matrix of a rigid spacecraft, v and ω denote the spacecraft's translational and angular velocity, ρ is the distance from the mass center of the spacecraft to the point where the force is applied, and u_1 and u_2 represent the control force and control torque. For any vector $\eta = [\eta_1 \ \ \eta_2 \ \ \eta_3]^T$, the skew matrix $\eta^{\times}$ can be written as

$$\eta^{\times} = \begin{bmatrix} 0 & \eta_3 & -\eta_2 \\ -\eta_3 & 0 & \eta_1 \\ \eta_2 & -\eta_1 & 0 \end{bmatrix}. \quad (3)$$

The kinematic equations of a spacecraft in 6-DOF are given by

$$\dot{r} = -\omega^{\times}r + v, \quad (4)$$

$$\dot{q}^{\circ} = \frac{1}{2}\Omega(\omega)\cdot q^{\circ}, \quad (5)$$

where r denotes the spacecraft position, and q° is the unit quaternion, which describes the attitude motion of a spacecraft without singularities [6]. The unit quaternion q° is defined as

$$q^{\circ} = \begin{bmatrix} q_0 \\ q \end{bmatrix} = \begin{bmatrix} \cos\frac{\phi}{2} \\ e\sin\frac{\phi}{2} \end{bmatrix}, \quad (6)$$

where e is a unit vector called the Euler axis, ϕ denotes the magnitude of the Euler axis rotation, and q_0 and q represent the scalar and vector components of q°, respectively, and are subject to the constraint:

$$q^T q + q_0 = 1. \quad (7)$$

Then the attitude kinematics of (5) can be rewritten as

$$\dot{q} = \frac{1}{2}(q_0 I + q^{\times})\cdot\omega,$$
$$\dot{q}_0 = -\frac{1}{2}q^T\cdot\omega, \quad (8)$$

where $q^{\times}$ is the skew matrix of q.

2.2. Error-Based Spacecraft Motion Equations. Substituting (4) and (8) into (1) yields

$$m\ddot{r}_e + 2mQ\dot{r}_e + mPr_e + m\Sigma = u_1, \quad (9)$$

where $r_e = r - r_d$, $v_e = v - v_d$, r_d and v_d represent the desired position and translational velocity, $Q = [E\dot{q}]^{\times}$, $E = [(1/2)(q_0 I + q^{\times})]^{-1}$, $P = [[E\ddot{q}]^{\times} + [E\dot{q}]^{\times}[E\dot{q}]^{\times}]$, and $\Sigma = [\dot{v}_d + [K\dot{q}]^{\times}v_d]$. The error quaternion q_e° is defined as

$$q_e^{\circ} = q_d^{\circ} \otimes q^{\circ}, \quad (10)$$

where q_d° represents the desired attitude quaternion and $\otimes$ denotes the quaternion multiplication [7, 8]. Hence, the attitude kinematics in terms of the error quaternion is given by

$$\dot{q}_e = \frac{1}{2}(q_{e0}I + q_e^{\times})\cdot\omega_e,$$
$$\dot{q}_{e0} = -\frac{1}{2}q_e^T\cdot\omega_e, \quad (11)$$

where $\omega_e = \omega - \omega_d$, $q_e^{\times}$ is the skew matrix of q_e, and ω_d denotes the desired attitude angular velocity [8]. Substituting (11) into (2) yields

$$J^*\ddot{q}_e + \Xi\dot{q}_e + M^T G = M^T(\rho^{\times}u_1 + u_2), \quad (12)$$

where $M = [(1/2)(q_{e0}I + q_e^{\times})]^{-1}$, $G = J\dot{\omega}_d + \omega_d^{\times}J\omega_d$, $J^* = M^T JM$, and $\Xi = -J^*\dot{M}^{-1}M - M^T[JM\dot{q}_e]^{\times}M - M^T[J\omega_d]^{\times}M + M^T\omega_d^{\times}JM$. It is assumed that $q_{e0} \neq 0$ and M is then invertible.

In practice, the mass and the inertia matrix cannot be known exactly, and only nominal ones are available. Furthermore, the environmental perturbations, including gravity gradient, atmospheric drag, geomagnetic torque, and solar radiation, always affect translation and attitude maneuver. Hence the uncertainties and perturbations are taken into account, and (9) and (12) can be, respectively, rewritten as

$$m_0\ddot{r}_e + 2m_0Q\dot{r}_e + m_0Pr_e + m_0\Sigma = u_1 + \delta_1, \quad (13)$$

$$J_0^*\ddot{q}_e + \Xi_0\dot{q}_e + M^T G_0 = M^T(\rho^{\times}u_1 + u_2) + \delta_2, \quad (14)$$

where $\delta_1 = f_1 - \Delta m(\ddot{r}_e + 2Q\dot{r}_e + Pr_e + \Sigma)$, $\delta_2 = M^T(f_2 + (\Delta JM\dot{M}^{-1}M - [\Delta JM\dot{q}_e]^{\times}M - [\Delta J\omega_d]^{\times}M + \omega_d^{\times}\Delta JM)\dot{q}_e - \Delta JM\ddot{q}_e - \Delta J\dot{\omega}_d - \omega_d^{\times}\Delta J\omega_d)\cdot m = m_0 + \Delta m$, m_0 and Δm denote the nominal and uncertain parts of m, $J = J_0 + \Delta J$, J_0 and ΔJ are the nominal and uncertain parts of J, f_1 and f_2 are the bounded perturbations, and J_0^*, Ξ_0, and G_0 are nominal functions of J^*, Ξ and G, which can be readily obtained by (12) and thus omitted here. According to (13) and (14), the error-based spacecraft motion equations in the presence of uncertainties and perturbations can then be given by

$$C_1\ddot{x}_e + C_2\dot{x}_e + C_3x_e + C_4 = Ku + \delta, \quad (15)$$

where $C_1 = \begin{bmatrix} m_0 I & 0 \\ 0 & J_0^* \end{bmatrix}$, $C_2 = \begin{bmatrix} 2m_0Q & 0 \\ 0 & \Xi_0 \end{bmatrix}$, $C_3 = \begin{bmatrix} m_0P & 0 \\ 0 & 0 \end{bmatrix}$, $C_4 = \begin{bmatrix} m_0\Sigma \\ M^T G_0 \end{bmatrix}$, $K = \begin{bmatrix} I & 0 \\ M^T\rho^{\times} & M^T \end{bmatrix}$, $x_e = \begin{bmatrix} r_e \\ q_e \end{bmatrix}$, $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $\delta = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}$, and I is an identity matrix.

2.3. Definition and Lemma

Definition 1. Consider the following autonomous system:

$$\dot{x} = f(x,t) \quad f(0,t) = 0 \quad x \in R^n, \tag{16}$$

where $f : R \rightarrow R^n$ is non-Lipschitz continuous on an open neighborhood R of the origin $x = 0$. The equilibrium $x = 0$ of (16) is finite-time convergent if there is an open neighborhood U of the origin and a function t_s, such that every solution trajectory $x(t)$ of (16) starting from the initial point $x_0 \in U_0/\{0\}$ is well defined and unique in forward time for $t \in [0, t_s)$ and satisfies

$$\lim_{t \rightarrow t_s} x(t, x_0) = 0. \tag{17}$$

Here t_s is called the settling time of the initial state x_0. The equilibrium of (16) is finite-time stable if it is Lyapunov stable and finite-time convergent [9, 10].

Lemma 2. *For any vector* $\nu = [\nu_1 \ \nu_2 \ \cdots \ \nu_n]^T$, *if* $0 < \kappa < 2$, *then the following inequality holds [8]:*

$$\|\nu\|^{\kappa} \leq \sum_{i=1}^{n} |\nu_i|^{\kappa}, \tag{18}$$

where $\|\cdot\|$ *denotes the vector Euclidean norm.*

Lemma 3. *Consider the nonlinear system* (14). *Suppose that* χ *is a* C^1 *function defined on a neighborhood* $U \subset R^n$ *of the origin, and the real number* $c > 0$ *and* $0 < \partial < 1$, *such that (1)* χ *is positive definite on* U; *(2)* $\dot{\chi} + c\chi^{\partial}$ *is negative semidefinite on* U. *Then, there exists an area* $U_0 \subset R^n$ *such that any* χ *which starts from* U_0 *can reach* $\chi = 0$ *in finite time. The settling time* t_s *satisfies [8]*

$$t_s \leq \frac{\chi_0^{1-\partial}}{c(1-\partial)}. \tag{19}$$

3. Controller Design

3.1. Control Objective. The aim of robust finite-time control applied to spacecraft with coupled translation and attitude dynamics is to design a controller such that ($r_e = 0$, $v_e = 0$, $q_e = 0$, $\omega_e = 0$) can be achieved in finite time in the presence of model uncertainties and perturbations.

3.2. Finite-Time Controller. For the system (15), we propose the following controller and Theorem 4:

$$u = \left(C_1^{-1}K\right)^{-1}\left(C_1^{-1}C_2\dot{x}_e - \alpha\dot{x}_e - \beta\frac{d}{dt}x_e^{a/b} + C_1^{-1}C_3x_e + C_1^{-1}C_4 - k\,\mathrm{sgn}(s)\right), \tag{20}$$

where $s = \dot{x}_e + \alpha x_e + \beta x_e^{a/b}$, a, b are positive odd numbers with $b > a$, and $\alpha, \beta, k > 0$.

Theorem 4. *For system* (15), *controller* (20) *can drive the system state* x_e *to the equilibrium in finite time.*

Proof. The candidate Lyapunov function is defined as

$$V_1 = \frac{1}{2}s^T \cdot s. \tag{21}$$

Computing the first-order derivative of V_1 yields

$$\begin{aligned}\dot{V}_1 &= s^T \cdot \dot{s} = s^T\left(\ddot{x}_e + \alpha\dot{x}_e + \beta\frac{d}{dt}x_e^{a/b}\right)\\ &= s^TC_1^{-1}\left(Ku + \delta - C_2\dot{x}_e - C_3x_e - C_4 + \alpha C_1\dot{x}_e + \beta C_1\frac{d}{dt}x_e^{a/b}\right).\end{aligned} \tag{22}$$

Substituting the controller (20) and employing Lemma 3 yield

$$\begin{aligned}\dot{V}_1 &= s^T\left(-k\,\mathrm{sgn}(s) + C_1^{-1}\delta\right) \leq -\lambda s^T\,\mathrm{sgn}(s) \leq -\lambda\|s\|\\ &= -\sqrt{2}\lambda V_1^{1/2},\end{aligned} \tag{23}$$

where $\|C_1^{-1}\delta\| \leq \Omega$, $\lambda = k - \Omega \geq 0$, and Ω, λ are positive constants. Then $s = 0$ is achieved in finite time according to Lemma 3. The system is therefore transformed as

$$\dot{x}_e = -\alpha x_e - \beta x_e^{a/b}. \tag{24}$$

Another candidate Lyapunov function is defined as

$$V_2 = \frac{1}{2}x_e^T \cdot x_e. \tag{25}$$

Computing the first-order derivative of V_2 yields

$$\begin{aligned}\dot{V}_2 &= x_e^T \cdot \dot{x}_e = x_e^T\left(-\alpha x_e - \beta x_e^{a/b}\right)\\ &\leq -\beta x_e^T \cdot x_e^{a/b} \leq -2^{(1+\varepsilon)/2}\beta V_2^{(1+\varepsilon)/2},\end{aligned} \tag{26}$$

where $\varepsilon = a/b$, and then $x_e = 0$ is achieved in finite time as well as $\dot{x}_e$ can converge to 0. According to (11), $\omega_e = 0$ is also achieved in finite time. Namely, ($r_e = 0$, $v_e = 0$, $q_e = 0$, $\omega_e = 0$) is achieved in finite time. □

Remark 5. The controller (20) contains a nonlinear element $\beta(d/dt)x_e^{a/b}$, such that singularity arises if

$$\frac{d}{dt}\left(x_e^{a/b}\right) = -\alpha\frac{a}{b}x_e^{a/b} - \beta\frac{a}{b}x_e^{2a/b-1}, \tag{27}$$

and the inequality $a/b > 0.5$ is therefore required to avoid controller singularity.

3.3. Observer-Based Finite-Time Controller. In practice, the model uncertainties and perturbations can affect the stability of the closed-loop system. Previous studies have proposed various approaches to solve this problem [11–15]. In Theorem 4, $k \geq \Omega$ is required. If there exist larger uncertainties and perturbations, k should be then increased to guarantee the stability of the closed-loop system, which will enlarge the chattering. To alleviate the chattering problem and deal with the coupled dynamics, the observer approach is widely investigated [16–18]. Linear observers are designed for hybrid systems which contain coupling dynamics in [16]. A disturbance observer is proposed to reduce inherent chattering in [17]. We propose the following finite-time observer and Theorem 6. The system's dynamics and kinematics in (15) become

$$\dot{y}_1 = y_2, \qquad \dot{y}_2 = \tau + d - F(y_2), \tag{28}$$

where $y_1 = x_e$, $y_2 = \dot{x}_e$, $\tau = C_1^{-1}Ku$, $d = C_1^{-1}\delta$, and $F(y_2) = C_1^{-1}C_2\dot{x}_e + C_1^{-1}C_3x_e + C_1^{-1}C_4$. The observer is given by

$$\dot{\hat{y}}_2 = \tau + \hat{d} - F + \mu_1 \operatorname{sgn}(s_1) + \mu_2 s_1^{\sigma}, \qquad \dot{\hat{d}} = \mu_1 \operatorname{sgn}(s_1) + \mu_2 s_1^{\sigma}, \tag{29}$$

where $\hat{(\cdot)}$ denotes the observed value of $(\cdot)$, $s_1 = y_2 - \hat{y}_2$, $0 < \sigma < 1$, $\mu_1, \mu_2 > 0$. Then we propose the following controller and Theorem 6:

$$u = \left(C_1^{-1}K\right)^{-1}\left(C_1^{-1}C_2\dot{x}_e - \alpha\dot{x}_e - \beta\frac{d}{dt}x_e^{a/b} + C_1^{-1}C_3x_e + C_1^{-1}C_4 - k\operatorname{sgn}(s) - \hat{d}\right). \tag{30}$$

Theorem 6. *For system* (15)*, controller* (30) *can drive the system state x_e to the equilibrium in finite time.*

Proof. The proof includes two consecutive steps:

(1) d can converge to $\hat{d}$ in finite time;

(2) under the condition of Step (1), $(x_e = 0, \dot{x}_e = 0)$ can be further achieved in finite time.

Proof of Step (1). The candidate Lyapunov function is defined as

$$V_3 = \frac{1}{2}s_1^T \cdot s_1. \tag{31}$$

Computing the first-order derivative of V_3 yields

$$\begin{aligned}\dot{V}_1 &= s_1^T \cdot \dot{s}_1 = s_1^T\left(\dot{y}_2 - \dot{\hat{y}}_2\right) \\ &= s_1^T\left(d - \hat{d} - \mu_1\operatorname{sgn}(s_1) - \mu_2 s_1^{\sigma}\right),\end{aligned} \tag{32}$$

where d is a bounded vector, and then it can be assumed that $\|d - \hat{d}\| \le \mu_1$. Equation (32) can be rewritten as

$$\dot{V}_1 \le -\mu_2 s_1^T s_1^{\sigma} \le -2\mu_2 V_1^{(1+\sigma)/2}, \tag{33}$$

where $0 < ((1+\sigma)/2) < 1$. Then $s_1 = 0$ is achieved in finite time by Lemma 3; namely $y_2 = \hat{y}_2$ holds in finite time t_s. According to (28) and (29), it can be concluded that $\dot{y}_2 = \dot{\hat{y}}_2$ and $d = \hat{d}$ hold in finite time t_s. Hence the candidate Lyapunov function is defined as

$$V_3 = \frac{1}{2}s^T \cdot s. \tag{34}$$

Computing the first-order derivative of V_3 yields

$$\begin{aligned}\dot{V}_3 &= s^T \cdot \dot{s} = s^T\left(\ddot{x}_e + \alpha\dot{x}_e + \beta\frac{d}{dt}x_e^{a/b}\right) \\ &= s^T C_1^{-1}\left(Ku + \delta - C_2\dot{x}_e - C_3x_e - C_4 + \alpha C_1\dot{x}_e + \beta C_1\frac{d}{dt}x_e^{a/b}\right) \\ &= s^T\left(-k\operatorname{sgn}(s) + d - \hat{d}\right)\end{aligned} \tag{35}$$

d can converge to $\hat{d}$ in finite time t_s, then the following inequality holds after t_s:

$$\dot{V}_3 = s^T\left(-k\operatorname{sgn}(s)\right) \le -\sqrt{2}kV_3^{1/2}, \tag{36}$$

$s = 0$ is therefore achieved in finite time according to Lemma 3, and the system is transformed to (24). The proof of Step (2) can be readily accomplished by following the line of Theorem 4 and thus omitted here. Actually, $x_e = 0$ is the terminal attractor of system (24). □

Remark 7. It can be seen that the existence of sign function in the controllers (20), (30), and the observer (29) can lead to chattering problem. To alleviate the undesirable high-frequency chattering, the following function is adopted to replace sign function [8]:

$$\Delta(z) = \begin{cases}\operatorname{sgn}(z) & \text{if } |z| \ge \xi \\ \dfrac{|z|^{\iota}}{|z|^{\iota} + \xi} & \text{if } |z| < \xi,\end{cases} \tag{37}$$

where ξ, which is a positive constant, denotes the thickness of boundary layer, and $0 < \iota < 1$. In fact, chattering phenomenon also exists in switched systems [19, 20].

4. Numerical Simulation

In this section, simulations of a rigid spacecraft in 6-DOF are presented to illustrate the above observer-based controller (30) with the function (37). The spacecraft parameters are given by

$$\begin{gathered}J_0 = \begin{bmatrix}1000 & -50 & -10 \\ -30 & 1000 & -40 \\ -20 & -40 & 800\end{bmatrix}\text{kg}\cdot\text{m}^2, \\ m_0 = 1000\,\text{kg}, \\ \Delta J \le 10\%\,J_0, \qquad \Delta m \le 10\%\,m_0.\end{gathered} \tag{38}$$

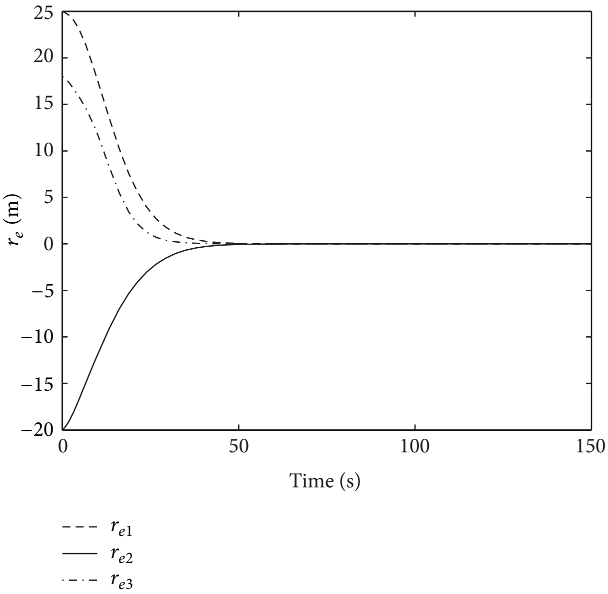

Figure 1: Position errors.

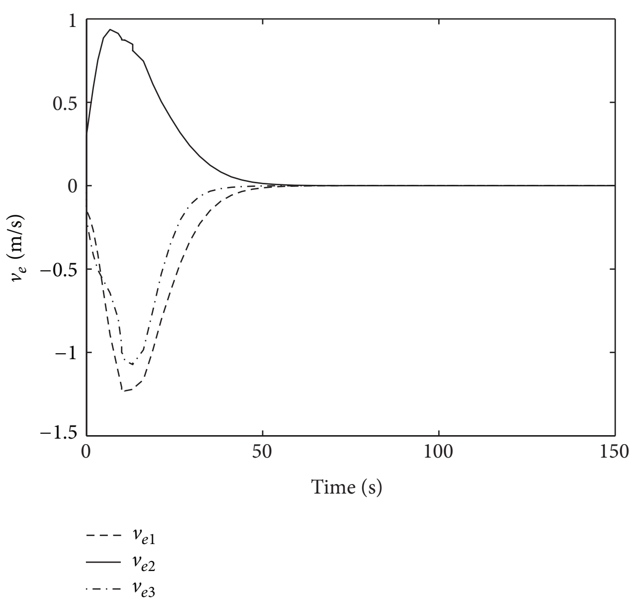

Figure 2: Velocity errors.

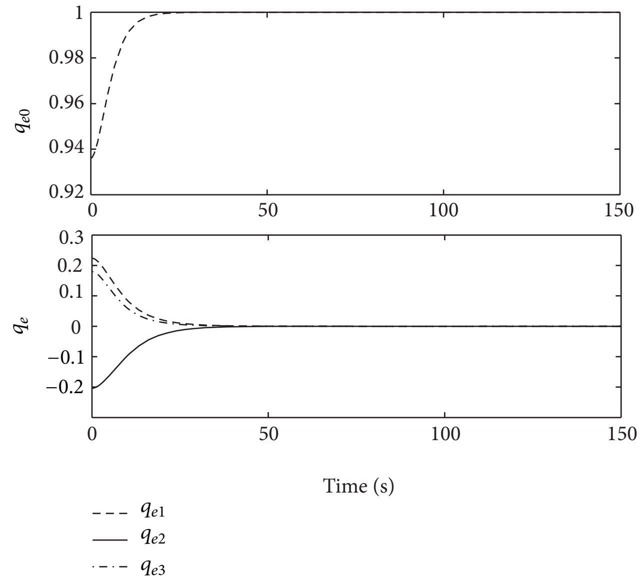

Figure 3: Euler parameter errors.

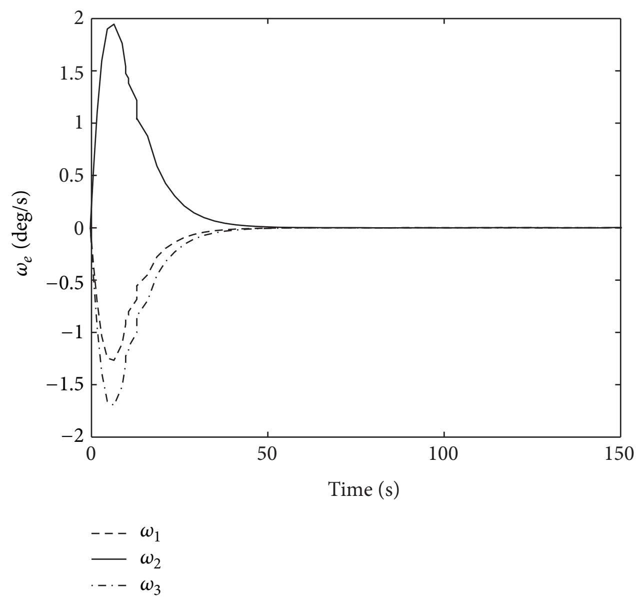

Figure 4: Angular velocity errors.

The initial position and attitude are

$$r(0) = \begin{bmatrix}25 & -20 & 18\end{bmatrix}^T \text{ m}, \qquad v(0) = 0_{3\times1} \text{ m/s},$$
$$q^{\circ}(0) = \begin{bmatrix}0.93 & 0.22 & -0.21 & 0.19\end{bmatrix}^T, \qquad \omega(0) = 0_{3\times1} \text{ deg/s},$$
$$\rho = 0_{3\times1} \text{ m}, \tag{39}$$

and the desired position and attitude are

$$r_d = 0_{3\times1} \text{ m}, \qquad v_d = 0_{3\times1} \text{ m/s},$$
$$q_d = 0_{3\times1}, \qquad q_{0d} = 1, \qquad \omega_d = 0_{3\times1} \text{ deg/s}. \tag{40}$$

The parameters of controller (30) and the disturbance are

$$\alpha = 0.05, \qquad \beta = 0.2, \qquad k = 0.12,$$
$$a = 7, \qquad b = 9,$$
$$\mu_1 = 0.12, \qquad \mu_2 = 2, \qquad \sigma = 0.68,$$
$$\xi = 0.001, \qquad \iota = 0.6,$$
$$f = \begin{bmatrix}\left(5\times10^{-3}\right)_{3\times1} \\ \left(6\times10^{-3}\right)_{3\times1}\end{bmatrix} \sin(0.025t) \begin{pmatrix}\text{m/s}^2 \\ \text{N}\cdot\text{m}\end{pmatrix}. \tag{41}$$

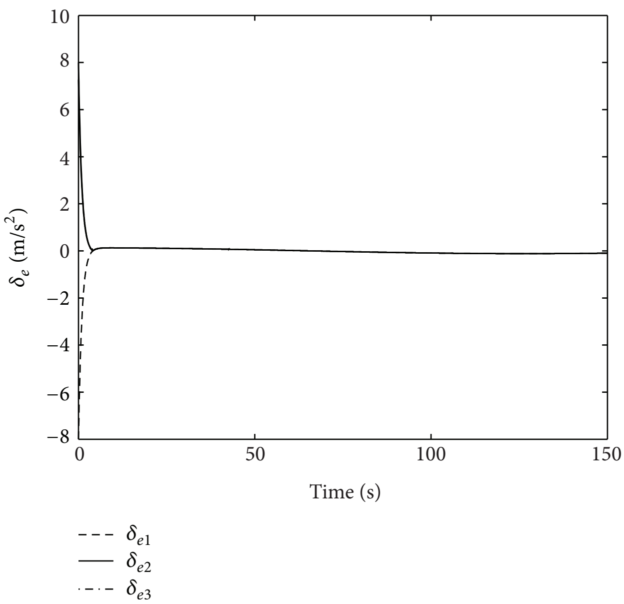

Figure 5: Observed value error.

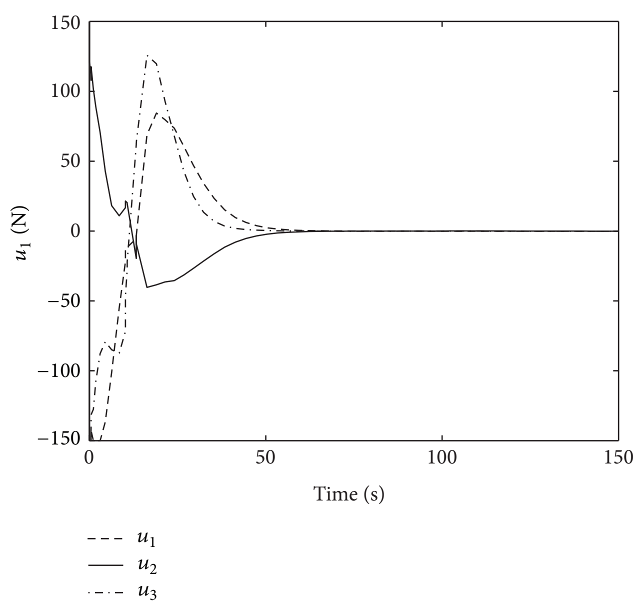

Figure 6: Control force.

The simulation results are given as follows. Figures 1 and 2 show the position and translational velocity errors, while Figure 3 presents the error quaternion and Figure 4 is the angular velocity errors. As shown, r_e, v_e, q_e, and ω_e converge to zero in 50 seconds. The observed value error $\delta_e = \delta - \widehat{\delta}$ is given in Figure 5, which represents the observed error of δ_1. The finite-time observer can estimate δ_1 in a relatively short period. The control force and control torque are shown in Figures 6 and 7, respectively. As can be seen, the chattering problem is effectively alleviated.

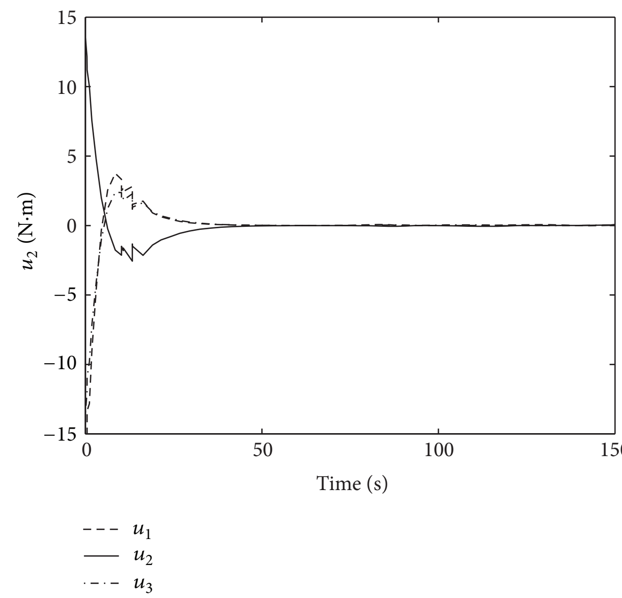

Figure 7: Control torque.

5. Conclusion

Robust finite-time control for spacecraft with couple translation and attitude dynamics is addressed in this paper. Two finite-time controllers are proposed in the presence of model uncertainties and environmental perturbations. It should be noted that the controllers have demonstrated superior performance, which can drive spacecraft position and attitude to the desired trajectory in finite time rather than in the asymptotic sense. To alleviate the chattering and increase the robustness of closed-loop system, a finite-time observer is designed. The observer can estimate the value of uncertainties and perturbations in finite time and then guarantee the global finite-time stability. Numerical simulations are finally presented to demonstrate that the proposed controllers have fast maneuvers performance and are robust to model uncertainties and environmental perturbations.

List of Symbols

a, b, ε:	Positive odd numbers
$F(y_2)$:	System state matrix
J:	Inertia matrix of a rigid spacecraft
$\mathring{q}$:	Unit quaternion
$\mathring{q}_d$:	Desired quaternion
q_{e0}, q_e:	Scalar and vector components of $\mathring{q}_e$
t_s:	Settling time
u_2:	Control torque
Q, P, Σ:	Gain matrixes
J^*, Ξ, K:	Gain matrixes
s:	Switching surface
v, v_d:	Spacecraft's translational velocity, desired translational velocity
V_i:	Lyapunov function
α, β:	Positive odd numbers

δ_e: Observed value error
ω, ω_d: Spacecraft's angular velocity, desired angular velocity
η, ν: Any vector
ϕ: Magnitude of the Euler axis rotation
c, ∂: Real number
ξ: Thickness of boundary layer
$(\cdot)^T$: Transpose of matrix or vector $(\cdot)$
$\hat{(\cdot)}$: Observed value of $(\cdot)$
e: Unit vector called the Euler axis
I: Identity matrix
m: Mass of a rigid spacecraft
q_0, q: Scalar and vector components of $\mathring{q}$
$\mathring{q}_e$: Error quaternion
r, r_d, r_e: Spacecraft position, desired position, position error
u_1: Control force
u: Control vector
C_1, C_2, C_3, C_4: Gain matrixes
G: Gain vector
s_1: Error of y_2 and $\hat{y}_2$
τ: Generalized control vector
k, μ_i, σ, ι: Positive numbers
d, δ: Generalized disturbance vector
x_e, y_i: State vector
ρ: Distance from the mass center of the spacecraft to the point where the force is applied
η_i, ν_i: ith scalar components of η,ν
κ, Ω, λ: Constants
χ: Function defined on a neighborhood
$(\cdot)^{\times}$: Skew matrix of vector $(\cdot)$
$(\cdot)_0, \Delta(\cdot)$: Nominal and uncertain parts of $(\cdot)$
Sgn: Sign function.

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