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## EXISTENCE FOR NONOSCILLATORY SOLUTIONS OF FORCED HIGHER-ORDER NONLINEAR NEUTRAL DYNAMIC EQUATIONS

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**Abstract.** In this paper, we first study the existence of nonoscillatory solutions of dynamic equation  $[x(t) + p(t)x(\tau(t))]^{\Delta^m} + \sum_{i=1}^k p_i(t)f_i(x(\tau_i(t))) = q(t)$  on a time scale  $\mathbb{T}$ . By using Krasnosel'skii's fixed point theorem and some new techniques, we obtain sufficient conditions for the existence of nonoscillatory solutions for general  $p_i(t)$ ,  $f_i(x)$  and  $q(t)$  which means that they are allowed oscillate. Then, we extend our results to equations of the form  $[x(t) + p(t)x(\tau(t))]^{\Delta^m} + F(t, x(\delta(t))) = q(t)$ . We establish sufficient and necessary conditions for the existence of nonoscillatory solutions of this equation. Our results not only generalize and improve the known results stated for differential and difference equations using the time scale theory, but also improve some of the results for dynamic equations on time scales. Some examples are included to illustrate the results.

**Keywords.** dynamic equation; higher order; nonoscillation; time scales; neutral.

**AMS (MOS) subject classification:** 34K11, 39A10, 39A99.

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