

EMITTANCE OPTIMIZATION USING PARTICLE SWARM ALGORITHM*

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Abstract

In this paper we use a swarm intelligence algorithm, Particle Swarm Optimization (PSO), to optimize the emittance directly. Some constraint conditions such as beta functions, fractional tunes and dispersion function, are considered in the emittance optimization. We optimize the strengths of quadrupoles to search for low emittances. Here an FBA lattice studied in the design of the Hefei Advanced Light Source storage ring is used as the test lattice. The PSO is shown to be beneficial in the optimization.

INTRODUCTION

Today more low-emittance synchrotron light sources are in early operation, construction or design around the world, such as SSRF and NSLS-II, to satisfy the requirements of synchrotron radiation science. The synchrotron radiation users prefer high-brightness light sources with low emittance. So some existing light sources also have their upgrade plans to reduce the emittance. A major upgrade to the Hefei Light Source at our laboratory is underway to achieve lower emittance.

For lattice designers, the traditional method used to find the desired lattice with low emittance and satisfying lattice functions is not straightforward, and the solutions are usually based on their experiences. And, the method does not guarantee that the obtained solution is best. Presently, some direct methods have been used for lattice design. The scanning method [1] is very simple and easy to implement, but it is very time-consuming. When there are only several variables, one can use this method to search for optimal solutions. But for more variables, one has to consider the computer resources. Artificial Intelligence (AI) algorithms can well solve this problem.

At present, Genetic Algorithms (GAs) [2] have been successfully applied to linear optics optimization for some light sources, as well as nonlinear optimization. In addition, Particle Swarm Optimization (PSO), a swarm intelligence algorithm, is also an alternative for global optimization. We first used the PSO algorithm [3, 4] to optimize sextupoles for enlarging dynamic aperture and momentum aperture, and it was successful. Then naturally we applied it to linear optics optimization. In this paper, we will present our preliminary work on the linear optics optimization using PSO.

PARTICLE SWARM OPTIMIZATION ALGORITHM

Particle Swarm Optimization is an Artificial Intelligence algorithm proposed by James Kennedy and R. C. Eberhart in 1995 [5], motivated by the social behavior of organisms such as bird flocking. The PSO algorithm is easy to understand and implement, has only a few parameters to adjust, and converges very fast. So it has been developing very quickly, and has been successfully applied to solve a wide variety of problems in different domains.

In PSO, each potential solution is called “particle”. Each particle has its position and velocity, and has a simple memory which stores its personal best solution so far. Each particle updates its velocity and position according to its own experience, and the experience of neighboring particles. The basic equations of motion for particles are described as follows:

$$\begin{aligned} v_{id}(t+1) &= w \times v_{id}(t) + c_1 \times r_1 \times (p_{id}(t) - x_{id}(t)) + \\ &\quad c_2 \times r_2 \times (p_{gd}(t) - x_{id}(t)), \\ x_{id}(t+1) &= x_{id}(t) + v_{id}(t+1), \end{aligned} \tag{1}$$

where x_{id} , v_{id} , and p_{id} are respectively the position, velocity and best position so far of the i^{th} particle in the d^{th} dimension, p_{gd} is the global best position so far of the entire swarm in the d^{th} dimension, w is the inertia weight, c_1 is the cognitive learning factor, c_2 is the social learning factor, and r_1 and r_2 are two uniformly distributed random variables in the range $[0, 1]$. This is the global version of PSO, where the neighborhood of each particle is the entire swarm. In the local version of PSO, the neighborhood is a subset of the swarm.

Parameter selection is very important for the performance of PSO. We adopt the PSO with constriction factor introduced by Clerc. The velocity update can be expressed as follows:

$$\begin{aligned} v_{id}(t+1) &= \chi \left\{ \begin{aligned} &v_{id}(t) + c_1 \times r_1 \times (p_{id}(t) - x_{id}(t)) + \\ &c_2 \times r_2 \times (p_{gd}(t) - x_{id}(t)) \end{aligned} \right\}, \tag{2} \\ \chi &= \frac{2}{\left| 2 - \varphi - \sqrt{\varphi^2 - 4\varphi} \right|}, \varphi = c_1 + c_2 > 4. \tag{3} \end{aligned}$$

Here χ is the constriction factor. The recommended parameter values for the above equations are: $\varphi=4.1$ ($c_1=c_2=2.05$), $\chi=0.729$. Thus, the values of the parameters

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in Equation 1 are obtained: $w=0.729$, $c_1=c_2=0.729 \times 2.05=1.49445$.

The objective function is called fitness function in the PSO algorithm, and its value is called fitness value. The PSO algorithm minimizes the fitness value. The steps of the PSO algorithm are as follows:

- (1) Initialize a population of particles with random positions and velocities on D dimensions in the problem space.
- (2) For each particle, evaluate the fitness function.
- (3) For each particle, compare its current fitness value with the fitness value of its previous best position. If its current fitness value is lower than its previous best fitness value, then update its previous best position and corresponding fitness value with its current position and fitness value.
- (4) Determine the current best particle of the swarm with the lowest fitness value. If the fitness value is lower than the fitness value of the global best position, then update the global best position and corresponding fitness value with the best particle's position and fitness value.
- (5) Update the velocity and position of each particle according to Equation 1.
- (6) Loop to (2) and repeat until a criterion is met, usually a sufficiently good fitness value or a maximum number of iterations.

Note that all positions should not be beyond the problem space, and that all velocities should not be beyond the maximum velocity limit.

EMITTANCE OPTIMIZATION

In the lattice design, we want to find the lattice not only having low emittance but also having satisfying lattice functions. In our optimization, the emittance is the objective function, and the lattice functions are treated as constraints. So it is a constrained optimization problem.

The general form of an optimization problem with constraints can be expressed as follows:

$$\begin{aligned} & \text{minimize } f(\mathbf{x}) \\ & \text{s.t. } \begin{cases} g_j(\mathbf{x}) \leq 0, & j = 1, 2, \dots, J; \\ h_k(\mathbf{x}) = 0, & k = 1, 2, \dots, K; \\ x_{i_{\min}} \leq x_i \leq x_{i_{\max}}, & i = 1, 2, \dots, N. \end{cases} \end{aligned} \quad (4)$$

Here, $f(\mathbf{x})$ is the objective function to be optimized, $g_j(\mathbf{x})$ and $h_k(\mathbf{x})$ are respectively the inequality and equality constraints imposed on the design, and \mathbf{x} is the set of decision variables with lower and upper bounds of $x_{i_{\min}}$ and $x_{i_{\max}}$.

Presently, in our emittance optimization, the decision variables are quadrupole strengths, and the constraints are beta functions, fractional tunes and dispersion function. For examples, if we want the horizontal and vertical beta functions (β_x, β_y) to be in the ranges $[a, b]$ and $[c, d]$, respectively, there are four inequality constraints:

$$a - \beta_x \leq 0, \quad \beta_x - b \leq 0, \quad c - \beta_y \leq 0, \quad \beta_y - d \leq 0. \quad (5)$$

And, if we want (β_x, β_y) to be e and f , respectively, there are two equality constraints:

$$\beta_x - e = 0, \quad \beta_y - f = 0. \quad (6)$$

The key point in the process of solving constrained optimization problems is to deal with the constraints. Here our method is based on the method of double fitness values [6, 7], which has been used to optimize the dynamic aperture in our previous nonlinear optimization.

In the method of double fitness values, the constraint violations are treated as a fitness function. And, the objective function is the major fitness function. Thus, using this method, every particle has two fitness values in the PSO algorithm. The two fitness values of the i^{th} particle are described as:

$$\begin{aligned} F_o(i) &= f(\mathbf{x}_i), \\ F_v(i) &= \sum_{j=1}^J a_j \max(0, g_j(\mathbf{x}_i)) + \sum_{k=1}^K b_k |h_k(\mathbf{x}_i)|. \end{aligned} \quad (7)$$

Here, $F_o(i)$ is the value of the objective function; $F_v(i)$ is the weighted sum of constraint violations, described by the maximum value of 0 and $g_j(\mathbf{x}_i)$, and the absolute value of $h_k(\mathbf{x}_i)$; a_j and b_k are weight factors.

Some infeasible solutions with small constraint violations are possibly more preferable than feasible solutions. Thus, allowing some infeasible solutions is helpful for searching for optimal solutions. So, a constant $\varepsilon > 0$ is introduced, and the criteria for comparison are as follows:

- (1) if the values of $F_v(i)$ and $F_v(j)$ corresponding to particle i and particle j , respectively, are both less than or equal to ε , the particle having lower value of F_o is better;
- (2) if $F_v(i)$ and $F_v(j)$ are both greater than ε , the particle having lower value of F_v is better;
- (3) if F_v of particle i or j is greater than ε , and F_v of particle j or i is less than or equal to ε , the particle having lower value of F_v is better.

Note that the PSO algorithm minimizes F_o and F_v .

Based on the above criteria, in our emittance optimization, for particle i , if it does not have a stable solution, a large constant (for example, 10000 (unit: nm•rad)) is assigned to $F_o(i)$, and another constant is assigned to $F_v(i)$. If it has a stable solution, but $F_v(i)$ is greater than ε , a moderate constant (for example, 100 (unit: nm•rad)) is assigned to $F_o(i)$. If it has a stable solution, and $F_v(i)$ is less than or equal to ε , $F_o(i)$ is obtained according to the formula for calculating the emittance.

As an example of application, we optimize the emittance of an FBA lattice studied in the design of the storage ring of Hefei Advanced Light Source (HALS). The lattice has twenty super-periods, and we employ some combined-function (dipole-quadrupole) magnets.

In our optimization, there are eight variables of quadrupole strengths. The basic constraint conditions are:

- (1) maximum horizontal and vertical beta functions $\beta_x, \beta_y < 30\text{m}$,
- (2) horizontal beta function at center of straight $10\text{m} < \beta_x < 15\text{m}$,
- (3) vertical beta function at center of straight $\beta_y < 5\text{m}$,
- (4) fractional tunes < 0.5 ,
- (5) maximum dispersion $\eta < 0.1\text{m}$,
- (6) non-dispersive straight.

We used the PSO algorithm with a population size of 10000 and 1000 iterations, to optimize the emittance. The minimum emittance that can be reached is $68\text{pm}\cdot\text{rad}$, and the lattice functions of one period of one obtained solution are shown in Fig.1.

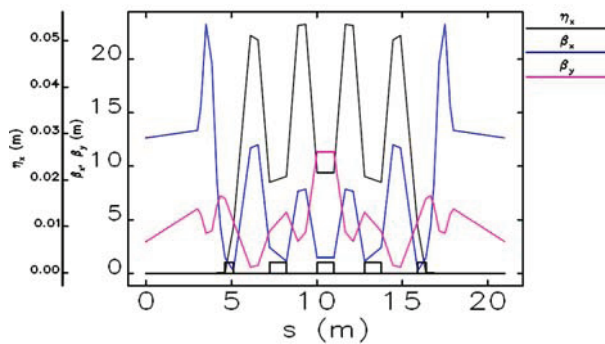


Figure 1: Lattice functions of the non-dispersive lattice.

At our laboratory, Genetic Algorithms are also applied to the emittance optimization [3]. We have found that, using the PSO algorithm we can get as good results as using GAs.

CONCLUSIONS

The PSO algorithm has been developing very quickly since it was proposed in 1995. We first used the PSO to optimize the dynamic aperture. Then we applied it to the emittance optimization, and as shown above, it is beneficial in the optimization. But the work presented here is only the preliminary work. We will improve the work in future.

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