



.....

**VIBRATION ANALYSIS AND CONTROL OF PIEZOELECTRIC
SMART STRUCTURES BY FEEDBACK CONTROLLER ALONG-
WITH SPECTRA PLUS SOFTWARE**

Kishor B. Waghulde^{1*}, Dr. Bimlesh Kumar²,

¹Research Scholar and Associate Professor,

J.T.Mahajan College of Engineering Faizpur

Tal. Yawal, Dist. Jalgaon – 425 524. (Maharashtra), India

*Corresponding author E-mail: kishorwaghulde@gmail.com

Mobile: 09423490910, Fax: 02585-248181

²Principal, J.T. Mahajan College of Engineering, Faizpur

Tal. Yawal, Dist. Jalgaon – 425 524. (Maharashtra), India

E-mail: bimlesh2000@rediffmail.com

ABSTRACT

The locations of actuators and sensors over a structure determine the effectiveness of the controller in controlling vibrations. If we need to control a particular vibration mode, we have to place actuators and sensors in locations with high control. In many cases of vibration control, low frequency modes are considered to be important. Hence, we only need to consider a certain number of modes in the placement of actuators and sensors. We extended the methodology for finding optimal placement of general actuators and sensors over a flexible structure. For vibration analysis ANSYS software is used. Experimentation is done for control vibration and to find optimal position of piezoelectric actuator/sensor over a thin plate. To obtain frequency response from PZT actuators and sensors, Spectra plus software is used.

Key words: Vibration Analysis, Actuators and Sensors, Feedback controller Optimal Placement, Finite Element Method, Spectra plus software.

INTRODUCTION

Active vibration control in distributed structures is of practical interest because of the demanding requirement for guaranteed performance. This is particularly important in light-weight structures as they generally have low internal damping. An active vibration control system requires sensors, actuators, and a controller. The design process of such a system encompasses three main phases such as structural design, optimal placement of sensors and actuators and controller design. In vibration suppression of structures, locations of sensors

and actuators have a major influence on the performance of the control system. It is well known that misplaced sensors and actuators lead to problems such as the lack of observability or controllability. Active vibration control is defined as a technique in which the vibration of a structure is reduced by applying counter force to the structure that is appropriately out of phase but equal in force and amplitude to the original vibration. As a result two opposing forces cancel each other, and structure essentially stops vibrating. Techniques like use of springs, pads, dampers, etc have been used previously in order to control vibrations. These techniques are known as ‘Passive Vibration Control Techniques’. They have limitations of versatility and can control the frequencies only within a particular range of bandwidth. Hence there is a requirement for ‘Active Vibration Control’. ‘Active Vibration Control’ makes use of ‘Smart Structures’ [3,5]. This system requires sensors, actuators, a source of power and a compensator that performs well when vibration occurs. Smart Structures are used in bridges, trusses, buildings, mechanical systems, space vehicles, telescopes, and so on. The analysis of a basic structure can help improve the performance of the structures under poor working conditions involving vibrations. “A Smart Structure” means a structure that can sense an external disturbance and respond to that with active control in real time to maintain the mission requirements. A Smart Structure typically consists of a host structure incorporated with sensors and actuators coordinated by a controller. The integrated structured system is called Smart Structure because it has the ability to perform self diagnosis and adapt to environmental change. One promising application of such smart structure is the control and suppression of unwanted structural vibrations. Figure 1 depicts the schematic representation of the basic elements of a smart structure [2,4].

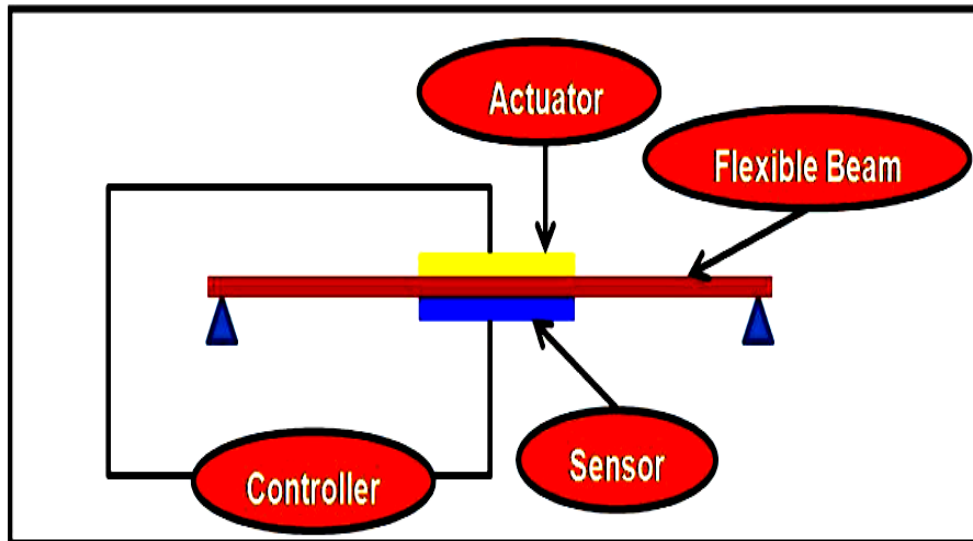


Figure 1: Schematic Representation of the Basic Elements of a Smart Structure

Optimal Placement of Piezoelectric Actuators

Consider the placement of a single actuator, say the j^{th} actuator. In this case, we consider only a single input version of system G , i.e. the transfer function from j^{th} actuator signal to the structural deflection. Each modal contribution depends on the location of the j^{th} actuator on the structure. Thus, if we intend to find the optimal placement for the actuator, the

contribution of mode (m, n) due to the j^{th} actuator is $\|\tilde{G}_{mnj}\|_2^2$ where,

$$\tilde{G}_{mnj} = \frac{P_{mnj}}{\sqrt{\rho h}(s^2 + 2\zeta_{mn}\omega_{mn}s + \omega_{mn}^2)} \quad (1)$$

A function f_{mnj} can be defined as:

$$f_{mnj}(x_{1j}, y_{1j}) = \|\tilde{G}_{mnj}\|_2 = \left| \frac{\bar{A}_j \Psi_{mnj}(x_{1j}, y_{1j})}{\sqrt{\rho h}} \right| \left\| \frac{1}{s^2 + 2\zeta_{mn}\omega_{mn}s + \omega_{mn}^2} \right\|_2 \quad (2)$$

The modal controllability is

$$M_{mn}(x_{1j}, y_{1j}) = \frac{f_{mnj}(x_{1j}, y_{1j})}{\alpha_{mnj}} \times 100\% \quad (3)$$

Where $\alpha_{mnj} = \max_{(x_{1j}, y_{1j}) \in R_1} f_{mnj}(x_{1j}, y_{1j})$ is the set of all possible actuator location [1,5].

If only the I_m lowest frequency modes are taken into account, the spatial controllability is

$$S_c(x_{1j}, y_{1j}) = \frac{1}{\beta_j} \sqrt{\sum_{i=1}^{I_m} f_{m_i n_i j}(x_{1j}, y_{1j})^2} \times 100\% \quad (4)$$

Where $\beta_j = \max_{(x_{1j}, y_{1j}) \in R_1} \sqrt{\sum_{i=1}^{I_m} f_{m_i n_i j}(x_{1j}, y_{1j})^2}$.

Several higher frequency modes can be chosen to reduce control spillover and the spatial controllability contributed by these modes S_{c2} is

$$S_{c2}(x_{1j}, y_{1j}) = \frac{1}{\beta_{2j}} \sqrt{\sum_{i=I_m+1}^{\bar{I}} f_{m_i n_i j}(x_{1j}, y_{1j})^2} \times 100\% \quad (5)$$

Where $\beta_{2j} = \max_{(x_{1j}, y_{1j}) \in R_1} \sqrt{\sum_{i=I_m+1}^{\bar{I}} f_{m_i n_i j}(x_{1j}, y_{1j})^2}$ and \bar{I} again corresponds to the highest frequency mode that is considered for the control spillover reduction. Hence, the optimization problem for the placement of piezoelectric actuators can be set up.

Optimal Placement of Piezoelectric Sensors

For the case where $k_{31} = k_{32}$ and $g_{31} = g_{32}$, the voltage induced in the j^{th} piezoelectric sensor $V_{sj}(t)$ in (5.23) can be shown to be:

$$V_{sj}(t) = \frac{k_{31j}^2}{C_j g_{31j}} \left(\frac{h + h_{pj}}{2} \right) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Psi_{mnj}(x_{1j}, y_{1j}) q_{mn}(t) \quad (6)$$

Where Ψ_{mnj} is

$$\Psi_{mnj} = \left[\int_{y_{1j}}^{y_{2j}} \frac{\partial \Phi_{mn}(x_{2j}, y)}{\partial x} dy - \int_{y_{1j}}^{y_{2j}} \frac{\partial \Phi_{mn}(x_{1j}, y)}{\partial x} dy \right] + \int_{x_{ij}}^{x_{2j}} \frac{\partial \Phi_{mn}(x, y_{2j})}{\partial y} dx - \int_{x_{ij}}^{x_{2j}} \frac{\partial \Phi_{mn}(x, y_{1j})}{\partial y} dx$$

If we compare V_{sj} to the general description for the j^{th} sensor output v_j , we can observe that

$$V_{sj}(t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mnj} q_{mn}(t) \quad (7)$$

Where C_{mnj} is obtained from (6). Then we will need the following theorem: Consider $G_{vu}(s, x_u, y_u)$, then

$$\|G_{vu}\|_2^2 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \bar{f}_{mn}^2 \quad (8)$$

Where,

$$\bar{f}_{mnj}(x_1, y_1) = \sqrt{\frac{1}{\rho h} \left\| \frac{C_{mnj}}{s^2 + 2\zeta_{mn}\omega_{mn}s + \omega_{mn}^2} \right\|_2^2} \quad (9)$$

and \bar{f}_{mn} is a function of the sensor locations that are expressed by (x_1, y_1)

Hence, the observability can be defined as:

$$K_{mn}(x_{1j}, y_{1j}) = \frac{\bar{f}_{mnj}(x_{1j}, y_{1j})}{\bar{\alpha}_{mnj}} \times 100\% \quad (10)$$

Where

$$\bar{f}_{mnj} = \left| \frac{k_{31j}^2}{C_j g_{31j} \sqrt{\rho h}} \left(\frac{h + h_{pj}}{2} \right) \Psi_{mnj}(x_{1j}, y_{1j}) \right| \left\| \frac{1}{s^2 + 2\zeta_{mn}\omega_{mn}s + \omega_{mn}^2} \right\|_2$$

$$\bar{\alpha}_{mnj} = \max_{(x_1, y_1) \in R_1} \bar{f}_{mnj}(x_{1j}, y_{1j}) \quad (11)$$

The spatial observability is

$$S_o(x_{1j}, y_{1j}) = \frac{1}{\bar{\beta}_j} \sqrt{\sum_{i=1}^{I_m} \bar{f}_{m_i n_i j}(x_{1j}, y_{1j})^2} \times 100\% \quad (12)$$

Where, $\bar{\beta}_j = \max_{(x_1, y_1) \in R_1} \sum_{i=1}^{I_m} \bar{f}_{m_i n_i j}(x_{1j}, y_{1j})^2$.

Moreover, the spatial observability for observation spillover reduction is

$$S_{oj}(x_{1j}, y_{1j}) = \frac{1}{\bar{\beta}_{2j}} \sqrt{\sum_{i=I_m+1}^I \bar{f}_{m_i n_i j}(x_{1j}, y_{1j})^2} \times 100\% \quad (13)$$

Where $\bar{\beta}_{2j} = \max_{(x_1, y_1) \in R_1} \sum_{i=I_m+1}^I \bar{f}_{m_i n_i j}(x_{1j}, y_{1j})^2$.

The optimization problem for the placement of piezoelectric sensors is

$$\begin{aligned} & \max_{(x_1, y_1) \in R_1} S_o(x_1, y_1) \\ & \text{subject to: } K_{m_i n_i}(x_1, y_1) \geq b_i, i = 1, 2, \dots, I_m \\ & S_{o2}(x_1, y_1) \leq c \end{aligned} \quad (14)$$

Now, consider a particular case where identical piezoelectric patches are used as actuators and sensors. M and K are clearly similar since they are both linearly proportional to $|\Psi_{mnj}(x_{1j}, y_{1j})|$ which is the only function that depends on the locations of patches. Hence, the spatial controllability and spatial observability are also similar.

$$\begin{aligned} M_{mn}(x_{1j}, y_{1j}) &= K_{mn}(x_{1j}, y_{1j}) \\ S_c(x_{1j}, y_{1j}) &= S_o(x_{1j}, y_{1j}) \\ S_{c2}(x_{1j}, y_{1j}) &= S_{o2}(x_{1j}, y_{1j}) \end{aligned} \quad (15)$$

The above results have an implication on the optimal placement of a collocated

piezoelectric actuator/sensor pair. Suppose we place a piezoelectric actuator on a structure that gives certain levels of spatial controllability and modal controllability. Placing a piezoelectric sensor at the same location would yield similar levels of spatial observability and modal observability. Thus, we only need to optimize the placement of either actuator or sensor. A similar optimization procedure can be used to find the optimal position and size of each collocated actuator/sensor pair to obtain optimal spatial controllability, while maintaining sufficient modal controllability levels [1,6,7,8].

Feedback Controller

There are two radically different approaches to disturbance rejection: feedback and feed-forward. Although this text is entirely devoted to feedback control, it is important to point out the salient features feedback controller. The principle of feedback is represented in Fig.2; the output y of the system is compared to the reference input r , and the error signal, $e = r - y$, is passed into a compensator $H(s)$ and applied to the system $G(s)$. The design problem consists of finding the appropriate compensator $H(s)$ such that the closed loop system is stable and behaves in the appropriate manner [7].

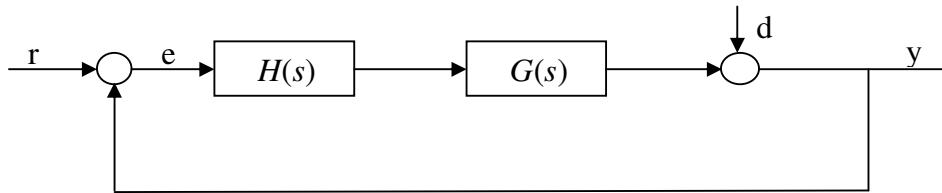


Figure 2: Principle of Feedback Controller

There are many techniques available to find the appropriate compensator, and only the simplest and the best established will be reviewed in this text. They all have a number of common features:

- ❖ The bandwidth ω_c of the control system is limited by the accuracy of the model; there is always some destabilization of the flexible modes outside ω_c (residual modes). The phenomenon whereby the net damping of the residual modes actually decreases when the bandwidth increases is known as spillover.
- ❖ The disturbance rejection within the bandwidth of the control system is always compensated by an amplification of the disturbances outside the bandwidth.
- ❖ When implemented digitally, the sampling frequency ω_s must always be two orders of magnitude larger than ω_c to preserve reasonably the behavior of the continuous system. This puts some hardware restrictions on the bandwidth of the control system.

Experimental Analysis

We have considered a thin rectangular plate with simply-supported edges. The piezoelectric actuator/sensor pair has similar properties in x and y directions, i.e. $k_{31}=k_{32}$, $g_{31}=g_{32}$ and $d_{31}=d_{32}$ are mounted on the plate. Fig.3 shows position of circular actuators on the Al plate. The position of respective sensors are mounted on same position on opposite size of the plate. The sizes of the circular patches are fixed and they are oriented in similar directions with respect to the plate. The properties of the plate and piezoelectric patches used are in Table1.

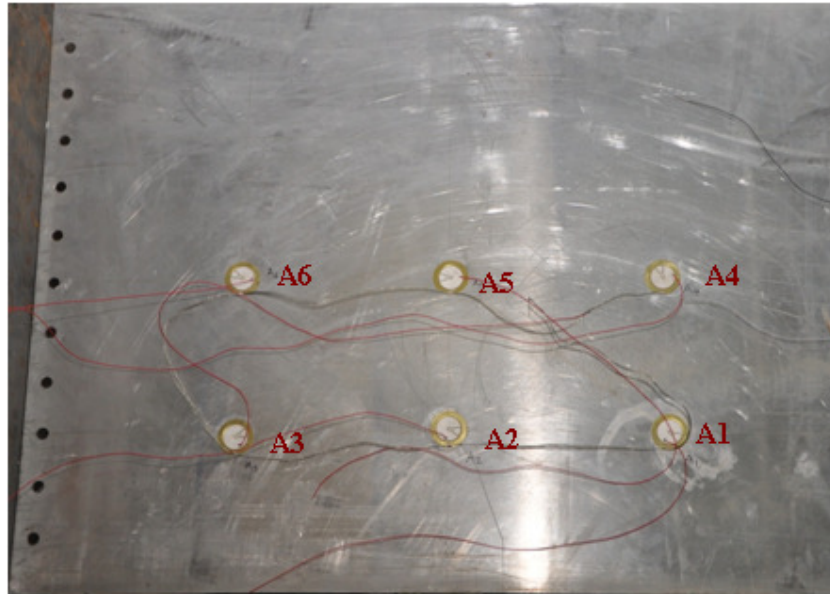


Figure 3: Plate with Circular PZT Patches as Actuators

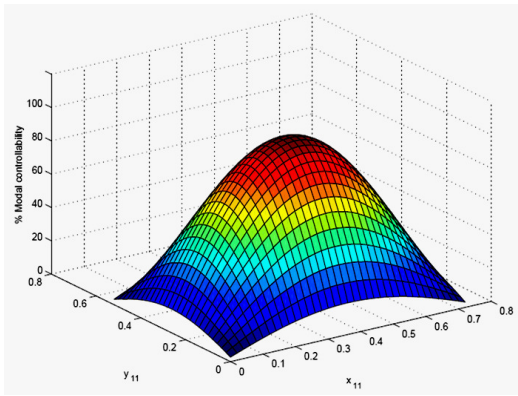
Table 1: Properties of the Piezoelectric Laminate Plate

Plate Young's modulus, E	:	$7 \times 10^{10} \text{ N/mm}^2$
Plate Poisson's ratio, ν	:	0.3
Plate density, ρh	:	11.0 kg/m^2
Piezoceramic Young's modulus, E_p	:	$6.20 \times 10^{10} \text{ N/mm}^2$
Piezoceramic Poisson's ratio, ν_p	:	0.3
Charge constant, d_{31}	:	$-3.20 \times 10^{-10} \text{ m/V}$
Voltage constant, g_{31}	:	$-9.50 \times 10^{-3} \text{ Vm/N}$
Capacitance, C	:	$4.50 \times 10^{-7} \text{ F}$
Electromechanical coupling factor, k_{31}	:	0.44

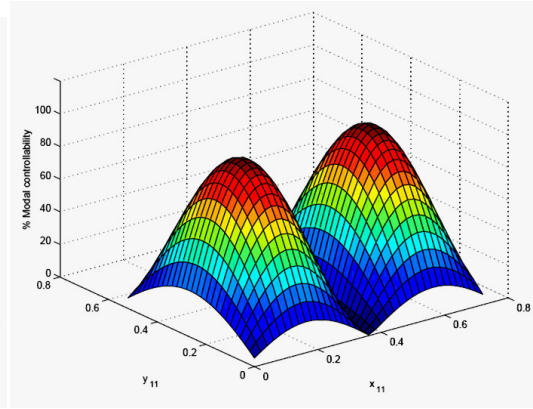
The natural frequencies are found out by ANSYS and experimental method that are tabulated in Table 2. Figures 4 to 6 shows the modal controllability, Fig. 7 shows Spatial Controllability and Fig. 8 shows Spatial Controllability of the first five modes versus the piezoelectric actuator location. This location corresponds to placing the actuator in the middle of the plate.

Table 2: First Six Modes of the Plate

No.	Mode (m,n)	Simulation ω_n (Hz)	Experiments ω_n (Hz)	Error (%)
1	(1,1)	41.9	41.8	0.2
2	(2,1)	87.1	85.9	1.4
3	(1,2)	122.4	121.1	1.1
4	(3,1)	162.4	159.2	2.0
5	(2,2)	167.6	164.3	2.0
6	(3,2)	242.9	234.5	3.6

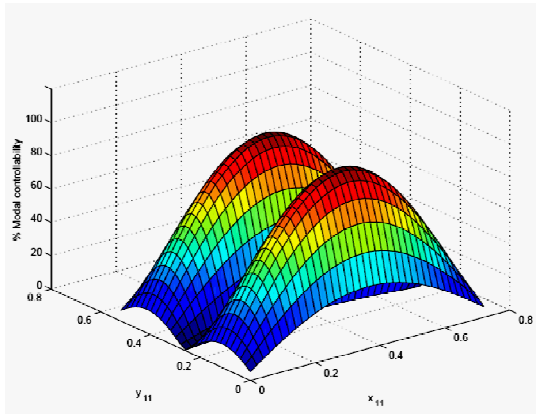


(a) Mode (1,1)

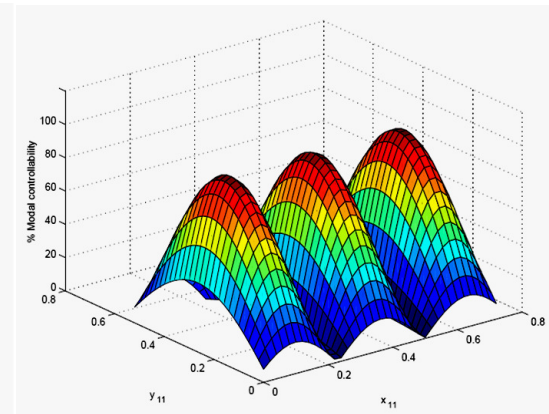


(b) Mode (2,1)

Figure 4: Modal Controllability - Modes 1 and 2

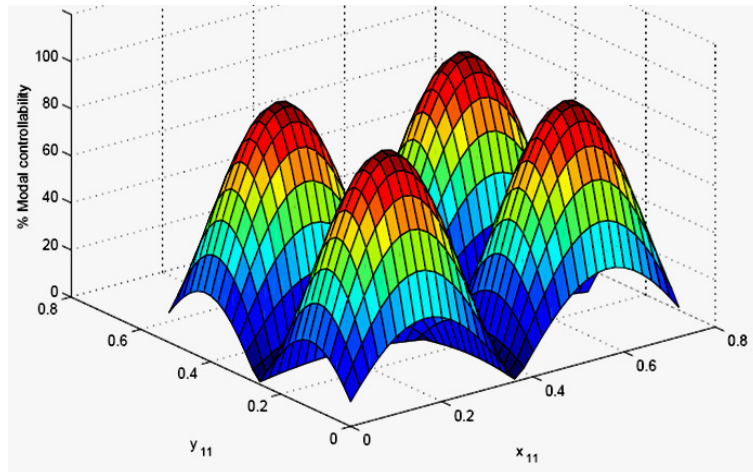


(a) Mode (1,2)



(b) Mode (3,1)

Figure 5: Modal Controllability - Modes 3 and 4



(a) Mode (2,2)

Figure 6: Modal Controllability – Modes 5

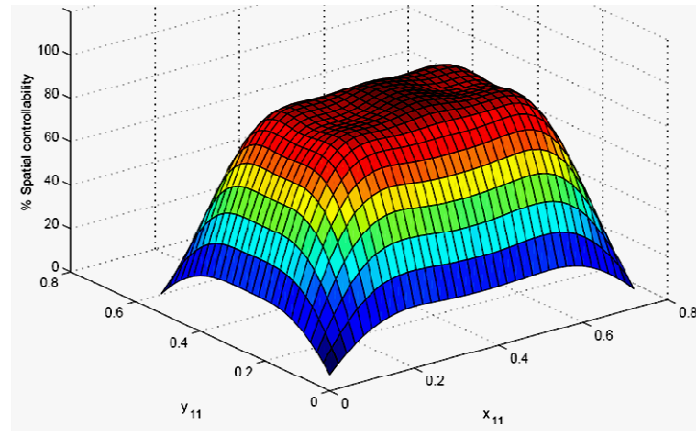


Figure 7: Spatial Controllability S_c - Based on the First Five Modes

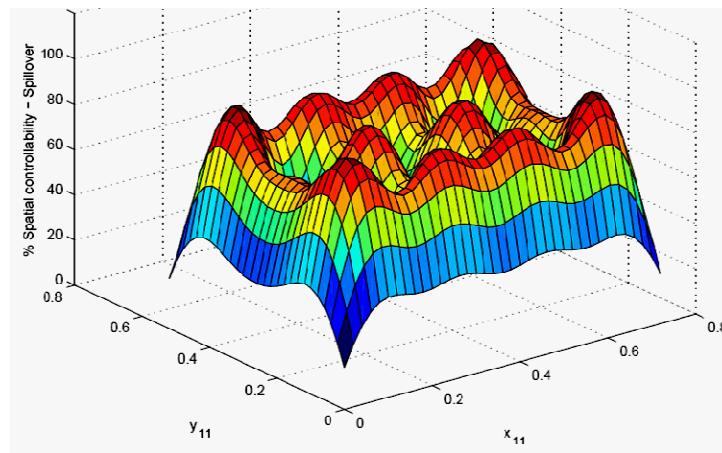


Figure 8: Spatial Controllability S_{c2} (Control Spillover Reduction)

Multi-waveform Signal Analyzer were used to obtain frequency responses from the piezoelectric laminate patches. Important parameters of the plate, such as resonance frequencies voltage variations, were obtained from an industrial personal computer (IPC) with Spectra plus software for the data acquisition and system control. To control the vibration Feedback controller is used. Figure 9 shows the experimental setup for simply supported beam. The time response curve for without controller is shown in Figure 10. Figure11 (a) to 11 (f) shows time response curve with controller for different locations. The frequency response curve without controller is shown in Figure 12(a). Figure 12 (b) shows combine results for all the locations of actuators along with feedback controller and Figure 13 shows the result of frequency response without controller with average of all six positions.



Figure 9: Experimental Setup

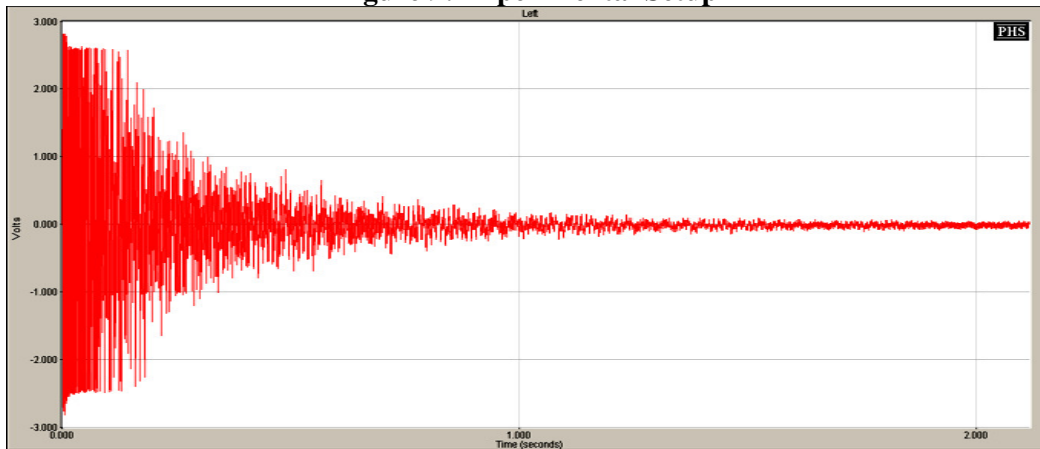


Figure 10: Time Response Curve for Simply Supported Plate without Controller

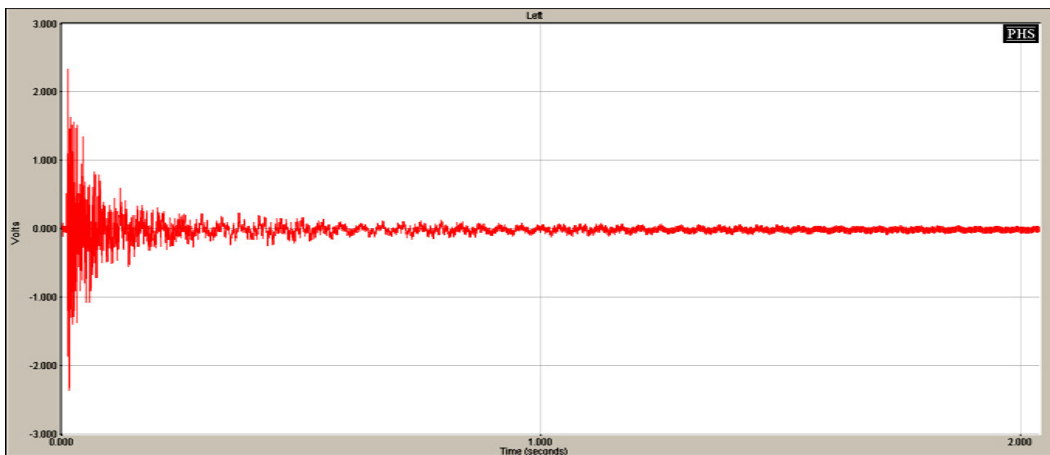


Figure 11(a): Time Response Curve with Controller (Actuator Location A1)

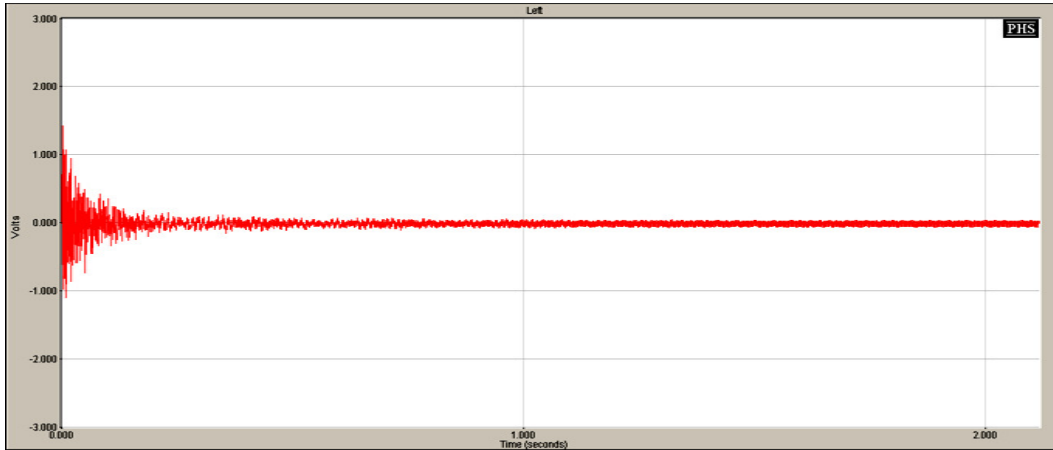


Figure 11(b): Time Response Curve with Controller (Actuator Location A2)

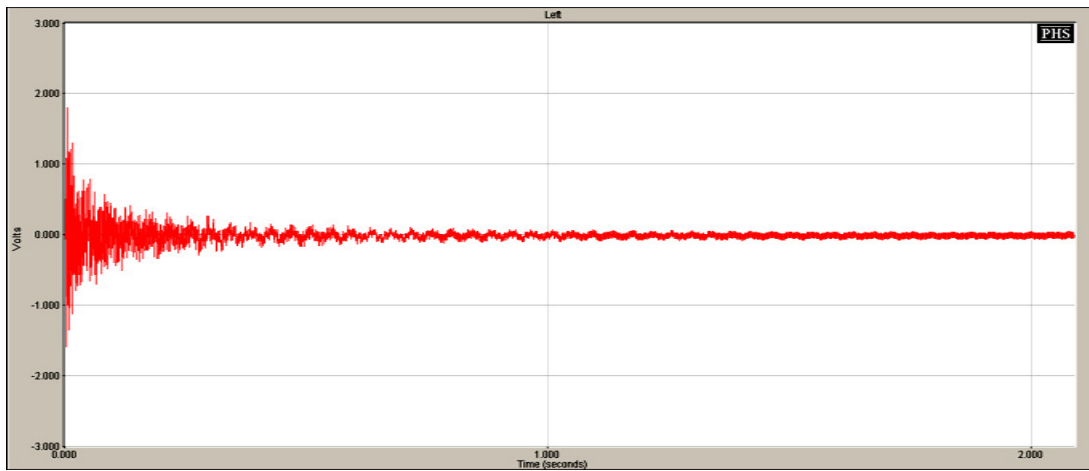


Figure 11(c): Time Response Curve with Controller (Actuator Location A3)

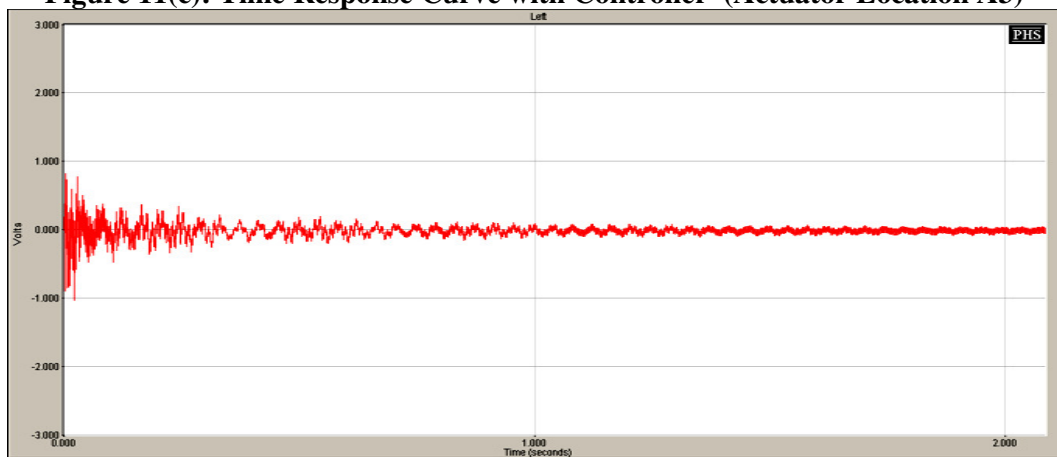


Figure 11(d): Time Response Curve with Controller (Actuator Location A4)

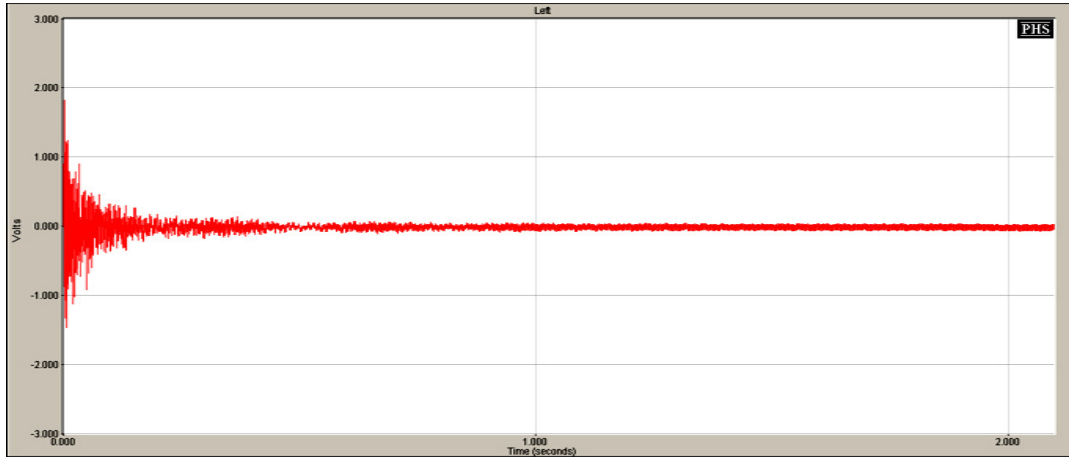


Figure 11(e): Time Response Curve with Controller (Actuator Location A5)

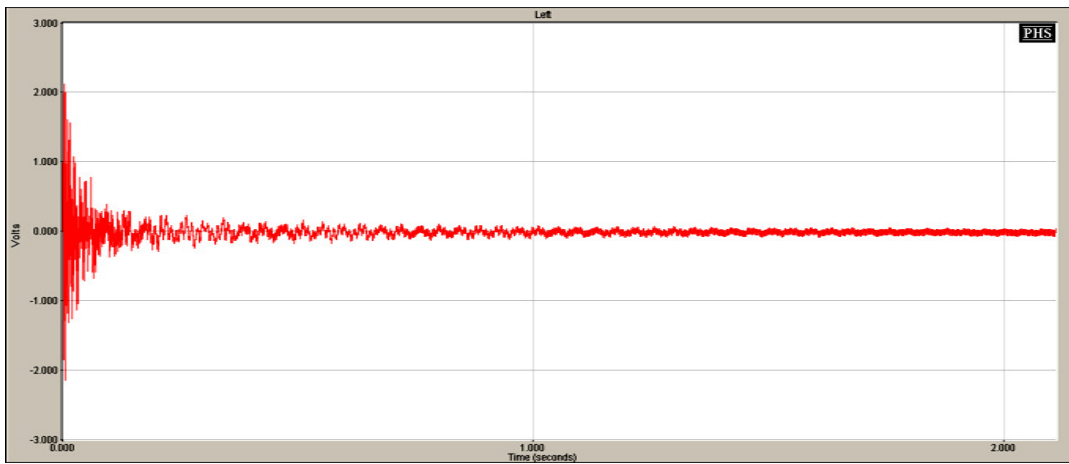


Figure 11(f): Time Response Curve with Controller (Actuator Location A6)

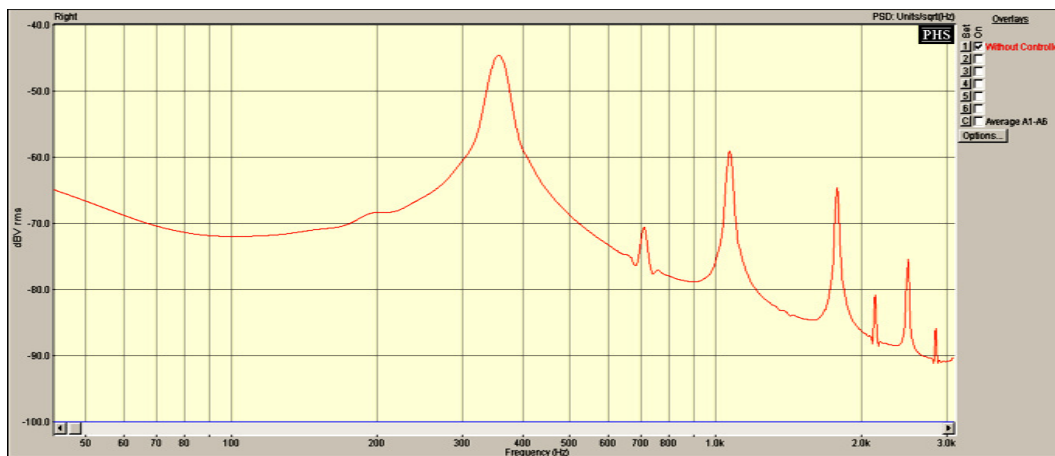


Figure 12(a): Frequency Response Curve without Controller

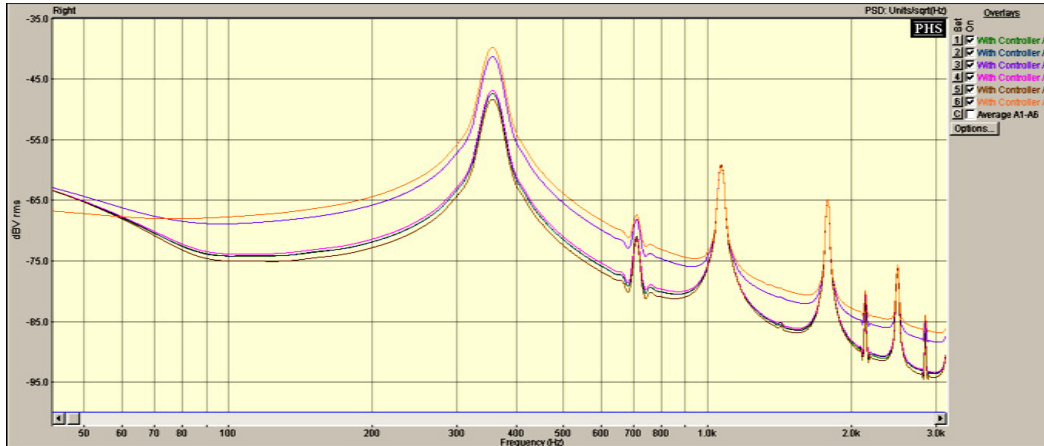


Figure 12(b): Frequency Response Curve with Controller

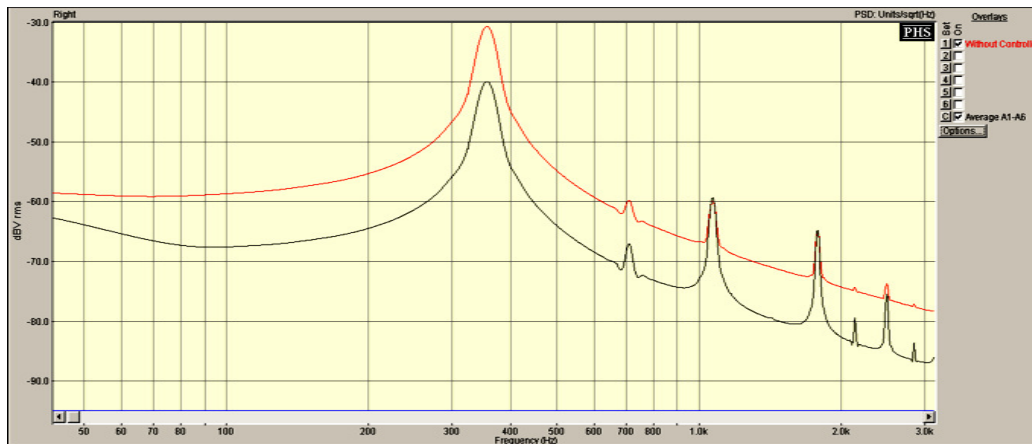


Figure 12(b): Frequency Response Curve without Controller and Average of A1 to A6

Due to the symmetry of the problem, the readings from the sensor is the same for the actuator locations that are parallel position along the width of the plate. That can be observed from the figures for pairs at locations A1 and A4, A2 and A5, A3 and A6. Now, considering the output varying the actuator location along the length of the plate shows that the settling time has the best values when the actuator is placed at locations A2 and A5. Also it can be seen that the settling time value is better in the actuator locations A1 and A4 than locations A3 and A6. In general, it can be said that placing the actuator middle of the plate gives better results than placing it closer to the both ends. From frequency response curve it clear that the vibration is controlled by using feedback controller. Also in this case the control is more at position A2 and A5 than all other positions.

CONCLUSIONS

This paper presents the numerical analysis and experimental results of vibration suppression of a simply supported plate bonded with circular PZT sensors and actuators. From the numerical studies we know that the spatial controllability and modal controllability were used for optimal placement of collocated piezoelectric actuator–sensor pairs on a thin flexible

plate. The optimization methodology allowed the placement of collocated actuator–sensor pairs for effective average vibration reduction over the entire structure. It was shown that the optimization methodology could be used for a collocated actuator–sensor system. From experimental results we can know that the presented method of optimal placement for the simply supported plate is feasible. For the plate with PZT's system, the actuator location was varied along the length and the width of the plate. For the case with the feedback controller, the settling time for the actuator locations at the center of the plate gave better results in attenuating the structural vibrations. By using signal generator and feedback controller the proposed control method can suppress the vibration effectively, especially for vibration decay process and the smaller amplitude vibration.

REFERENCES

1. Dunant Halim and S.O. Reza Moheimani (2003), “An optimization approach to optimal placement of collocated piezoelectric actuators and sensors on a thin plate”, Elsevier Science Ltd, Mechatronics-13, pp 27-47.
2. Zhi-cheng Qiu, et al (2007), “Optimal placement and active vibration control for piezoelectric smart flexible cantilever plate”, Journal of Sound and Vibration-301, pp 521–543.
3. Ismail Kucuk, et al (2011), “Optimal vibration control of piezolaminated smart beams by the maximum principle”, Computers and Structures-89, pp 744–749.
4. U. Ramos, and R. Leal (2003), “Optimal location of piezoelectric actuators”, AMAS Workshop on Smart Materials and Structures SMART’03, Jadwisin, pp.101–110.
5. V. Sethi and G. Song (2005), “Optimal Vibration Control of a Model Frame Structure Using Piezoceramic Sensors and Actuators”, Journal of Vibration and Control-11, pp 671–684.
6. Jinhao Qiu and Hongli Ji (2010), “The Application of Piezoelectric Materials in Smart Structures in China”, International Journal. of Aeronautical & Space Science 11(4), pp. 266–284.
7. A. Preumont, (2011), “A book on Vibration Control of Active Structures-An Introduction”, Third Edition, Springer Verlag Berlin Heidelberg.
8. Alberto Donoso, Jose Carlos Bellido, “Distributed Piezoelectric Modal Sensors for Circular Plates” Journal of Sound and Vibration, 319, 2009,pp. 50–57.
9. K. Ramkumar, and et al., “Finite Element Based Active Vibration Control Studies on Laminated Composite Box Type Structures under Thermal Environment”, International Journal of Applied Engineering Research ISSN 0973-4562 Volume 6, Number 2, 2011 pp. 221–234.
10. Limei Xu, and et al, “Size Optimization of a Piezoelectric Actuator on a Clamped Elastic Plate”, IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, Vol. 56, No. 9, September 2009, pp. 2015-2022.
11. L. Azrar, S. Belouettar, J. Wauer, “Nonlinear Vibration Analysis of Actively Loaded Sandwich Piezoelectric Beams with Geometric Imperfections” Journal of Computers and Structures, 86, 2008, pp. 2182–2191.
12. Dr. Chandrashekhar Bendigeri, “Studies on Electromechanical Behavior of Smart Structures by Experiment and FEM”, International Journal of Engineering Science and Technology, ISSN : 0975-5462 Vol. 3, No., 3 March 2011, pp. 2134-2142.