USING DIGITAL PHOTOGRAPHY TO SUPPORT TEACHING AND LEARNING OF PROPORTIONAL REASONING CONCEPTS

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Abstract

Proportional reasoning entails multiplicative relationships in situations of comparison. Successful proportional reasoners can recognise a proportional situation as distinct from a non-proportional one; they have a sense of co-variation and they have a range of strategies for solving proportional problems.

As educators we realise that teaching proportional reasoning cannot solely rely on asking students to complete symbolic and mechanical methods, such as the cross-product algorithm. To develop proportional reasoning, students must have regular opportunities to experience the underlying concepts. These concepts include foundational aspects of proportional reasoning, such as fractional thinking, multiplicative thinking (as opposed to additive), relative thinking (as opposed to absolute), as well as concepts of rate and scale.

As part of a large multi-state project in Australia to enhance middle years students' numeracy through a focus on proportional reasoning, 120 teachers participated in a series of professional learning workshops. These teachers generally reported feeling confident teaching the algorithmic aspects of proportional reasoning but a number of them specifically asked for assistance with the conceptual development of their students' proportional reasoning.

In response, the researchers developed a series of activities with the teachers, where digital cameras were used in the school environs to capture images that represented examples of proportional reasoning concepts. In small groups, the teachers moved around the school taking their photos and then reported back to the workshop, showing their images through a data projector while they explained the concepts they felt their images captured.

This presentation articulates the ways that the digital cameras were used by the teachers to capture and report on the proportional reasoning concepts, and their thoughts and aspirations as to how they would use the cameras to develop the proportional reasoning of their students.

Keywords: proportional reasoning, digital cameras, images, middle school.

1 INTRODUCTION

Proportional reasoning is an important component of numeracy (mathematical literacy). It entails multiplicative relationships in comparative situations requiring a sense of co-variation i.e. as one aspect of the relationship varies so to does the other, either in direct or inverse proportion [1,2]. It is important in the middle years of schooling to develop students' proportional reasoning to maximize their success in subjects, of which such reasoning is a foundational component, such as science, technology, engineering and mathematics. Proportional reasoning is also important in our daily lives as numerate citizens and in many professions, such as chemists, builders or chefs. Some of the key elements of proportional reasoning include multiplicative, fractional and relational thinking. To acquire these skills it is suggested that students' develop conceptual understanding before proportional procedures and/or algorithms are taught [3].

2 BACKGROUND TO THE STUDY

This paper reports on a study that was developed in response to teachers' requests originating from a large multi-state Australian research project that investigated the enhancement of students' numeracy through a focus on proportional reasoning [see 4,5,6]. Several teachers involved in the broader project

were interested in ways to develop their students' understanding of the underlying concepts of proportional reasoning without necessarily focusing on algorithmic or procedural skills. These teachers felt that their students could not always recognize the difference between a proportional and non-proportional situation, could not determine the type of proportional situation, and often lacked the specific language to competently engage in or describe the proportional situations. The teachers felt that focusing on procedural or algorithmic approaches to solving problems involving proportion was neither sufficient nor effective for enhancing students' underlying conceptual understanding. The teachers' desire to develop their students' conceptual understanding either prior to or alongside the procedural aspects of proportional reasoning aligned with commonly espoused pedagogical practices [7]. In response to these requests from teachers, the authors of this paper developed a research sequence to investigate means of engaging students in a variety of proportional and non-proportional situations with an emphasis on conceptual development rather than procedural. Phase One, which has been reported elsewhere [8] trialed the classroom use of researcher supplied photographic images as conversation starters to portray real life examples of these varied proportional situations. At the completion of this phase some teachers commented that while the activity was valuable, they felt that 'researcher supplied' images were potentially not as suitable to their classroom context as images that they or their students might generate themselves. These comments led to Phase Two, which required the participating teachers to gather photographic images from the school environment that they felt portrayed examples of proportional situations involving concepts such as multiplicative, relative or fractional thinking, ratios, rates or scale. Phase Three (yet to be reported) will involve students gathering photographic representations of proportional situations. This paper reports Phase Two of this research sequence.

2.1 Conceptual understanding of proportional reasoning

Conceptual knowledge reflects an understanding of mathematical relationships constructed internally and connected to existing ideas. In contrast, procedural knowledge reflects understanding of rules and skills needed to complete mathematical tasks [9]. Teaching the procedural skills involving standard methods of calculations in proportional situations does not allow teachers to address the underlying concepts inherent in proportional situations. Deep understanding is required in order to successfully reason proportionally. When the concept is not fully developed, students often make systematic errors in proportional reasoning [10]. Students often misuse proportionality, applying proportional calculations to non-proportional situations, such as doubling the area of a circle when the diameter is doubled [11].

In a study by Miller and Fey [12], a focus on the conceptual aspects of proportional reasoning as a means of strengthening student performance was shown to be successful. Further, Fujimura [13] noted that building on children's intuitive knowledge is an accepted means of introducing ratio and proportion. He identified three means of doing this: using everyday contexts, incorporating students' own representations and methods for solving problems, and asking students to express their own ideas and pictorial presentations.

2.2 Proportional reasoning concepts

The research literature contains descriptions of several main problem types of proportional reasoning situations that are frequently encountered in the middle years of schooling curricula. These include rate problems (involving both commonly used rates, such as speed, and rate situations in which the relationship between quantities is defined within the question, such as pizza pieces per child); part-part-whole (e.g., ratio problems in which two complementary parts are compared with each other or the whole); and stretchers and shrinkers (growth or scale problems) [14]. Concepts such as relative and absolute comparisons are also pivotal in students' understanding of proportionality. For example, if Country A, with a third of the population of Country B, wins half as many Olympic medals as Country B, it can be said that while absolutely, Country B has the most medals, relatively Country A with the lower population has performed better. In addition to the need for a focus on a range of proportional situations, as previously mentioned, it is also important to provide opportunities for students to practice differentiating proportional and non-proportional situations [11].

3 THE STUDY

3.1 Participants

The participants of Phase Two of the research sequence were teachers who attended a series of professional learning workshops on enhancing numeracy through a focus on proportional reasoning. There were 120 teachers who completed this phase. Participants were all middle years teachers from primary and secondary schools. The half-day workshops pertinent to this phase were held in five locations in two Australian states over a period of two weeks.

3.2 The digital equipment

The digital cameras used in this study were 'flip' cameras. These cameras have a built-in USB that connects directly to a computer. The images are then quickly and easily downloaded onto the computer from which they can be viewed through a data projector.

3.3 Procedure

At professional learning workshop teachers were informed of the results of Phase One of this research sequence. They viewed the images that were part of the Phase One intervention and heard examples of the teachers' and students' feedback about using images to initiate discussions involving proportional reasoning concepts inherent in the images. The responses of the teachers and students of the Phase One intervention were reported. Teachers were also informed that the teacher participants of Phase One believed that they could make the activity more beneficial to their students by contextualizing the images to suit their classes.

Phase Two was designed to give the teacher participants an opportunity to use digital cameras to capture examples of proportional situations around the school environs. The teachers worked in groups of two or three. Each group had a digital camera. The groups were given approximately 20 minutes to move around the school to take pictures.

On their return to the workshop, each group in turn connected their digital cameras to the laptop and data projector. The groups shared their images with the rest of the workshop attendees. As each image was projected the group members explained the inherent proportional situation that they had captured in the image. Discussion was encouraged between the presenting group and the audience. This allowed the teachers to develop ideas about how they might generate discussions of the images with their students.

3.4 Data collection

The images of proportional situations that each group captured were collected by the researchers and stored on a computer hard drive. The names of each group's members were recorded with the images they captured. The key points of each group's presentation to the rest of the workshop were recorded by one of the researchers as field notes. Key elements of the ensuing discussion were also noted.

The images were collated and analysed for commonalities to determine the success of using images to capture proportional situations in the school environs and to also determine the extent to which the types of proportional situations (e.g., multiplicative or relational) were portrayed through the image.

The field notes of the workshop presentations and discussions were analysed to determine themes of interest.

4 RESULTS

All groups from all workshops were successful in using the digital camera to capture images that portrayed some of the key elements of proportional situations. Some groups took over ten images in the time provided while all groups took at least five.

4.1 Images

The most frequently captured element of proportional reasoning through image was that of multiplicative relationships. Some examples of these include, a) a series of bicycle racks, depicting the number of racks multiplied by the number of places for each bicycle in the rack, b) a cross hatched

metal drain cover where the vertical metal bars crossing with the horizontal bars created a multiplicative opportunity to determine the number of intersections of the bars, and c) windows, in groups of four across the front of a building.

The concept of relative thinking was often captured through the combination of two images. The first image was of an object without context, making it difficult to determine the size. The second image had the same object but contextualized with a well-known object thus allowing the viewer to determine the size of the original object relative to the newly inserted object. Examples of this include, a) First Image: a non-descript rectangle of metal placed on a blank background followed by the second image of the metal object placed next to a woman's shoe, and b) First Image: a rubbish bin without surrounding context followed by the second image including a person standing beside the rubbish bin.

Other examples of images that portrayed elements of proportional reasoning were

1. Scale:
    a. Three dimensional scale; a cylindrical drinking container next to a cylindrical hot water urn.
    b. Two dimensional scale: a rectangular book placed flat against a much larger window crating a situation of comparison of area.
    c. One Dimensional Scale: the length/height of a person and their shadow compared to the shadow of a tree to determine its height.
2. Ratio: Examples of cooking: maintaining the correct ratio of ingredients for catering for large numbers of students
3. Fractional Thinking: Windows with multiple panes of glass; the bottom row being used to display student work, the upper panes left clear, thus creating a fraction of used and unused windows, depicting part-part-whole fractional relationships.

It was noted that the inherent proportional situation in some of the images was clearly obvious to the audience. On some occasions the audience preempted the group's speaker by suggesting what the image portrayed before the speaker was able to explain. At other times the image had to be explained by the presenters to make the proportional situation clear to the audience.

4.2 Dialogue

The field notes recorded dialogue in two main aspects; the first was the presenters' explanations of the images that they were presenting and the second was the ensuing discussion with the audience about the images and their applicability and possible benefit to students.

The explanations of the images presented generally showed a sound understanding of the concepts of proportional reasoning and the accompanying language. A number of groups expressed initial concern that they would not find any examples of proportional situations in the school's environs, but were pleased that this was not the case. The general consensus was that finding proportional situations around the school was enjoyable and that using the digital cameras to capture the image was a powerful learning experience for them.

Other dialogue of note pertaining to the capturing and presenting of the images include,

1. Many groups expressed surprise at how many proportional situations could be found in the school environs.
2. Teachers reported that working in groups and finding, photographing and discussing the proportional situations was a powerful experience.
3. A number of teachers mentioned that they had never thought of using a digital camera to capture mathematical concepts and they felt that they could perhaps use the cameras in broader contexts beyond proportional reasoning.
4. Teachers regularly expressed that presenting the images to an audience meant that this required them to understand the concepts to be photographed and to have the language to clearly explain the images they had captured.

Beyond the presentation and explanations of images the discussions within the workshop groups almost always turned to how they (the teachers) could use this kind of activity with their students. A number of key points were raised through these discussions.

1. All teachers agreed that students could easily use the digital cameras, though some teachers from some schools expressed concern about 'trusting' students with the cameras outside the classroom.
2. The general consensus was that the students would need to have solid backgrounds in the proportional concepts before they began to photograph examples around the school.
3. Not all concepts would need to be addressed before this activity could be conducted. For instance, if a class had been working on multiplicative thinking, in contrast to additive, they could attempt this activity with that specific concept in mind.
4. This would be a good culminating activity for students who had been learning particular concepts in class.
5. The collaboration required to work as a group to identify, photograph and explain the proportional situations was a powerful aid to learning.
6. Teachers felt that with the prevalence of smart phones and digital cameras, students could easily extend the range of the photographic task beyond the school area.
7. One teacher noted that this exercise would require higher order thinking skills, in that assuming the student had some background understanding of the proportional concepts, they would need a thorough understanding to be able to recognise these situations in a new context.
8. When prompted by the researchers, all participant teachers agreed that this activity seemed a powerful means of highlighting or developing the underlying concepts of proportional reasoning without reverting solely to procedural or algorithmic methods.

5 DISCUSSION

Teacher participants were asked to use digital cameras to capture images of proportional situations in the school environs in response to requests from some of them to learn ways of developing the concepts of proportion without exclusively focusing on procedural/algorithmic methods. The participants had seen examples of researcher generated images of proportional situations but they did not relate to the contextual requirements of specific classes. By engaging the teacher participants in this activity it was hoped that they would gain an understanding of the learning processes involved and consider how they would translate the activity into their own classroom contexts.

The reaction of many teachers was initially less than confident that they would be able to identify situations of proportion in the school environs. Therefore, the fact that all groups, in all workshops, were successful in identifying and capturing some appropriate images allayed this lack of confidence. The teachers were generally surprised by the abundance of proportional situations inherent in their school environment. In addition, the activity provided an opportunity for the teachers to consider ways to capture and use images with their students that were appropriate to their own contexts. Teachers stated that the exposure to the proportional concepts in the workshops had given them enough understanding to successfully complete the photographic task, and that the task was a powerful means of focusing on the concepts and applying one's knowledge. This experience aligns with the notion of conceptual knowledge noted by Young-Loveridge [9] in that it reflects the understanding of the mathematical relationships constructed internally and connected to existing ideas.

The images captured by the teachers showed that it was possible to find within the school environs examples of most of the component elements of proportion as described by Lamon [14] including multiplicative, fractional and relational thinking, ratio, rate and scale. From analysis of the images taken, it was clear that some of these elements were more readily identified than others (for example, scale and rate were not as frequently included as were relative or multiplicative situations), however, sharing the images with each other helped to broaden the possibilities for all.

The positive reaction of the teachers to the activity led them to discuss its applicability to their students. The teachers, all from middle years classes (Years 5-9) felt that their students could successfully use the digital cameras. They therefore felt that if their students had sufficient understanding of specific proportional concepts that they would attempt the activity with them. Teachers suggested that the activity could be conducted with their classes one concept at a time or that the task could be extended beyond the school area, for example by asking the students to record photographs at home or in their neighbourhoods. The teachers had, through their own experiences with the activity, decided that it was a task achievable by their students at an appropriate time in their learning.

The ensuing dialogue, once a group had presented their images and explanations, revealed that teachers generally felt that the activity had consolidated their understanding of the concepts and encouraged them to use the language associated with the concepts, such as relative thinking or multiplicative situations. Reflecting this, the teachers saw potential benefit for their students' learning through the collaborative process and the presentation to a group requiring understanding of concepts and language.

6 CONCLUSION

The teacher participants of this study had been attending a series of workshops over two years and had been exposed to considerable professional learning about proportional reasoning in particular and numeracy in general. The collection and presentation of images of proportional situations in the school environs gave the teachers an opportunity to test and clarify their own understanding of some of the concepts covered by the workshops. In so doing, the teachers had experienced a learning sequence similar in some ways to those of their students. The teachers in general felt that using the digital cameras to capture images of proportional situations and then presenting and explaining them was a successful means of focusing on the concepts of proportion and clarifying their understanding and associated language. The teachers felt that the learning experience would translate well into their middle years classes, with the learning benefits of being an engaging, collaborative activity that allowed conceptual development of proportional reasoning.

The next phase of this research will investigate student use of the digital cameras in activities that replicate those experienced by these participant teachers.

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