OPTIMUM PLACEMENT AND PROPERTIES OF TUNED MASS DAMPERS USING HYBRID GENETIC ALGORITHMS

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ABSTRACT

Tuned mass dampers (TMDs) systems are one of the vibration controlled devices used to reduce the response of buildings subject to lateral loadings such as wind and earthquake loadings. Although TMDs system has received much attention from researchers due to their simplicity, the optimization of properties and placement of TMDs is a challenging task. Most research studies consider optimization of TMDs properties. However, the placement of TMDs in a building is also important. This paper considers optimum placement as well as properties of TMDs. Genetic algorithms (GAs) is used to optimize the location and properties of TMDs. Because the location of TMDs at a particular floor of a building is a discrete number, it is represented by binary coded genetic algorithm (BCGA), whereas the properties of TMDS are best suited to be represented by using real coded genetic algorithm (RCGA). The combination of these optimization tools represents a hybrid coded genetic algorithm (HCGA) that optimizes discrete and real values of design variables in one arrangement. It is shown that the optimization tool presented in this paper is stable and has the ability to explore an unknown domain of interest of the design variables, especially in the case of real coding parts. The simulation of the optimized TMDs subject to earthquake ground accelerations shows that the present approaches are comparable and/or outperform the available methods.

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1. INTRODUCTION

In order to reduce the response of buildings due to lateral loads many devices have been proposed. One of the methods is by using tuned mass dampers (TMDs). TMDs are a kind of vibration absorbers which is relatively easy to be implemented. By adding a small additional mass, where the stiffness and damping are designed in a proper way, the vibration of building can be reduced. The use of vibration absorber dates to the 1900s when Frahms applied a US patent. Frahm model is applied to a main spring- mass without damping that is attached with a small spring-mass without damping in order to reduce displacement of the main mass under harmonic loading (Soong and Dargush [1]). The idea of Frahm is extended to structures as a general application of TMDs, where all properties including damping are included in the application. Nowadays many buildings utilize TMDs (Holmes [2], Soong and Dargush [1], Villaverde [3]) such as in John Hancock Tower (Boston, USA), Citicorp Centre (New York, USA), Sydney Tower (Sydney, Australia), Chiba Tower (Chiba, Japan), Fukuoka Tower (Fukuoka, Japan), Crystal Tower (Osaka, Japan), Deutsche Bundespost cooling tower (Nürnberg, Germany), Canadian National Tower (Toronto, Canada) and Taipei 101 (Taipei, Taiwan).

In order the TMDs work properly, the properties of TMDs have to be designed so that the response of buildings can be reduced. Several papers have proposed many methods to design TMDs. Most of the proposed methods in TMDs design consider the main structure as SDOF structure in which a closed form formula is derived. The inclusion of damping in the main system has been considered in Warburton and Ayorinde [4] and Thompson [5]. In Warburton [6] various types of excitations have been considered and the optimum parameters have been tabulated for various simple systems. In another report, Asami et al. [7] considered an optimum design of TMDs for a system under random excitation. Rana and Soong [8] extended Warburton results and tabulated the results for various parametric structures. Chang [9] and Chang and Qu [10] extended the Warburton method for random loading. Note that most of the research only considers SDOF structure as the model. Optimization of TMD for multi degree of freedom (MDOF) systems, such as buildings, but are modeled as SDOF systems has been discussed in Kaynia *et al.* [11], Villaverde [3], Villaverde and Koyama [12], Sadek *et al*. [13]. Because the structures are modeled as SDOF systems the TMDs will tune to a single mode system only.

Although it is possible to find a closed form solution of the damper for MDOF structures, the complexity of parameters involved in the structures preclude its practicality. Therefore, the practical approach is to build a design procedure through the numerical optimization process. Hadi and Arfiadi [14] developed an optimization procedure to optimize TMDs for MDOF structures by using binary coded genetic algorithm (BCGA). In this case physical properties of the structures are used directly without necessarily converting the structures into a single mode model. Therefore, the vibration mode to be tuned does not necessarily to be known beforehand. Other optimization methods have also been proposed such as developed by Lee *et al.* [15] that employing frequency domain approach; Li and Xiong [16] for the extension to multi TMDs and by Wang *et al.* [17] for the limitation of the TMD response using two stage optimizations.

Most optimization problems consider the optimal properties of TMDs only. In some cases placement of TMDs are also important. This paper presents the procedure to optimize the location and properties of TMDs in one optimization program.

2. OPTIMIZATION TECHNIQUE USING GENETIC ALGORITHMS

Genetic algorithm (GA) is a stochastic algorithm that mimics natural phenomena as operators in the processing. The idea behind the mechanics of GA is to resemble the adaptive process in nature based on Darwinian's survival of the fittest mechanisms. GA has been used to obtain the optimum design of the function and has shown its superiority in obtaining nearly global optimum solution of the complex problems. Originated by Holland in 1960s (Goldberg [18] Holland [19], Michalewicz [20]). GA has been used to obtain optimum value in many areas. In GA the solution is considered as a population of candidates. These candidates experience evolutionary process based on survival of the fittest mechanisms. The candidates of solutions change their chromosomal content to produce offspring through crossover and mutation processes such that they strive to survive into the next generation. Naturally, a better individual will survive and be selected into the next generation, which is reflected by its fitness. Some less fit individuals will die and be replaced by most fit ones. Although some less fit individuals will also pass into the next generation, on average the fitness of the current generation is better than the fitness of the previous generations. The fitness is a representation of the objective function of the real problems to be solved. Since the measure of the optimality is defined by the fitness of individuals, GA does not need the gradient information to optimize the cost function. This condition makes GA suitable to be used for hard and complex optimization problems, and is capable of obtaining global optimum solution in a simple way.

In the initial development of GA, individual as design variable is represented by using a string containing 0 and 1. A string can be converted to an integer number which later can be converted further to a real number. The length of the string represents the decimal precision for a particular lower and upper bound of defined numbers. Therefore, the length of string bounds the range of real numbers. Because of this, the approximate design, *i.e.,* the upper and lower bounds of design variable, should be supplied by designers.

The limitation of using BCGA is that the length of the string should be estimated beforehand. In that case the approximate design value of the optimized variable should be supplied by the designer. For the problem where the minimum and maximum values of design variables are known or can be predicted, BCGA is the appropriate choice. The problem of discrete location like how to place the damper at a particular storey of high rise building is better represented by binary coding. However, many design variables may not be best represented by integer number. Although BCGA has the capability to represent real values with some techniques, the length of the string limits its capability to explore the unknown domain (Arfiadi and Hadi [21]). The properties of some design variables in general are real numbers. Therefore real coded genetic algorithm (RCGA) is best suited to obtain real variables. In RCGA individuals as candidates of design variables are represented by real numbers. RCGA has been applied in many areas of optimization including aerospace

design, biotechnology, industrial engineering, microware, water resources management and control (Herrera *et al.* [22]).

3. MDOF STRUCTURES WITH TMD

Consider a multi-degree-of-freedom (MDOF) structure with TMD attached at a particular floor. The structure is assumed as a shear building with the mass lumped at each floor level. The equation of motions can be written as

$$\mathbf{M_s}\,\ddot{\mathbf{X}}_{\mathbf{s}} + \mathbf{C_s}\,\dot{\mathbf{X}}_{\mathbf{s}} + \mathbf{K_s}\,\mathbf{X}_{s} = \mathbf{e_s}\ddot{x}_g \tag{1}$$

where

$$\mathbf{M_s} = diag[m_1 \quad m_2 \quad ... \quad m_N \quad m_d] \tag{2a}$$

$$\mathbf{C_s} = \mathbf{C_o'} + \mathbf{C_d'} \tag{2b}$$

$$\mathbf{K_s} = \mathbf{K_o'} + \mathbf{K_d'} \tag{2c}$$

$$\mathbf{X_s} = [x_1 \ x_2 \ ...x_N \ x_d]^{\mathrm{T}} \tag{2d}$$

$$\mathbf{e_s} = [-m_1 \ -m_2 \ ...-m_N \ -m_d]^{\mathrm{T}} \tag{2e}$$

where $\mathbf{M_s}$, $\mathbf{C_s}$, $\mathbf{K_s}$ are mass, damping and stiffness matrices of the structure, respectively; $\mathbf{X_s}$ is relative displacement with respect to ground, $\ddot{x}_g$ is acceleration of the ground due to earthquake, $\mathbf{e_s}$ is influence of the earthquake on the structure. The super dot represents derivative with respect to time. In Eq. (2a) m_i = the mass of the ith floor (i=1,2,.., N), m_d = the mass of the damper, $\mathbf{C_o'}$ in eq. (2b) is (N+1)×(N+1) damping matrix as a contribution of $\mathbf{C_o}$, $\mathbf{C_o} = $ N×N damping matrix of structure without TMD as a function of c_i, where c_i = the damping of the ith floor (i=1,2,.., N), $\mathbf{K_o'}$ in Eq.(2c) is (N+1)×(N+1) stiffness matrix as a contribution of $\mathbf{K_o}$, $\mathbf{K_o} = $ N×N damping matrix of a structure without TMD as a function of k_i = the stiffness of the ith storey (i=1,2,.., N).

TMDs contribution to the stiffness matrix, $\mathbf{K_d'}$, which is obtained from

$$\mathbf{K}_d = \begin{bmatrix} k_d & -k_d \\ -k_d & k_d \end{bmatrix} \tag{3}$$

where k_d = the stiffness of the damper, with destination vector to locate the damper stiffness

as follows

$$\mathbf{ID_d} = \begin{bmatrix} j_d & N+1 \end{bmatrix} \tag{4}$$

where j_d is the floor number at the location of TMD, and $(N+1)$ denotes degrees of freedom's number for the TMD. Note that j_d is not necessary to be the top floor. By using this representation, the assembly process may follow the assembly process in the stiffness matrix method. The same thing can be done for the contribution of the damping if TMDs by changing $\mathbf{K_d}$ and k_d in Eq. (2) by $\mathbf{C_d}$ and c_d, respectively. Similarly $\mathbf{C_o}$ and $\mathbf{K_o}$ are assembled to obtain $\mathbf{C_o'}$ and $\mathbf{K_o'}$ with the appropriate destination vector.

The equation of motions can be converted into a state space equation:

$$\dot{\mathbf{Z}} = \mathbf{A}\,\mathbf{Z} + \mathbf{E}\,\mathbf{w} \tag{5}$$

where

$$\mathbf{Z} = \begin{Bmatrix} \dot{\mathbf{X}}_\mathbf{s} \\ \ddot{\mathbf{X}}_\mathbf{s} \end{Bmatrix}, \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_\mathbf{s}^{-1}\mathbf{K}_\mathbf{s} & -\mathbf{M}_\mathbf{s}^{-1}\mathbf{C}_\mathbf{s} \end{bmatrix}, \mathbf{E} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{M}_\mathbf{s}^{-1}\mathbf{e}_\mathbf{s} \end{Bmatrix}, \text{and } \mathbf{w} = \ddot{x}_g \tag{6a-d}$$

with the output equation

$$\mathbf{z} = \mathbf{C_z}\mathbf{Z} \tag{7}$$

It is to be noted that displacements, velocity, absolute acceleration or their combination may be included to the output vector $\mathbf{z}$.

The objective of the problem is to minimize $\mathbf{z}$, which can be reflected by H_2 norm of the system. It can be seen that the problem of passive control in this framework introduces a new interest in the design output, *i.e.*, the regulated output can be chosen in a flexible manner as the response quantities that we want to minimize. Therefore, the designer has the flexibility to choose the response to be minimized in the performance index. In this paper relative displacement of a structure relative to ground is taken as the regulated output.

The H_2 norm objective function in this paper can be cast as follows: obtain TMD location (j_d) and properties (c_d and k_d) in Eq. (5) with the regulated output according to Eq. (7) such that to minimize H_2 norm which can be obtained as (Doyle *et al.* [23], Lublin *et al.* [24])

$$J = \left[tr\left(\mathbf{C_z}\mathbf{L}_c\mathbf{C}_z^\mathrm{T}\right)\right]^{1/2} = \left[tr\left(\mathbf{E}^\mathrm{T}\mathbf{L_o}\mathbf{E}\right)\right]^{1/2} \tag{8}$$

Matrices $\mathbf{L}_c$ and $\mathbf{L_o}$ can be obtained by using Lyapunov equations:

$$\mathbf{A}\,\mathbf{L}_c + \mathbf{L}_c\mathbf{A}^\mathrm{T} + \mathbf{E}\,\mathbf{E}^\mathrm{T} = \mathbf{0} \tag{9a}$$

or

$$\mathbf{A}_\mathbf{l}^\mathrm{T}\mathbf{L_o} + \mathbf{L_o}\mathbf{A} + \mathbf{C_z^T}\mathbf{C_z} = \mathbf{0} \tag{9b}$$

4. DAMPER OPTIMIZATION

To optimize the TMD properties, Hadi and Arfiadi [14] utilized binary coded genetic algorithm (BCGA). In BCGA, individuals or variables are represented by string containing 0 and 1. Since BCGA was used the range of the properties of the TMD should be supplied by designers. This may be considered as drawback of using BCGA. When the designers do not have experience concerning the range of the design values, the procedure may fail to obtain the optimum values. The application of BCGA in optimization of size and placement of linear viscous dampers has been used also by Ashahina *et al.* [25] to two and three dimensional structures. The use of distributed GA (DGA), where the population of binary GA is divided into several populations, is used by Mohebbi and Ghanbarpour [26] in order to optimize multi TMDs. Arfiadi [27] used BCGA to obtain optimum placement of damper, where the properties of dampers have been decided from the manufacture.

In this paper, optimum placement and properties of TMD is carried out at the same time by using hybrid coded genetic algorithm (HCGA). The TMD location as a discrete point is best suited to be optimized by using BCGA, while the damper properties as real variables are optimized by using RCGA. The optimization is run in a single program as hybrid optimization for hybrid design variables.

For the optimum placement of damper, the location of the damper is represented by binary string containing 0 and 1 as discrete variable. For particular individual the representation is depicted in Figure 1. A string is converted to integer by using (Michalewicz [20]):

$$t_i = \sum_{j=o}^{r} h_j \times 2^j \tag{10}$$

where h_j = string-*j* from right (0 or 1), *r* = length of string – 1 as shown in Figure 2. Because this is a discrete problem, it is not necessary to convert the integer into a real number.
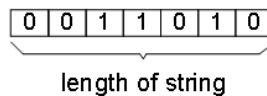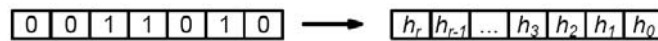

Figure 1. Representation of individual in BCGA

Figure 2. Transformation to integer

The individuals will experience mutation and crossover. Two parents selected for crossover resulted in new offspring as shown in Figure 3. The selection of the individuals for the crossover is done by using roulette wheel based procedure.
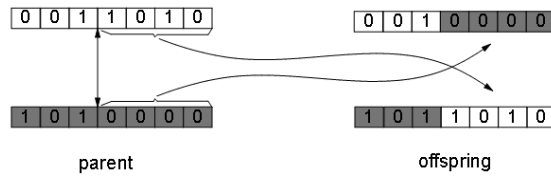
Figure 3. One point crossover for BCGA

An individual selected for mutation, the bit string is changed from to 0 and 1 depending on its initial bit string as shown in Figure 4.

Figure 4. Mutation for BCGA

For optimization of TMD properties, RCGA with real numbers are used directly. For example for an initial individual that has four design variables, four random numbers are generated as depicted in Figure 5.

Figure 5. Individual with four design variables in RCGA

Although there are many mutation and crossover procedures available, the crossover and mutation used in this paper are taken as follows.

For the individuals G_1 and G_2 taken for crossover, the resulting offspring $G_1^{'}$ and $G_2^{'}$ follow what is so called balanced crossover (Herrera *et al.* [28]) as follows (Figure 6) :

$$G_1^{'} = a\left(G_1 - G_2\right) + G_1 \tag{11a}$$

$$G_2^{'} = a\left(G_2 - G_1\right) + G_2 \tag{11b}$$

where a = random numbers between 0 and 1. It can be seen that for RCGA by using this crossover method the domain of interest for the optimization is not necessary to be known *a priori*, as the crossover has the ability to explore the unknown domain (Arfiadi and Hadi [21]). This is the capability of RCGA to explore the unknown domain of interest as opposed to BCGA. As can be seen below in the example, the designer might guess initial values for the design variables very arbitrarily without affecting the final design.
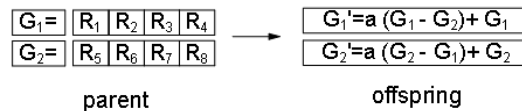

Figure 6. A balanced crossover for RCGA

For the mutation, the resulting design variable after simple mutation is (Figure 7):

$$\mathbf{G}_p' = \begin{bmatrix} R_1 & R_2 & \cdots & R_j' & \cdots & R_N \end{bmatrix} \tag{12a}$$

$$R_j' = \alpha\, a\, R_j \tag{12b}$$

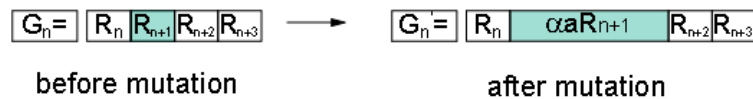where $\alpha > 1$, and a = random number between 0 and 1.

Figure 7. A simple mutation

Note that in the GA used in this paper elitist strategy was utilized, where the best fit individuals always survive and pass into the next generation (Grefenstette [29]). In addition in each generation, some new born random individuals are always inserted in the population replacing random old individuals (Arfiadi and Hadi [21]). The flowchart for the HCGA is depicted in Figure 8.
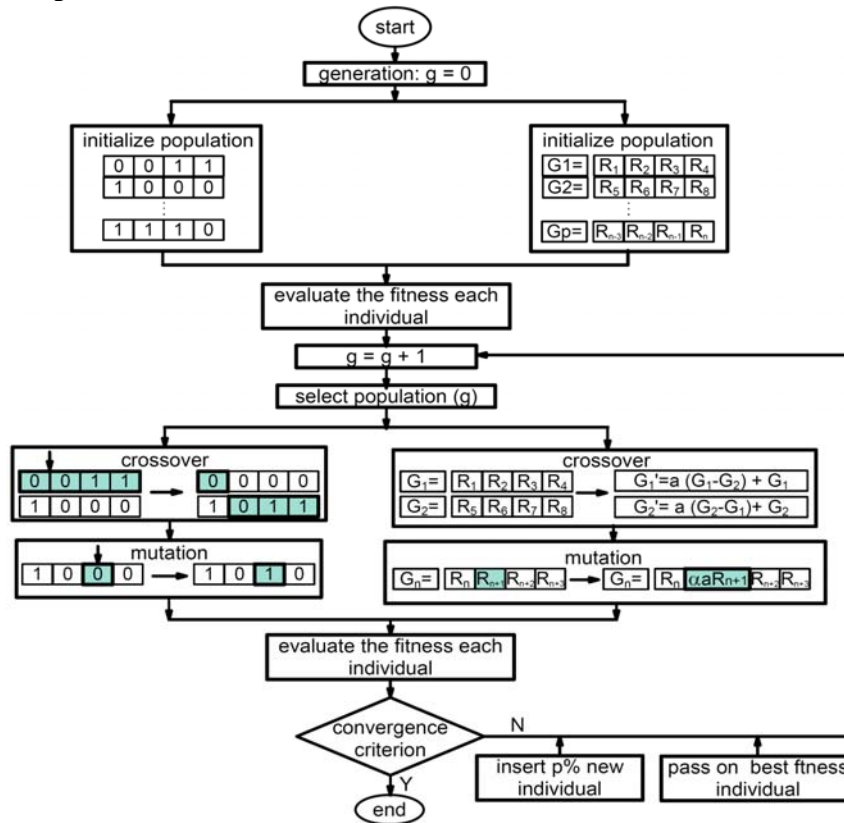

Figure 8. Flowchart of HCGA to be used in this paper

5. APPLICATION

Consider a 10 storey building as shown in Figure 9. Building is assumed to be a shear building, where the properties are shown in Table 1. A stiffness proportional damping is assumed for the structure with the damping ratio of the fundamental mode equals to 2 %. The TMD is used to reduce vibration of the structure. The location and properties of TMD are optimized using HCGA. The location of TMD is represented by BCGA while the properties are suitable to be represented by RCGA. A combination of them in one run formed HCGA. The mass of TMD is taken as 115 t which is about 2 % of the total mass of building.

Table 1. Structural data

Floor	Mass (t)	Stiffness (kN/m)
10	489.3	367187.5
9	535.3	367187.5
8	535.3	367187.5
7	548.8	1048724.2
6	562.3	1048724.2
5	562.3	1048724.2
4	562.3	1048724.2
3	567.6	1410587.5
2	572.9	1410587.5
1	572.9	1410587.5

The properties of HCGA is taken as follows:

maximum generation = 400, population size = 30, probability of crossover = 0.45, probability of mutation = 0.1, the new born individual inserted in each generation = 5% of the total population.

The objective is to determine the optimum value of the stiffness k_d and the damping c_d of the damper that minimizes the H_2 norm transfer function from the external disturbance to the regulated output. Relative displacements of the structure with respect to the ground are taken as regulated outputs such that

$$\mathbf{z} = \begin{bmatrix} \mathbf{I}_{10 \times 10} & \mathbf{0}_{10 \times 11} \end{bmatrix} \mathbf{Z}$$

(13)

where $\mathbf{I}$ = identity matrix and $\mathbf{0}$ = matrix containing zeros.

Because GA tries to maximize the fitness, the performance index used in the program is modified as:

$$J = C_w \times \frac{1}{\text{norm } H_2} \tag{14}$$

where C_w = constant to scale the objective function. In this example $C_w = 10$. As the exploration in RCGA is unlimited to a particular boundary, it is possible that after mutation and crossover, the stiffness and damping of TMD have negative values. To reduce HCGA in exploring the unwanted domain, the resulting objective function is penalized by assigning the performance index to a smallest value that can be accepted by the program, when the resulting properties of the dampers have negative values.

The location of the damper is represented by a binary string that can represent between 1 and 10, while initial values of k_d and c_d can be taken arbitrary. To optimize TMD, four runs were carried out. In every run different starting (initial) values of k_d and c_d were taken as shown in Table 2. After 400 generations, the evolving best fitness individual in each run is depicted in Figure 10.

Table 2. Initial k_d and c_d in each run

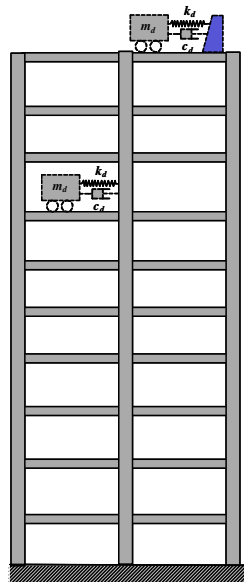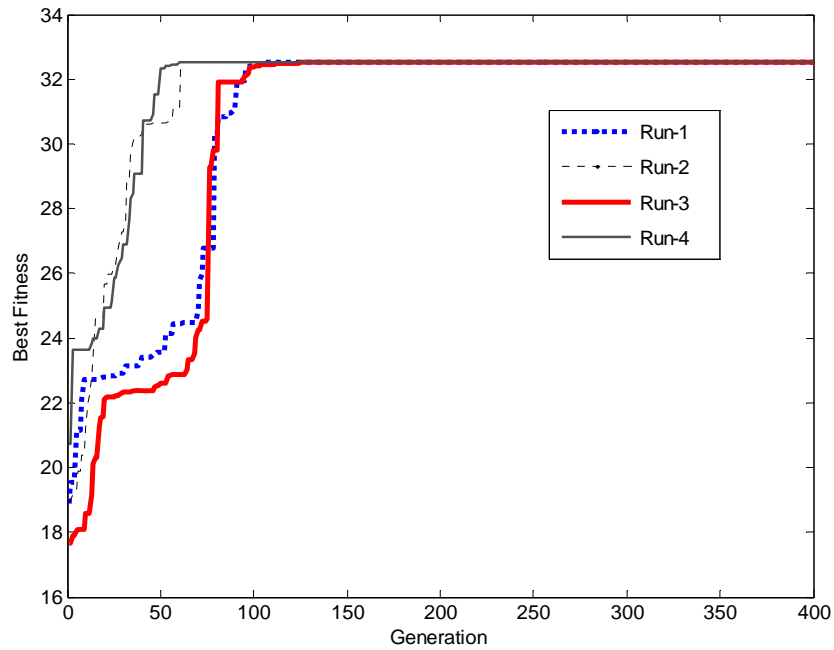| Run	Lower bound	Upper bound
1	0	100
2	90	100
3	12	20
4	0	10000

Figure 9. A 10 storey building

Figure 10. Evolving best fitness individual in each run

The resulting of optimization by using HCGA are: c_d = 175.033 kN-s/m, k_d = 4540.369 kN/m, and location of TMD is at the 10th floor.

It can be seen from the simulation that although different starting values of damper properties were used, HCGA was able to obtain the same results. This shows also that the HCGA used in this paper is a stable optimization tool. In addition although the initial domain of optimum value is located very far from the domain of interest, the real coded GA is able to explore such an unknown domain. This can be seen for runs 1 to 3 that start with the initial design variables which are far from the design variables. Although the initial domain of interest is outside of the domain of design variables, HCGA converge to almost the same results. This may not be achieved if we use BCGA, as it has to be supported by the initial guess that has to be in the range of the domain of interest of design variables as shown in Hadi and Arfiadi [14].

In order to assess the stability of the HCGA, a simulation was carried out when the damper location is set at floor 10. RCGA similar to Arfiadi and Hadi [21] was then used to obtain the optimum value of the properties of TMD. In this optimization the fitness scale C_w is taken as unity, while the initial guess of TMD properties are taken as the same as in the previous simulation. The evolving best fitness is shown in Figure 11. The results of simulation after 400 generations results in the same TMD properties, although the history of best fitness is slightly different.
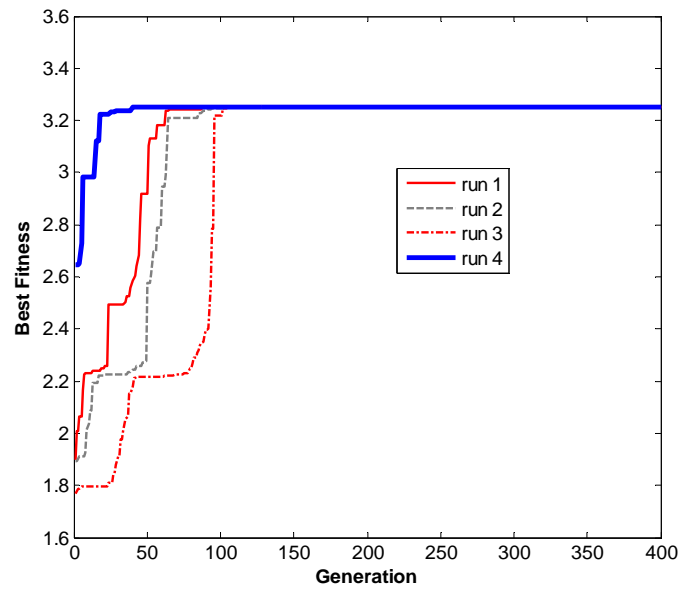
Figure 11. Evolving best fitness when TMD is located at the 10th floor $C_w = 1$

To see the effectiveness of TMD in reducing vibration due to earthquake, the structure is simulated subjected to El Centro 1940 NS, Kobe 1995, Hachinohe 1968 and Northridge 1994 ground accelerations. The results of simulation are shown in Figures 12 to 15 for the time history responses subject to El Centro 1940 NS, Kobe 1995, Hachinohe 1968 and Northridge 1994 ground accelerations, respectively, while the peak displacements in each floor can be seen in Figures 16 to 19.
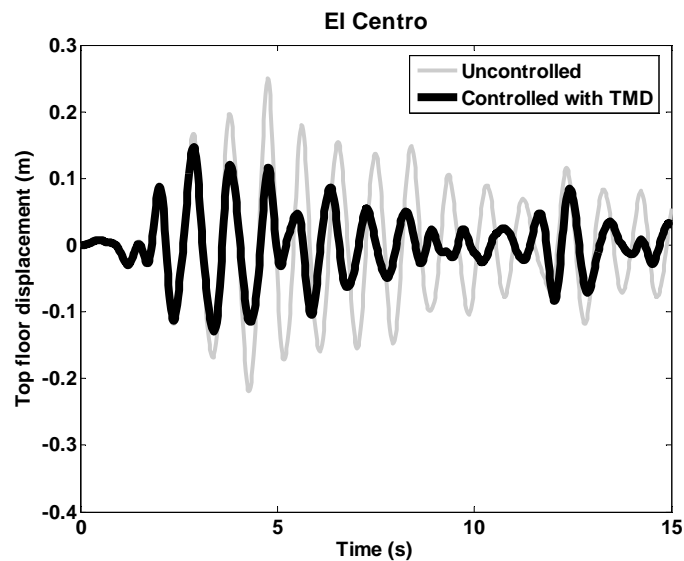

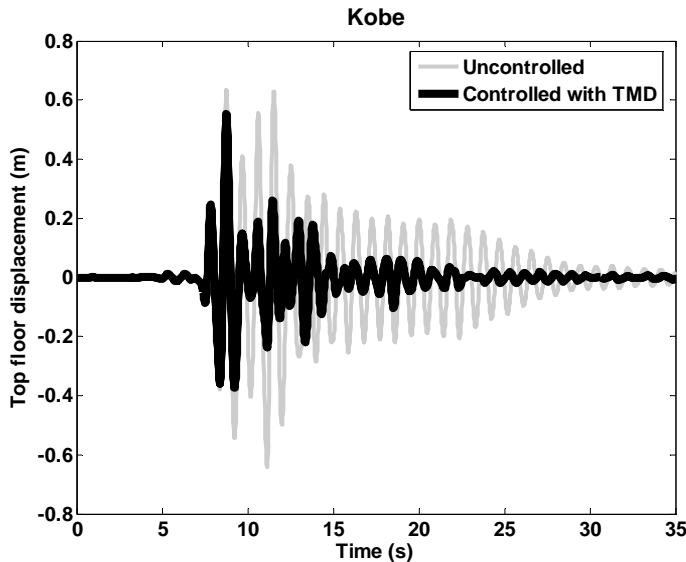Figure 12. Time history response of top floor due to El Centro earthquake

Kobe

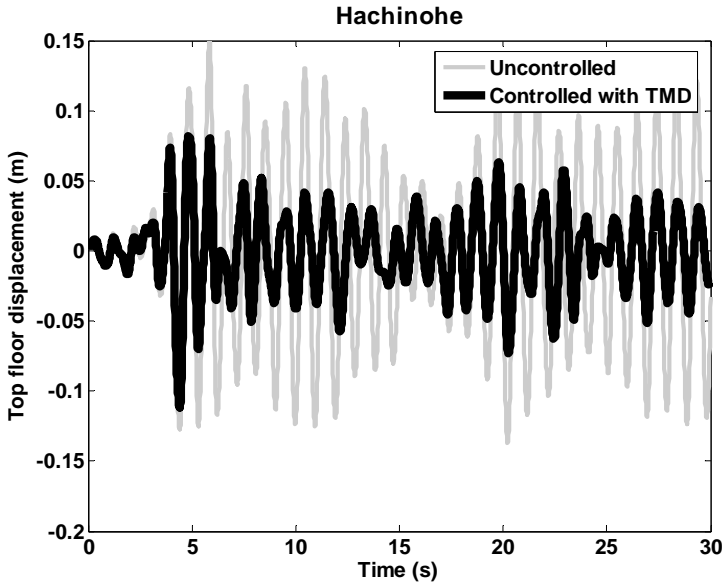Figure 13. Time history response of top floor due to Kobe earthquake

Hachinohe

Figure 14. Time history response of top floor due to Hachinohe earthquake
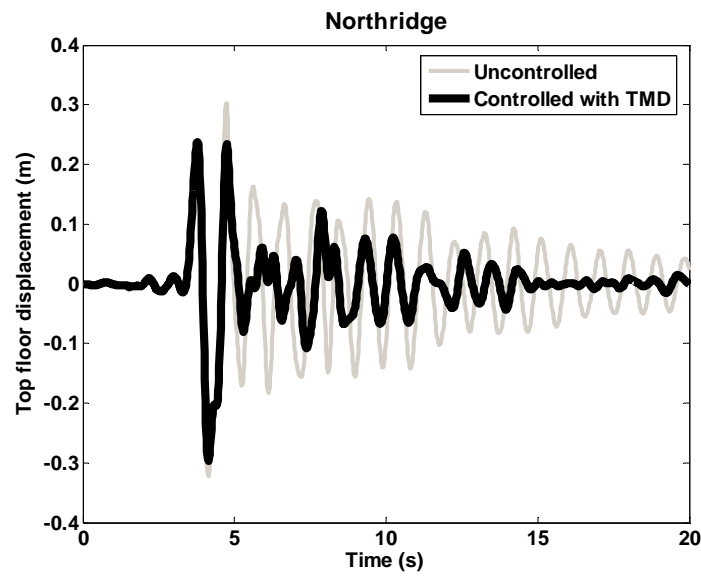
Figure 15. Time history response of top floor due to Northridge earthquake
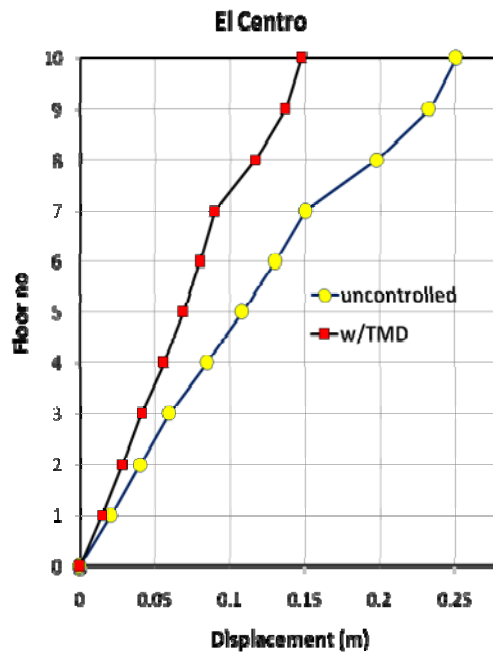

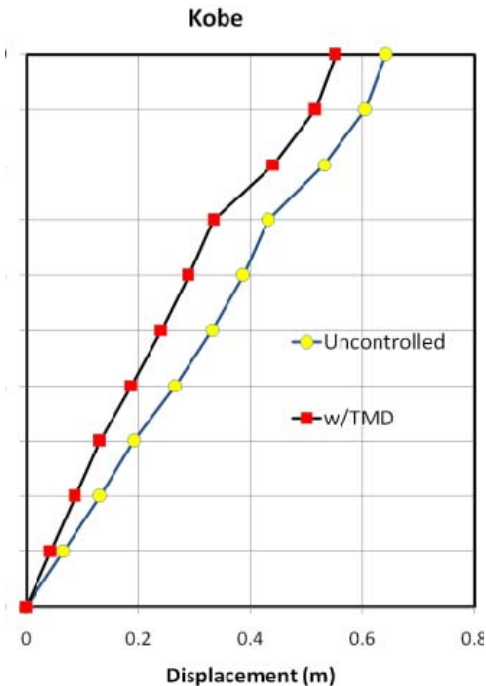Figure 16. Peak displacement due to El Centro earthquake

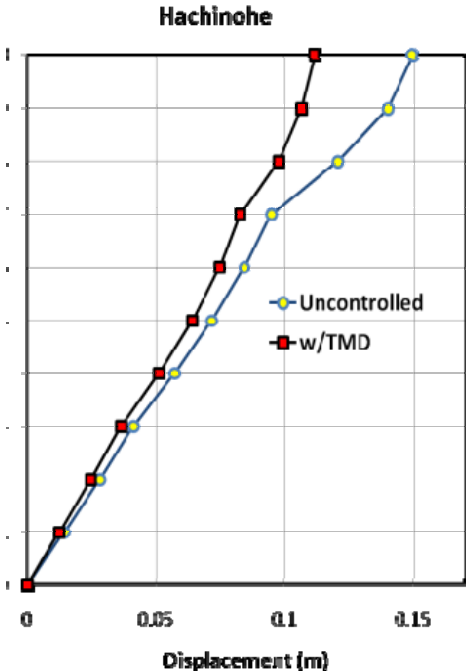Figure 17. Peak displacement due to Kobe earthquake

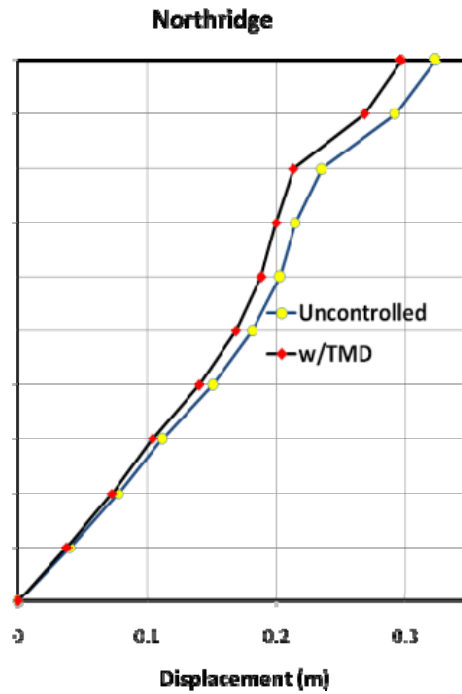Figure 18. Peak displacement due to Hachinohe earthquake

Figure 19. Peak displacement due to Northridge earthquake

6. COMPARISON WITH OTHER METHODS

The comparison result with Den Hartog [30], Warbuton [6] and Sadek *et al.* [13] were calculated. In Den Hartog and Warburton methods, the structure is converted to a single degree of freedom system then damper parameters are computed.

The formula of Den Hartog [30] was based on the SDOF undamped structure with harmonic external load. According to Den Hartog the optimum tuning frequency (α_{opt} = $\omega_{TMD}/\omega_{structure}$) can be expressed as:

$$\alpha_{opt} = \frac{1}{1+\mu} \qquad (15)$$

whereas the optimum damping ratio of the damper ξ_{dopt} is formulated as

$$\xi_{dopt} = \sqrt{\frac{3\mu}{8(1+\mu)}} \qquad (16)$$

μ is the mass ratio of damper.

To use the formula, the MDOF structure is then converted to SDOF structure following procedure in Soong and Dargush [1] by normalizing the mode shape at the location of TMD to be 1 unit. From the computation the resulting first mode is:

$$\phi_1^T = \begin{bmatrix} 0.0917 & 0.1818 & 0.2686 & 0.3788 & 0.48 & 0.5697 & 0.6457 & 0.8198 & 0.9405 & 1 \end{bmatrix}$$

The first modal mass:

$$M_1 = \phi_1^T \mathbf{M} \phi_1 = 208.8 \text{ t}$$

The mass ratio:

$$\mu = \frac{m_d}{M_1} = \frac{115}{208.8} = 0.0572$$

The optimum frequency ratio from Eq. (15):

$$\alpha_{opt} = \frac{1}{1+\mu} = 0.9459$$

From which we can obtain

$$\omega_d = \alpha_{opt}\omega_1 = 6.3202 \text{ rad/s}$$

and

$$k_d = m_d\omega_d^2 = 4593.6 \text{ kN/m}$$

From Eq. (16):

$$\xi_{dopt} = \sqrt{\frac{3\mu}{8(1+\mu)}} = 0.1425$$

such that

$$c_d = 2m_d\omega_d\xi_d = 207.14 \text{ kN-s/m}.$$

Another approach to be compared here is according to Warburton [6] where several design formulas have been derived for the optimum design of the absorber attached on SDOF undamped structures. To facilitate comparisons, the formula based on the white-noise excitation was taken. Based on this design, the optimum tuning frequency of the damper is formulated as:

$$\alpha_{opt} = \frac{1}{1+\mu}\sqrt{1-\mu/2} \tag{17}$$

and the optimum damping ratio of the damper is formulated as:

$$\xi_{dopt} = \sqrt{\frac{\mu(1-\mu/4)}{4(1+\mu)(1-\mu/2)}} \tag{18}$$

Similar to the previous approach the equivalent SDOF model was then determined and used to find the optimum parameters of TMD. From Eqs. (17) and (18) we obtain:

$$\alpha_{opt} = 0.9322 \text{ and } \xi_{dopt} = 0.1172$$

from which we obtain:

$$k_d = 4462.1 \text{ kN/m and } c_d = 167.91 \text{kN-s/m}$$

In Sadek *et al.* [13] the optimization for MDOF structures are taken based on eqs.

$$\alpha_{opt} = \frac{1}{1+\mu\phi_{ij}}\left(1-\xi\sqrt{\frac{\mu\phi_{ij}}{1+\mu\phi_{ij}}}\right) \tag{19}$$

$$\xi_{dopt} = \phi_{ij}\left(\frac{\xi}{1+\mu} + \sqrt{\frac{\mu}{1+\mu}}\right) \tag{20}$$

In Eqs. (19) and (20) the mode shapes are modified such that to have a unit participation factor. In this case the mode shape becomes:

$$\phi_1^T = \begin{bmatrix} 0.1324 & 0.2623 & 0.3876 & 0.5466 & 0.6926 & 0.822 & 0.9317 & 1.1828 & 1.357 & 1.4428 \end{bmatrix}$$

The first modal mass becomes:

$$M_1 = \phi_1^T \mathbf{M}\phi_1 = 4181.9 \text{ t}$$

The mass ratio is:

$$\mu = \frac{m_d}{M_1} = \frac{115}{4181.9} = 0.0275$$

The optimum value from Eq. (19):

$$\alpha_{opt} = \frac{1}{1+\mu\phi_{ij}}\left(1-\xi\sqrt{\frac{\mu\phi_{ij}}{1+\mu\phi_{ij}}}\right) = 0.95808$$

where ϕ_{ij} = amplitude of the *i*th mode vibration being considered for a unit participation factor computed at the location of the TMD (*j*th degree-of-freedom) =1.4428.

From Eq. (20):

$$\xi_{dopt} = \phi_{ij}\left(\frac{\xi}{1+\mu} + \sqrt{\frac{\mu}{1+\mu}}\right) = 0.2641$$

From which we obtain the stiffness and damping of TMD, respectively, as:

$$k_d = 4713.2 \text{ kN/m} \text{ and } c_d = 388.91 \text{kN-s/m}$$

The results of present approach, Den Hartog, Warburton and Sadek *et al.* are presented in Table 3.

To gain the comparison, the peak displacement due to El Centro 1940 NS excitation is plotted in Figure 20. From these results it can be shown that the reduction in peak response of the present approach is almost similar to the one in Den Hartog and Waburton approach. Note also that the results of present approach out-perform of Sadek approach. However, compared to those methods, the method developed in this paper has the flexibility as it can optimize not only the properties but also the placement of TMD which might not be possible
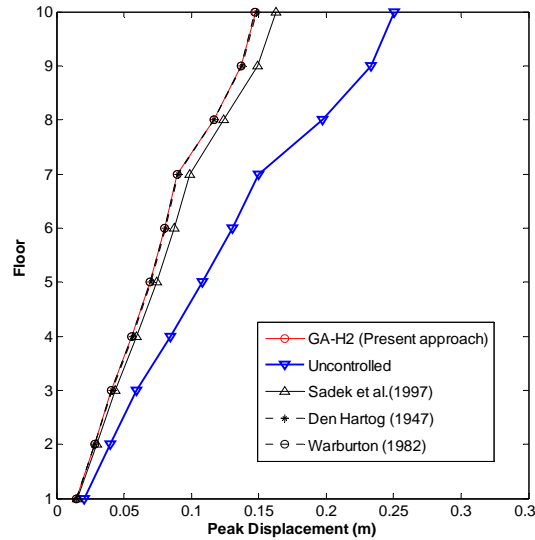
in those methods.

Figure 20. Comparison of peak displacements

Table 3. Optimum values of TMD parameters

Parameters	HCGA or RCGA (Present approach)	Den Hartog [30]	Warburton [6]	Sadek *et al.* [13]
k_d (kN/m)	4540.4	4593.6	4462.1	4713.2
c_d (kN-s/m)	175.0	207.14	167.91	388.91
ω_d (rad/s)	6.283	6.3202	6.2291	6.4019
ξ_d	0.1211	0.1425	0.1172	0.2641

7. CONCLUSIONS

Hybrid coded genetic algorithm has been used to optimize placement and properties of TMD. The location of TMD is represented by binary coded genetic algorithm while properties of TMD are represented by real coded genetic algorithm. The hybrid coded genetic algorithm is capable of obtaining the TMD location and properties. Four runs have been conducted for the optimization. It can be seen that in each run the program end up with the optimum result, although the initial values of design variable are taken arbitrary. This shows that real coded genetic algorithms are suitable where the designer does not have an experience on how the expected optimum value of design variables. It is to be noted that real coded GA will explore the unknown domain of interest which is very suitable for the optimization. The simulation results show that the TMD are able to reduce the structural vibration. If more reduction is expected active TMD may be used.

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