IC/99/116

United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency
THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

EXCITATION OF BREATHER (BION) IN SUPERLATTICE

S.Y. Mensah[1]
Department of Physics, Laser and Fibre Optics Centre,
University of Cape Coast, Cape Coast, Ghana
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy,

F.K.A. Allotey
Atomic Energy Commission, Kwabenya, Ghana
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy

and

N.G. Mensah
Department of Mathematics, University of Cape Coast, Cape Coast, Ghana.

MIRAMARE – TRIESTE
September 1999

[1]Regular Associate of the Abdus Salam ICTP.

Abstract

Soliton breather excitation in superlattice has been studied in this paper. It is observed that under certain conditions, the vector potential equation for the electromagnetic wave propagating through the superlattice assumes the sine-Gordon(sG) equation. The solution of which does not give only a soliton but also a soliton breather. The binding energy of the breather is calculated to be

$$E_b = 16\gamma(1 - \sin\nu), \quad \gamma = (1 - \frac{u^2}{v_0^2})^{-1/2}$$

where u is the velocity of the breather and v_0 is the velocity of the electromagnetic wave in the absence of electrons. As can be seen, when $\nu \to \frac{\pi}{2}$ the binding energy tends to zero, hence, the breather disintegrates into a soliton and antisoliton. It was further observed that the binding energy decreases with an increase in Δ (the half miniband width) for a given value of d (SL period). Similarly it also decreases with increase in d for a given value of Δ. Comparing the breather's rest energy E_b to that of soliton E_s i.e $E_b = 2E_s \sin\nu$. We noted that the breather's rest energy is less than that required to excite a soliton.

1 Introduction

The nonlinear interaction of quantum periodic semiconductor structures-quantum superlattices with electromagnetic radiation is an important direction of research in modern quantum electronics. This is as a result of latest technological advances made in the preparation of extremely high-quality periodic structures demonstrating the very strong nonlinear properties in the millimeter, submillimeter, infrared and optical electromagnetic field[1-4]. The presence of additional periodic potential (a superlattice potential) due to, for example, the periodic disribution of the chemical composition of the semiconductor gives rise to the subdivision of the energy spectrum of the electron carriers into a series of narrow (usually less than $0.1eV$) allowed and forbidden minibands. The nonlinearity of the superlattice is due to the fact that the electron energy in a miniband is bounded, which gives rise to an oscillatory motion of electrons in a magnetic field on one hand, and a strongly nonparabolic dispersion law, on the other hand, a consequence of which is an N shape current-voltage characteristic[5,6], dynamic localization of electrons (self induced transparency)[7] and absolute negative conductivity[8].

The miniband nature of the energy spectrum of the carrier electrons in SL facilitates the propagation of large amplitude nonlinear electromagnetic fields, particularly those exhibiting solitary wave solutions. The nonlinear fields e.g. the soliton, are now of great interest in many physical problems[9]. Free-field solitons have been investigated extensively namely for quantum fields[10], but in condensed matter we require knowledge of the behavior and integrity of solitons in the presence of "impurities" or applied fields. Kink solitons, or domain walls, occur in magnetic[11] and ferrodistortive[12] materials and in many Landau-Ginsburg expansion contexts[13].

The very existence of solitons is due to the fact that, under certain conditions, the equation for the vector potential **A** in a superlattice reduces to sine-Gordon (sG) equation. The sG equation has not only the soliton solution but also a solution in the form of a breather which can be interpreted as bound states of solitons and antisolitons.The solution of sG equation has deeply influenced our understanding of various condensed matter phenomena, notably among which are the charge transfers in quasi 1-D conductors [13,14], flow of flux quanta in Josephson junction [15] and in superlattice[16,17].

In this paper, we will study the excitation of breather in SL. The motivation was driven, primarily, by the promise of massively increased bit rates, through the application of ultrashort soliton pulses in long distance optical communication networks. Secondly, the interaction of ultrashort electromagnetic pulses with matter has recently attracted considerable research interest as a result of progress in laser physics, in the production of light pulses with τ_p as short as one oscillation period[18]. It will be shown that the energy needed to excite the breather may be less than that required to excite soliton. This is attributed to the large binding energy of the breather. The dependence of Δ and d on the binding energy of the breather will also be discussed.

The paper will be organised as follows: Section two, we briefly review the basic equations; section three, the derived sine-Gordon equation is solved and finally in section four, the results are discussed and conclusions made.

2 The sine-Gordon EM field equation of the superlattice

The exceptional peculiarities of nonlinear equations are impossible to study using linearisation procedures, even with a subsequent of small nonlinearisation using the perturbation theory based on an expansion in the normal linear modes. In this section we shall show how nonlinearity in SLs leads essentially to the sine-Gordon equation.

Proceeding as in [17], we assume that the characteristic length in which a significant change in the EM field is large enough compared with the de Broglie wavelength of the electrons or with the superlattice period. Therefore the electron current density can be written as

$$j = -e\sum_{p} f(p)v(p+\frac{e}{c}A(r,t)) \tag{1}$$

where $f(p)$ is the distribution function of the elctron canonical momentum p , $v(p)$ is the electron velocity, e the electron charge and $A(r,t)$ the vector potential. The key physical parameter describing the electron distribution in the bands is the dispersion relation, for superlattices the following dispersion law is most often considered [16,17]:

$$\epsilon_\nu(p) = \frac{p_\perp^2}{2m} + \epsilon_\nu - \Delta_\nu \cos(\frac{p_z}{\hbar}d) \tag{2}$$

In eqn.(2), $p_\perp$ and p_z are the transverse and longitudinal(relative to the superlattice axis) components of the quasi momentum, respectively, Δ_ν is the half width of the ν^{th} allowed miniband,

$$\epsilon_\nu = \frac{\hbar^2}{2m}(\frac{\pi}{d_o})^2\nu^2 \tag{3}$$

are the size-quantized levels in an isolated conduction film, $d = d_o + d_1$ (d_o is the width of the rectangular potentail wells and d_1 is the potential depth with a non zero quantum transparency) is the superlattice period.

We assume that electrons are confined to the lowest conduction miniband ($\nu = 1$) ans omit the miniband indices. This is to say that the field does not induce transitions between the filled and empty minibands. We further assume that the characteristic time for change in the field is short compared with the mean free time of electrons τ. We therefore ignore the collision of electrons with the lattice. The electron velocity is given by

$$v_z(p) = \frac{\partial\epsilon(p)}{\partial p_z} = \frac{\Delta d}{\hbar}\sin(\frac{p_z d}{\hbar}) \tag{4}$$

Substituting eqn. (4) in eqn.(1) and making the following transformation $p_z \to p_{oz} + \frac{e}{c}A_z$, we obtain for the non-degenerate electron gas the following expression for the z component of the current density j,

$$j_z = j_o \sin(\frac{e}{\hbar c} A_z d) \tag{5}$$

where

$$j_o = -\frac{e\Delta d}{\hbar} \sum_p f(p) \cos p_{oz} d = -n \frac{e\Delta d}{\hbar} \frac{I_1(\Delta/kT)}{I_o(\Delta/kT)} \tag{6}$$

With n the conduction electron density and $I_k(x)$ the Bessel function of imaginary argument. We evoke the classical Maxwell equation for the vector potential, i.e.,

$$\Delta A - \frac{1}{v_o^2} \frac{\partial^2 A}{\partial t^2} = -\frac{4\pi}{v_o} j \tag{7}$$

and substitute j_z from (5) to obtain the nonlinear field equation:

$$\frac{\partial^2 \phi}{\partial t^2} - v_o^2 \Delta \phi + \omega_o^2 \sin \phi = 0 \tag{8}$$

where

$$\phi = \frac{e}{\hbar c} A_z d, \; \omega_o^2 = \frac{m \omega_p^2 d^2 \Delta I_1(\Delta/kT)}{\hbar^2 I_o(\Delta/kT)} \tag{9}$$

Here v_o is the EM velocity in the absence of electrons and $\omega_p^2 = \frac{4\pi n}{m} e^2$ is the square of the Langmuir frequency.
Equation (8) is quite frequent in the literature of nonlinear processes where it is called sine-Gordon equation.

3 Solution of sine-Gordon Equation

We can transform eqn.(7) into a dimensionless equation by making the following transformation;

$$\xi = \frac{\omega_o}{v_o} x \quad and \quad \tau = \omega_o t \tag{10}$$

the result of which gives,

$$\frac{\partial^2 \psi}{\partial \tau^2} - \frac{\partial^2 \psi}{\partial \xi^2} + \sin \psi = 0 \tag{11}$$

By using the method proposed by Lamb [18] we seek the solution in the form,

$$\psi(\xi, \tau) = \tan \frac{1}{4} \phi \tag{12}$$

$$\psi(\xi, \tau) = \frac{f(\xi)}{g(\tau)} \tag{13}$$

With the help of eqn(12) we reduce eqn.(11) into the form,

$$(1 + \psi^2)(\psi_{\tau\tau} - \psi_{\xi\xi} + \psi) - 2\psi(\psi_\tau^2 - \psi_\xi^2 + \psi) = 0 \tag{14}$$

Here we use the trigonometry identity

$$\sin \phi = \frac{4\psi(1 - \psi^2)}{(1 + \psi^2)^2} \tag{15}$$

We can then seek to separate the variables of eqn.(14) by using the trial eqn.(13). This gives the following equation,

$$(f^2 + g^2) \left(\frac{f''}{f} + \frac{\ddot{g}}{g} \right) + f^2 - g^2 = 2(f'^2 + \dot{g}^2) \tag{16}$$

where $f' = \frac{\partial f}{\partial \xi}$ and $\dot{g} = \frac{\partial g}{\partial \tau}$
Differentiating eqn(16) with respect ξ and τ we get;

$$\left(g^2\right)^{\cdot}\left(\frac{f''}{f}\right)' + (f^2)'\left(\frac{\ddot{g}}{g}\right)^{\cdot} = 0 \tag{17}$$

for all ξ and τ.
Hence

$$\frac{1}{(f^2)'}\left(\frac{f''}{f}\right)' = -\frac{1}{(g^2)^{\cdot}}\left(\frac{\ddot{g}}{g}\right)^{\cdot} = 2\mu \tag{18}$$

here μ is the separation constant. After integrating eqn.(18) twice we get;

$$\frac{1}{2}f'^2 = \frac{1}{2}\mu f^4 + \frac{1}{2}c_1 f^2 + c_2 \tag{19}$$

$$\frac{1}{2}\dot{g}^2 = -\frac{1}{2}\mu g^4 + \frac{1}{2}d_1 g^2 + d_2 \tag{20}$$

where c_1, c_2, d_1 and d_2 are integration constants. Substituting eqns.(19) and (20) into eqn.(16) shows that $c_1 - d_1 = 1$ and $c_2 - d_2 = 0$. Therefore

$$f'^2 = \mu f^4 + (1+\lambda)f^2 + \theta \tag{21}$$

$$\dot{g}^2 = -\mu g^4 + \lambda g^2 - \theta \tag{22}$$

When $\mu = \theta = 0$ we obtain asingle phase solution, i.e. a soliton solution. Therefore eqns.(21) and (22) become;

$$f'^2 = (1+\lambda)f^2 \tag{23}$$

$$\dot{g}^2 = \lambda g^2, \tag{24}$$

the solutions of which are;

$$g = \exp(\eta\sqrt{\lambda}(\tau - \tau_o)) \tag{25}$$

$$f = \exp(\eta\sqrt{1+\lambda}(\xi - \xi_o) \tag{26}$$

Hence from eqns.(12) and (13) we obtain;

$$\phi(\xi, \tau) = 4\arctan\left\{\exp\left[\eta\gamma\left((\eta - \eta_o) - \alpha\tau\right)\right]\right\} \tag{27}$$

where
$\gamma = \sqrt{1+\lambda} = \frac{1}{\sqrt{1-\alpha^2}}$ and $0 \leq \alpha < 1$; $\alpha = \frac{u}{v_o}$; $\eta = \pm 1$.
The solution of eqn.(27) characterises a soliton kink for $\eta = 1$ and antisoliton for $\eta = -1$.

Substituting $\xi = \frac{\omega_o}{v_o}x$ and $\tau = \omega_o t$ into eqn.(27) we obtain;

$$\phi(x, t) = 4\arctan\left\{\exp\left(\eta\gamma\frac{\omega_o}{v_o}\left((x - x_o) - ut\right)\right)\right\} \tag{28}$$

Eqns.(21) and (22) can also be solved for $\lambda \neq 0$ and $\mu \neq 0$, the result of which becomes,

$$f'^2 = (1+\lambda)f^2 + \theta \tag{29}$$

$$\dot{g}^2 = \lambda g^2 - \theta \tag{30}$$

For $\lambda > 0$ the solutions for eqns.(29) and (30) are

$$f(\xi) = \eta\sqrt{\frac{\theta}{1+\lambda}} \sinh\left[\sqrt{1+\lambda}(\xi - \xi_o)\right] \tag{31}$$

$$g(\tau) = \eta\sqrt{\frac{\theta}{\lambda}} \cosh\left[\sqrt{\lambda}(\tau - \tau_o)\right] \tag{32}$$

We put $\xi_o = 0$ and $\tau_o = 0$ without loss of generality. Hence,

$$\phi(\xi,\tau) = 4\arctan\left[\frac{\alpha\sinh(\gamma\xi)}{\cosh(\alpha\gamma\tau)}\right]. \tag{33}$$

Eqn.(33) describes the interaction of two solitons moving with velocities u in opposite directions.

When $-1 < \lambda < 0$ and $\theta < 0$ the solution of eqns.(29) and(30) gives;

$$\begin{aligned} \phi(\xi,\tau) &= 4\arctan\left[\frac{\sin\left\{\Omega\tau/(1+\Omega^2)^{\frac{1}{2}}\right\}}{\Omega\cosh\left\{\xi/(1+\Omega^2)^{\frac{1}{2}}\right\}}\right] = \\ &= 4\arctan\left[\frac{\tan\nu\sin(\tau\cos\nu)}{\cosh(\xi\sin\nu)}\right] \end{aligned} \tag{34}$$

where $\Omega = 1/\tan\nu$.

The solution (34) represents a localised pulsating object-the bound state of a soliton and antisoliton called a breather (bion). The breather type solution satisfies the boundary condition $\phi(\xi,\tau) \to 0$ as $|\xi| \to \infty$. Hence, it is sometimes called $O\pi$-pulse. The internal frequency ω of pulsation of a localised breather is given by;

$$\omega = \omega_o \cos\nu \tag{35}$$

The spatial spread of the pulsing region is inversely proportional to the following value;

$$Q = \sin\nu = 1/(1+\Omega^2)^{\frac{1}{2}} = \left(1 - \frac{\omega^2}{\omega_o^2}\right)^{\frac{1}{2}} \tag{36}$$

The transition of the motionless breather to the breather which moves as a whole with a velocity u, is realized by the Lorentz transformation.

$$\tau \to \gamma(\xi - \frac{u}{v_0}\tau), \gamma \equiv 1/(1 - \frac{u^2}{v_0^2})^{\frac{1}{2}}; \xi \to \gamma(\xi - \frac{u}{v_0}\tau)$$

so the breather, moving with a velocity u is described by the following function

$$\psi(\xi,\tau) = 4\arctan\left[\frac{\tan v sin[(\tau - \frac{u}{v_0}\xi)\gamma\cos v]}{\cosh[\gamma(\xi - \frac{u}{v_0}\tau)\sin v]}\right] \tag{37}$$

Hence the frequency of pulsations in moving breathers is given by

$$\omega_B = \gamma\omega_0\cos\nu \tag{38}$$

4 Discussion

We have studied the excitation of a soliton and a breather (bion) in SL. We did so by solving for the current j_2 along the superlattice axis in the absence of scattering and evoked the Maxwell's equation for the eloctromagnetic wave. We obtained the usual sine-Gordon equation. The most popular version is the ID sG equation whose solutions are the single and double soliton solutions i.e kink (soliton) and breather (bion). The stability of the soliton solution of the (sG) equation is demonstrated in [19].

The soliton solution is usually obtained in the form

$$\phi(x,t) = 4\arctan\left\{exp\left[\pm\left(\frac{x-ut}{l(u)}\right)\right]\right\} \tag{39}$$

where, $l^2(u) = l_0^2\gamma$ and $l_0^2 = \frac{u_0^2}{\omega_0^2}$. This describes a smoothed out function with values $\phi_0 \to \pm\pi$ as $x \to \pm\infty$. The quantity l refers to the spatial extension of the kink spreading. The dependence of l on the kink translational velocity u is suggestive of its "particle-like" or soliton features. $\phi(x,t)$ is a soliton travelling perpendicular to the SL axis (along the x direction) at a constant velocity u. It is easy to express eqn.(39) in terms of an electric field E_z which can be written as

$$E_z = E_0 sech\left(\frac{(x-ut)}{l(u)}\right) \tag{40}$$

where E_0 is the amplitude. The velocity u and the spatial spread (l) of the soliton can be expressed in terms of the amplitude by

$$l = \frac{v_0}{\omega_0\sqrt{1+\beta^2}}, u = \frac{\beta v}{\sqrt{1+\beta^2}}$$

where $\beta = \frac{eE_0 d}{2\hbar\omega_0}$. An increase in the amplitude causes the soliton width to approach zero and its velocity to approach the velocity of light v_0 in a homogeneous medium.

The amplitude, width, and velocity of the soliton can be combined in an expression

$$\frac{u}{lE_0} = \frac{ed}{2\hbar}$$

which depends on the SL period. [17]

Equations (37) and (38) describe a bion with velocity u and frequency of pulsation $\omega_B = \gamma\omega_0 \cos v$ and a soliton. The parameters of SL appear in the frequency of the bion through ω_0. It is observed that ω_B increases as d and Δ increases and vise versa. Hence the frequency of the bion can be regulated by changing Δ and d.

The total energy and momentum of solitons and bions were calculated following the approach in [20]. The energy of the breather (bion) is expressed as;

$$E_B = 16\gamma \sin\nu = 16\gamma\left(1 - \frac{\omega_B^2}{\omega_0^2}\right)^{\frac{1}{2}}$$

and that of momentum as;

$$p_B = 16u\gamma \sin \nu$$

It is worthy to note that the calculation is done by taking $v_0 = 1$. The energy of the soliton is $E_s = 8\gamma$[19]. The rest energy of the breather E_B is expressed in terms of E_s as $E_B = 2E_s \sin \nu$. The total energy of free solitons and antisolitons is equal to $2E_s = 16\gamma$. Therefore, under the formation of breather there is a release of the following energy;

$$E_b = 16\gamma - 16\gamma \sin \nu = 16\gamma[1 - \sin \nu]$$

According to this expression, $\nu \to \frac{\pi}{2}$ the binding energy approaches zero and the bion disintegrates into a soliton and an antisoliton.

The graph of the ratio of the binding energy of the breather to that of the soliton energy is plotted against $\Delta^* = \frac{\Delta}{kT}$ for given values of d (see figure 1). We observed that as Δ^* increases $\frac{E_b}{E_s}$ decreases. Similarly $\frac{E_b}{E_s}$ decreases as d increases but it does so at a faster rate than Δ^*. This can be seen from the expression

$$\frac{E_b}{E_s} = \frac{\omega_B^2 \hbar^2 I_0(\Delta^*)}{\omega_p^2 m d^2 kT \Delta^* I_1(\Delta^*)}$$

We, therefore, suggest that optimal selection of d and Δ^* can ensure a good breather stability. In [21] it has been proposed that a breather propagation through SL can cause ionization of impurity centres. This is manifested in the damping of the breather and by recombination radiation. It is also noted in the same paper that propagation time of a breather is two or three orders of magnitude less than the propagation time of soliton.

In conclusion we have shown that a soliton and a breather can be generated and propagated in SL. We noted that the rest energy of the breather E_B is related to the soliton energy E_s as $E_B = 2E_s \sin \nu$. Hence the energy needed to excite the breather is much less than that required to excite the soliton. This makes it highly desirable to investigate the possibility of propagation of breather in SL. Interestingly the energy relation depends on the SL parameters (Δ, d). This enables the properties of the breather and soliton to be manipulated by changing the parameters.

Acknowledgements

This work is supported by grant from the Swedish Agency SIDA within the "Associateship" and "visiting Scientist" schemes of the Abdus Salam International Centre for Theoretical Physics at Trieste, Italy.

References

[1] A. A. Ignatov, J. Genzer, E.Scomburg et al., Ann Phys. **5** 173 (1995).

[2] M. Hadjazi, J. F. Palmer, A. Sibille et al., Electron Lett. **29**, 648 (1993).

[3] M. L. Wanke, S. J. Allen, K. Maranowski et al., in Physics of Semiconductors, M. Scheffer and R. Zimmerman (Eds), World Scientific, Singapore (1996), p. 1791.

[4] H. Schneider, K. Fujiwara, H.T. Grahn et al., Appl. Phys. Lett. **56**, 605 (1996).

[5] L. Esaki and R. Tsu, IBM J. Res. Dev. **14**, 61 (1970).

[6] S. Y. Mensah, F. K. A. Allotey and A. Clement Superlattices and Microstructures. **19** (1996).

[7] A. A. Ignatov and Yu. A. Romanov, Phys. Status Solidi B **73**, 327 (1976).

[8] A. A. Ignatov, E. P. Dodin and V. I. Shashkin. Mod. Phys. Lett. **5**, 1087 (1991).

[9] R. K. Dodd, J. C. Eilbech, J. D. Cibbon and H. C. Morris, "Solitons and Nonlinear Waves" (Academic press, 1982).

[10] R. Rajaraman, "Soliton and Instantons, an introduction to Solitons and Instantons in Quantum Field Theory" (North-Holland, 1987).

[11] M. B. Fogel, S. E. Trullinger, A. R. Bishop and J. A. Krumhansl, Phys. Rew. Lett. **36** (1976).

[12] J. A. Krumhansl and J. R. Schrieffer, Phys. Rev. **11**, 3535 (1975).

[13] M. J. Rice, A. R. Bishop, J. A. Krumhansl and S. E. Trullinger, Phys. Rev. Lett. **36**, 432 (1976).

[14] A. M. Dikande and T. C. Kofane, Phys. Script. **49**, 110 (1994).

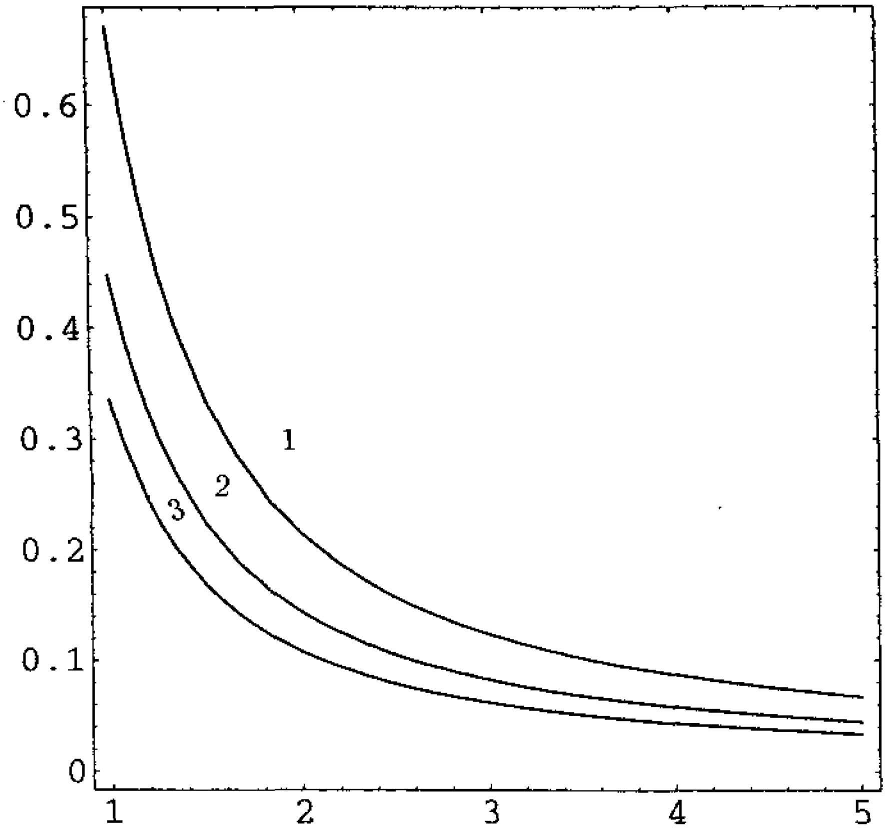
[15] See early developments in "Solitons in Actions", K. Longren and A. C. Scott, editors (Academic Press, 1977).

[16] A. P. Teterov, Solid State Commun. **54**, 421 (1985).

[17] E. M. Epshtein, Sov. Phys. Solid State **19**, 2020 (1977).

[18] S. A. Akmatov et al, Optics of Femto second Laser (Nauka, Moscow.

1989).
[19] A. C. Scott, Proc. IEEE **57**, 1338 (1969).
[20] S. Y. Mensah, F. K. A. Allotey and A.K.Twum, Preprint International Centre for Theoretical Physics, 336 (1995).
[21] S. V. Kryuchkov G. A. Syrodoev, Sov. Phys. Semicond. **5**, 573 (1990).



The plot of $\frac{E_b}{E_s}$ against Δ^* for:
1) $d = 100\mathring{A}$, 2) $d = 150\mathring{A}$, and 3) $d = 200\mathring{A}$.