Fatigue Life Predictions of Smooth and Notched Specimens Based on Fracture Mechanics

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An elastic plastic fracture mechanics solution for short fatigue cracks in smooth and notched specimens is presented which admits plasticity by replacing the conventional stress term with a strain term and accounts for the propagation of very short cracks by the introduction of an effective crack length which is equal to the actual length increased by length l_0, the length constant l_0 is characteristic of the material and material condition and is calculated from the smooth specimen endurance limit and the long crack threshold stress intensity. Crack growth results for cracks in both elastic and plastic strain fields of notched specimens when interpreted in terms of this strain based intensity factor showed excellent agreement with elastic long crack data. This intensity factor when combined with a propagation model that includes all stages of crack growth also successfully predicted the total fatigue life of the smooth and notched specimens studied here. The predicted propagation life of elliptical and circular notched specimens is in all cases within 50 percent of the actual fatigue lives.

Introduction

The fatigue life of many engineering structures is spent in initiating a crack at a notch root, propagating it through the region of stress concentration of the notch, and then propagating the crack across the remainder of the notched component cross section. The ratios of the numbers of cycles spent in these stages are found to vary with the material, notch geometry, specimen size, and the stress level [1-3]. Fatigue life predictions for notched components have usually been obtained by separating the total life into initiation and propagation [2-3]. This procedure requires the definition of a fatigue crack initiation length which serves as a bridge between initiation and propagation analysis [2-3]. Crack initiation lives have been obtained using the local strain approach and an approach based on the fracture mechanics parameter $\Delta K/\sqrt{\rho}$ [2-7]. Analytical techniques, based on fracture mechanics concepts have been developed for the purpose of estimating the crack propagation lives of notched and cracked members [8-10]. However, available fracture mechanics solutions have been shown [11-15] to be invalid for very small cracks and for cracks embedded inside the plastic regions generated by notches at high load levels.

In several investigations [3,16-18], it was observed that, in smooth specimens, fatigue cracks of a comparable size (a few thousandths of an inch long) initiated at an early stage in the fatigue life, and that the major portion of the fatigue life was spent in the propagation of these cracks to failure. Other observations [17] have shown that the initiation of a fatigue crack has to be associated with cyclic slip. This has led to a division of the fatigue life into the following phases: Cyclic slip, crack initiation, the growth of a microcrack, the growth of a macrocrack, and final failure. However, the transition from one phase to another cannot easily be defined and unfortunately not many studies are available on the microstructural level to aid in understanding the mechanisms of fatigue, especially the growth of small cracks. However, fracture mechanics solutions presented in previous papers [11-15] which appear to be applicable to both short and long cracks in smooth and notched specimens will be combined with a crack propagation model given in this paper to predict the fatigue crack propagation lives of smooth and notched specimens. These solutions have been shown [11-15] to predict accurately the growth of very small cracks in smooth and notched specimens and to be identical to the established conventional fracture mechanics predictions for long crack lengths and it is expected that they will give accurate predictions of the total life to fracture.

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Fracture Mechanics Analysis for Fatigue Cracks

Smooth Specimens. The following strain based intensity factor ΔK was developed [11-12] for a plastically strained short crack length l

$$\Delta K = \mathrm{F} E \Delta e \sqrt{\pi (l + l_0)} \tag{1}$$

where Δe and E are the applied strain range and the modulus of elasticity of the material respectively and l_0 is a constant for a given material and material condition. F is a geometry dependent constant which is of the order of unity for cracks of a size that is small compared to other geometric dimensions.

For elastic stress levels of the stress range ΔS, equation (1) becomes

$$\Delta K = F\Delta S\sqrt{\pi(l+l_0)} \qquad (2)$$

Since the threshold stress at very short crack lengths has been shown to approach the fatigue limit of the material $\Delta\sigma_e$ [11-15 and 19-20] l_0 may be obtained from equation (2) as:

$$l_0 = \left(\frac{\Delta K_{th}}{\Delta\sigma_e}\right)^2 \frac{1}{\pi} \qquad (3)$$

where F is approximated as unity and ΔK_{th} is the threshold stress intensity factor. As the crack length decreases, the length l_0 constitutes a continuously increasing fraction of the effective length until in the limiting case of an uncracked surface it represents the effective crack length at which the fatigue limit stress will just propagate a crack into the interior of the specimen. In addition to allowing for fatigue crack initiation at the fatigue limit, intensity factors defined by equations (1) and (2) also were shown to more accurately predict crack propagation rates for short cracks [11-15] than the more usual representation of stress intensity which deletes the term l_0.

The term l_0 which modifies the stress intensity appears to adequately account for two unique features of deformation near a free surface. The first of these is the effect of the boundary on the distribution of stresses around the crack tip. Talug and Reifsnider [21] pointed out that when the distance from the crack tip to the surface becomes very small the conventional ΔK based only on the singular part of the stress field is inadequate to fully describe the near crack tip stress field. The second feature is the reduced flow resistance of the surface grains of a metal compared to interior grains due to their lack of constraint by surrounding grains [11-15]. As yet there is no analytical method for combining the effect of these features on crack tip plasticity but the empirical calculation of l_0 as the effective crack length necessary to start a crack by exceeding the threshold stress intensity when the applied stress is equal to the endurance limit stress provides accurate predictions of crack propagation behaviour [11-15].

Notched Specimens. In using equation (1) to calculate intensity factors for cracks initiating from notch roots the nominal strain term Δe should be replaced by the local strain in the vicinity of the crack tip, $\Delta\epsilon$ and assuming no variation of crack length through the thickness, a value of unity is appropriate for F. This gives

$$\Delta K = E\Delta\epsilon\sqrt{\pi(l+l_0)} \qquad (4)$$

Estimates of the local strain $\Delta\epsilon$ in the plastic regime can be obtained from finite element plastic solutions [22] or may be derived from a relationship between concentration factors proposed by Neuber [23] which has been shown to give good estimates of local strain for a wide range of notched geometries [3,12-13]. The latter will be employed later in this section to determine $\Delta\epsilon$. However, when applied stress levels are low enough that the local strain $\Delta\epsilon$ remains elastic, the elastic stress concentration factor k' for a crack in a notched geometry may be used directly to determine the local stress or strain in terms of the nominal stress ΔS and equation (4) becomes

$$\Delta K = k' F_w \Delta S\sqrt{\pi(l+l_0)} = k'_{eq}\Delta S\sqrt{\pi(l+l_0)} \qquad (5)$$

Here the elastic stress concentration factor k' accounts for the increase in crack tip stress due to the notch. It is a function of $l/(l+c)$ as given in reference [24]. Here the crack length, l, is measured from the notch root and c is the notch depth. The quantity l_0 is again the material constant defined by equation (3). The stress concentration factor k' decreases from an initial maximum value approximately equal to $1.12\ k_t$, where k_t is the theoretical stress concentration factor, to a value of

$$\sqrt{\frac{l+c}{l}}$$

as the crack passes outside the field of influence of the notch. A long crack may be analyzed as a simple crack with a length equal to the actual crack length plus the notch depth. The value of the correction factor for finite plate width, F_w is equal to unity at zero crack length and increases with crack length [25]. It should be pointed out that at long crack lengths where the term l_0 is not significant equation (5) is identical to the conventional stress intensity used by many investigators [26-28] who applied linear elastic fracture mechanics to predict the growth of cracks in notches.

At stress levels causing local plasticity at notch roots, the elastic solution given by equation (5) is invalid and equation (4) must be used. Neuber's rule [23], may be used to obtain an approximate solution for $E\Delta\epsilon$ as follows:

$$k'_{eq}\Delta S = [\Delta\sigma\ \Delta\epsilon\ E]^{1/2} \qquad (6)$$

where k'_{eq} is the equivalent stress concentration factor defined by $(k' F_w)$ as given in equation (5), and $\Delta\sigma$ and $\Delta\epsilon$ are local stresses and strains in the vicinity of the crack tip. Values of the left-hand side of equation (6) for a given crack length and nominal stress may be computed using an elastic solution for k'_{eq} [24-25]. Since terms on the right hand side involve only material stress-strain response a base curve of $(\Delta\sigma\Delta\epsilon E)^{1/2}$ versus $\Delta\epsilon . E$ can be constructed from cyclic stress-strain data

Nomenclature

E = modulus of elasticity
F = geometry dependent constant
F_w = finite width correction factor
k' = elastic stress concentration factor for a crack in notched components, a function of crack length
k'_{eq} = equivalent stress concentration factor of a crack in notched components $k'_{eq} = k'.F_w$
k_t = stress concentration factor of uncracked members
N = number of cycles
R = ratio of minimum to maximum stress
da/dN = crack growth rate
l = crack length measured from the notch root
l_0 = material constant
a = total crack length including the depth of the notch
b = half the plate width
c = depth of the notch
ρ = notch root radius
A,C,m,M = crack propagation material constants
$\Delta\sigma_e$ = fatigue limit
$\Delta\sigma_{th}$ = threshold stress in smooth specimen
Δe = nominal strain range
ΔS = nominal stress range
$\Delta\epsilon$ = local strain range around crack tip in notched specimens
$\Delta\sigma$ = local stress range around crack tip in notched specimens
ΔK = stress or strain based intensity factor
ΔK_{th} = threshold stress intensity factor
ΔJ = the range of J integral
K_c = fracture toughness

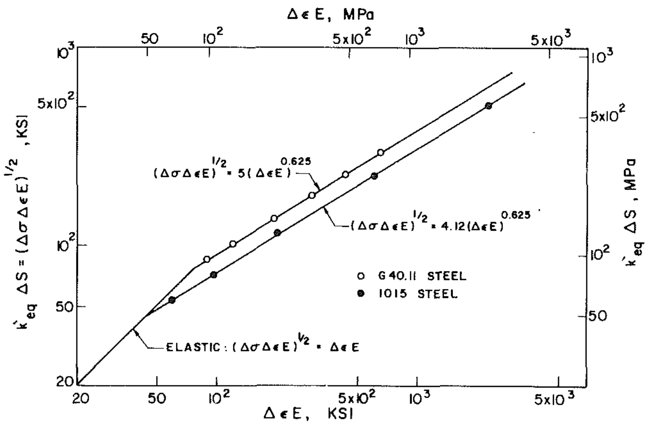

Fig. 1 Base curve $(\Delta\sigma\Delta\epsilon E)^{1/2}$ versus $E\Delta\epsilon$

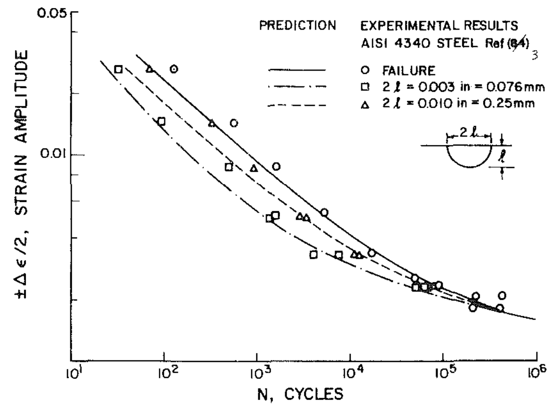

Fig. 2 Smooth specimen life predictions – AISI 4340 steel

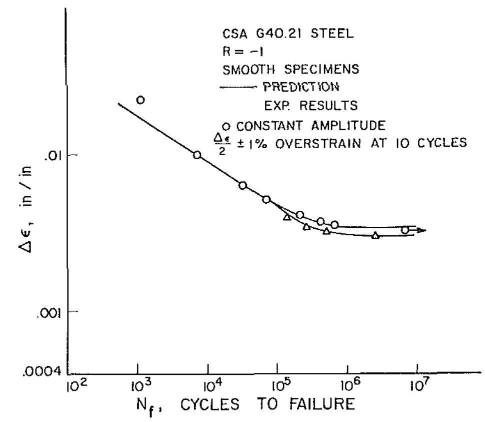

Fig. 3 Smooth specimen life prediction – G40.21 steel

for a given material as shown in Fig. 1. Hence values of $\Delta\epsilon.E$ corresponding to a given value of $k'_{eq}\ \Delta S$ can be obtained and inserted into equation (4) to determine the intensity factor. If the value of $k'_{eq}\ \Delta S$ is below the cyclic yield value for the material equation (4) reduces to equation (5).

In the present paper a further refinement is applied, is the calculation of the stress intensity factor for long cracks with their tips outside the plastic region around the notch root in the form of the Irwin [29] crack length correction which adds the plastic zone size to the crack length.

Fatigue Crack Propagation Model

Paris and Erdogan [30] first proposed the following fatigue crack propagation law:

$$\frac{da}{dN} = C(\Delta K)^m \tag{7}$$

where da/dN is the rate of crack growth per cycle and C and m are material constants which depend on environmental conditions. Subsequent investigators [31-33] have modified equation (7) to account for load ratio, and a variety of other factors including crack closure, threshold stress intensity and the critical stress intensity of fracture. One of these modifications proposed by McEvily [33] is given by the following equation:

$$\frac{da}{dN} = A[\Delta K - \Delta K_{th}]^M \left[\frac{K_c}{K_c - K_{\max}}\right] \tag{8}$$

where A and M are material constants, ΔK_{th} is the threshold stress intensity factor, and K_c is the fracture toughness determined as the value of ΔK at which the da/dN versus ΔK curve increases abruptly before fracture. Values of ΔK_{th} and K_c were obtained from fatigue tests for the materials tested in the present study [12-13]. Values of A and M were determined using a least squares computer program that provided a best fit of experimental data for cracked specimens.

Equation (8) is used in the present paper to predict crack propagation lives of smooth and notched specimens. This is achieved by a digital computer program which numerically integrates the equation cycle by cycle to obtain the crack increment and thence the crack length. Values of ΔK are estimated using equation (1) for smooth specimens while the procedure for notched specimens is that described in the previous section. For simplicity, it was assumed that the crack profile remained constant throughout the life of the smooth specimens. Hence the value of F given in equation (1) is assumed to be constant throughout the life. Also, it is assumed as suggested in the previous section, that there is no variation of crack length through the thickness of the notched specimens.

Smooth Specimen Life Predictions

Experimental results giving the number of fatigue cycles to a given crack size versus strain amplitude for small cracks in smooth specimens taken from reference [3] are plotted in Fig. 2. Good agreement between the predicted propagation life curves obtained from a numerical integration of equation (8) and the experimental data is shown in the figure. Values of ΔK are estimated using equation (1). For half circular surface cracks, a value of $F = 0.71$ is appropriate [3 and 16]. Figure 2 indicates that at high strain levels, the curves corresponding to different crack lengths are parallel and hence the fraction of life corresponding to a given crack size is independent of life. As the strain levels decrease, however, the curves corresponding to the various crack lengths converge and eventually from a single curve at the endurance limit. The same transition in behavior is also evident [17,34,35] in data for other materials. Dowling [3,16] and others [2] suggested that this transition is due to the change from propagation dominated behavior at short life to initiation dominated behavior at long life and is probably associated with the transition [16] from predominantly plastic deformation at short lives to predominantly elastic deformation at long lives. However, the present approach accurately predicts this transition.

Figure 3 also compares experimentally determined smooth specimen total fatigue life data ($R = -1$) for CSA G40.21 steel with the predicted propagation life curves obtained from a numerical integration of equation (8). Again a value of $F = 0.71$ is assumed in estimating ΔK via equation (1). There is good agreement between the experiments and the predictions. Life predictions for smooth specimens subject to an over-strain of ± 1 percent strain for 10 cycles followed by constant

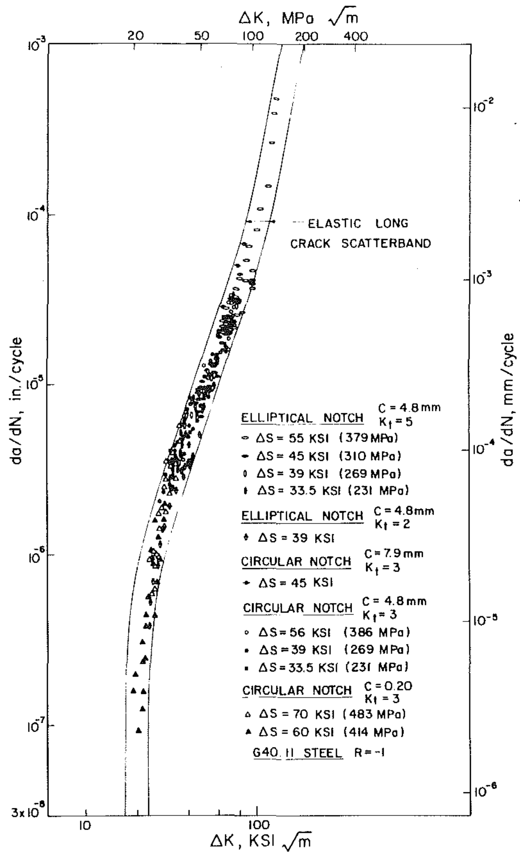

Fig. 4 Fatigue crack growth rates as a function of ΔK given by equation (4) in G40.11 notched steel plates

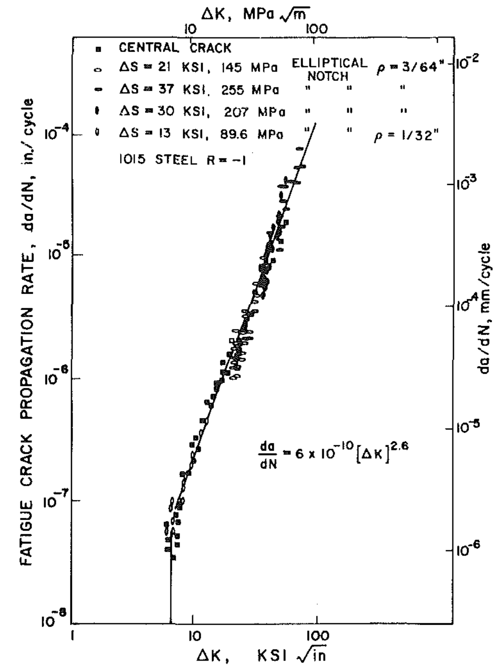

Fig. 5 Fatigue crack growth rates as a function of ΔK given by equation (4) in 1015 notched steel plates

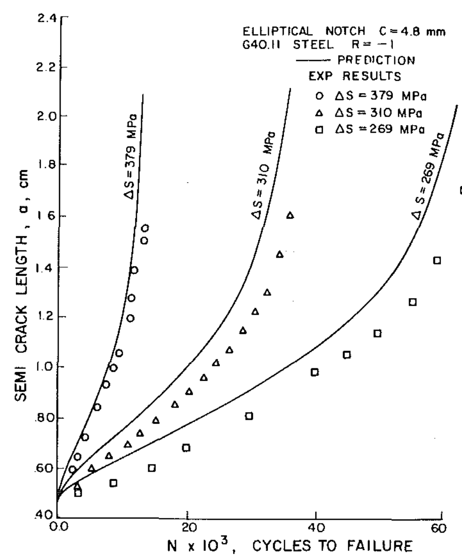

Fig. 6 Crack length versus N curves prediction

amplitude loading until failure are shown in Fig. 3. These predictions are achieved by integrating equation (8) first at the overstrain level for 10 cycles and then integrating the equation at the constant strain level starting with a crack length corresponding to that predicted at the end of 10 cycles at the overstrain level. The agreement between the predictions and the experimental results shown in Fig. 3 for the overstrained test results suggests that a linear summation of cycle ratios on the basis of crack length can be employed in estimating comulative fatigue damage and predicting the fatigue lives of test specimens subjected to variable amplitude loading.

It appears from the above results, that the crack growth model presented herein adequately predicts the fatigue life of smooth specimens. The accuracy of the predictions suggested that the term l_0 introduced into the intensity factor expressions adequately deals with the period spent in propagating short cracks where most of the life is spent. Also, the reduction of a three dimensional crack growth phenomenon to a two dimensional growth does not seem to have significantly affected the predictive accuracy of the crack growth model for smooth specimens.

Notched Specimen Life Prediction

Fatigue Crack Growth Rates. Constant amplitude load controlled testing ($R = -1$) at various stress levels was performed in a closed loop servo controlled electro hydraulic mechanical testing machine [12-13]. Circular and elliptical notched specimens were fabricated from two steels CSA G40.11 and ASTM A36. Details of the materials and specimens are given in references [12] and [13]. Crack propagation rates were determined from microscope measurements of crack lengths at various cycle numbers. Straight forward calculations of ΔK values for the G40.11 and 1015 steel data via equations (4) and (5) and the curves of Fig. 1 were performed and the results plotted versus da/dN in Figs. 4 and 5. The data obtained from differnt notches and for different load levels show excellent agreement with the long crack data for both 1015 steel and G40.11 steel. This agreement indicates that the solutions for the intensity factors given by equations (4) and (5) are accurate in predicting both elastic and inelastic short crack growth curves for notched specimens. Also, ΔJ solutions presented in reference [13] were shown to successfully correlate the data of Figs. 4-5 and it was shown that the parameter ΔJ given in references [11,13,15] and the parameter ΔK used in this paper are equivalent quantities for both elastic and inelastic strain levels and that either can be used to correlate or predict crack growth for smooth and notched specimens [11-15].

Crack Length Versus N Curves. Figures 6 to 8 show the relationship between the total number of cycles N and the

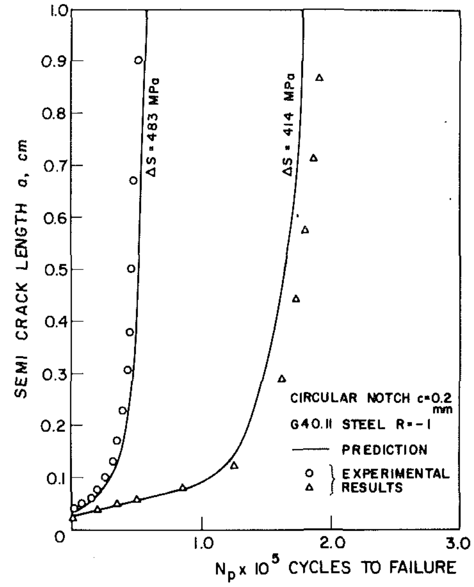

Fig. 7 Crack length verses *N* curves prediction

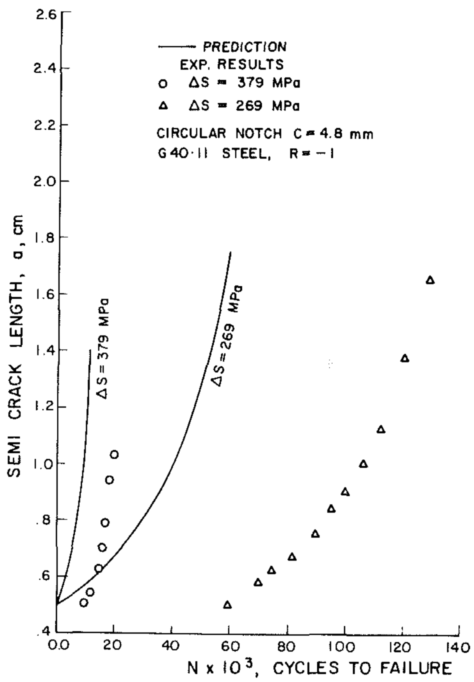

Fig. 8 Crack length verses *N* curves prediction

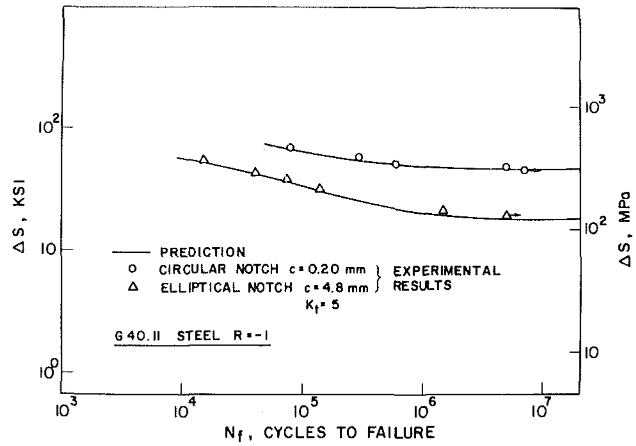

Fig. 9 Fatigue life prediction of notched specimens

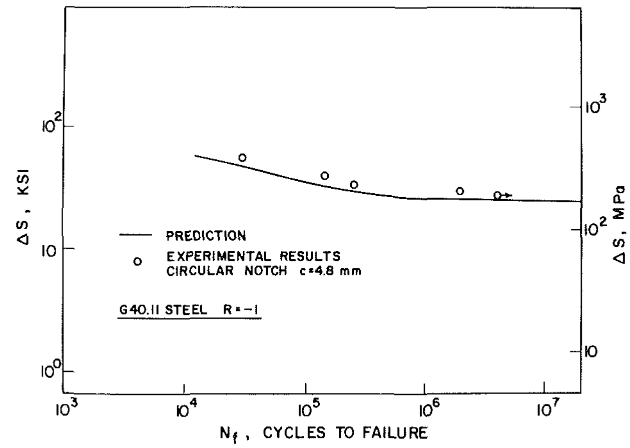

Fig. 10 Fatigue life prediction of notched specimens

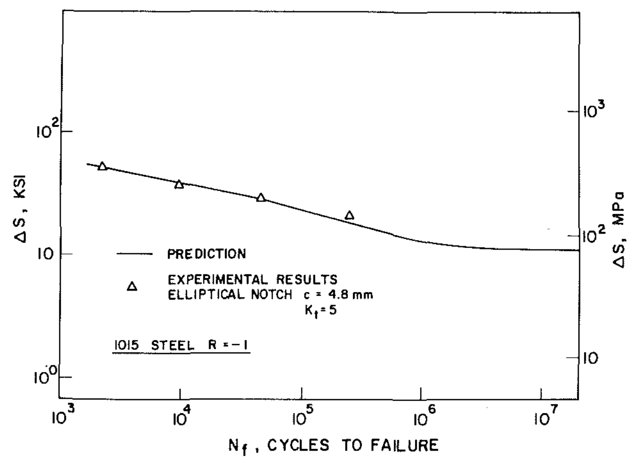

Fig. 11 Fatigue life prediction of notched specimens

measured crack length a for CSA G40.11 steel in elliptical and circular notched plates subjected to varying stress levels at $R = -1$. The crack length includes one half the depth of the notch, and therefore the graphs start with N equals zero corresponding to a crack length equal to the depth of the notch c. Predicted crack lengths are obtained based on the integration of equation (8) on a cycle by cycle basis. Good agreement between the predictions and the experimental results is shown in Figs. 6 and 7 for the 9.8mm diameter elliptical and 0.40mm diameter circular notches. Figures 6 and 7 indicate that the predicted propagation lives are within 15 percent of the actual lives for both geometries. This error reaches 50 percent for the 9.8mm diameter circular notched plates as shown in Fig. 8. The error may be due to the assumption of a straight crack front through the thickness of the specimens throughout the fatigue life which is not true for the biaxial stress state which exists at short crack lengths [36]. White [37] in fact observed that a large portion of the fatigue life in specimens having circular notches was spent in propagating small surface cracks at the root of these notches.

Fatigue Life Prediction. Figures 9-11 inclusive compare experimentally determined total fatigue life data for G40.11 steel and 1015 steel in elliptical and circular notched specimens with the predicted propagation life curves obtained from a numerical integration of equation (8) combined with solutions for ΔK given by equations (4) and (5). All the results shown in Figs. 9 to 11 are for completely reversed cycling ($R = -1$). The figures show that in general the predictions of propagation life are conservative and in good agreement with experimental total life for elliptical and circular notched specimens of the two steels studied. Although Fig. 10 indicates that the predicted result is 50 percent lower than the experimental results for the 4.8mm diameter circular notched specimens, predictions are within 15 percent of the experimental total fatigue lives at high stress levels for the circular and elliptical notched specimens shown in Figs. 9 and 11. At long lives the error reaches 50 percent but here the number of cycles spent in nucleating the cracks at the notch roots may be become important [3]. The discrepancy shown in Fig. 10 may be caused by the assumption of a straight crack front and or may be due to neglecting the portion of life spent in nucleating the cracks at the notch root. However, for a smaller circular notch ($c = 0.20$mm) the difference between the experimental and predicted results does not exceed 15 percent.

Figures 9 and 11 also suggest that, the crack growth model presented in this paper adequately predicts the fatigue life of notched specimens. This may be fortuitous in that the fatigue life of the notched specimens shown in these figures may be dominated by crack propagation. Such behavior is in fact observed [34,38] at sharp notches. Also, the accuracy of the predictions suggests that the term l_0 introduced into the intensity factor expressions and plasticity corrections adequately predicts the period spend in propagating small cracks at notches. In addition, the present approach avoids the difficulties of the previous procedures [2-3] in which fatigue life is separated into initiation and propagation requiring a definition for crack initiation length [2] or initiation data for a specific small crack length [3,16].

Conclusions

1. The intensity factor solutions given by equation (1) when combined with the suggested crack propagation model successfully predicted the fatigue life of smooth specimens for constant amplitude loading and for an overstrain followed by constant amplitude loading. Also the accurate predictions for overstrained tests suggests that a linear summation of cyclic ratios on the basis of crack length can be employed in estimating comulative fatigue damage and predicting fatigue lives under variable amplitude loading.

2. Strain based intensity factor solutions successfully correlated data for the growth of short cracks in plastically strained notches with elastic long crack results. Conversely short crack growth at inelasticly strained notches was accurately predicted from long crack data. These solutions when combined with equation (8) successfully predicted the total fatigue life of elliptical and circular notched specimens for the two materials studied. The predicted propagation life is in all cases within 50 percent of the actual fatigue lives of the notched specimens studied.

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