A. K. Sheikh
Asst. Professor,
Dept. of Mechanical Engineering,
University of Petroleum and Minerals,
Dhahran, Saudi Arabia
Assoc. Mem. ASME

L. A. Kendall
Assoc. Professor,
Dept. of Mechanical Engineering,
Washington State University,
Pullman, Wash. 99164
Assoc. Mem. ASME

S. M. Pandit
Assoc. Professor,
Dept. of Mechanical Engineering,
Michigan Technological University,
Houghton, Mich. 49931
Mem. ASME

Probabilistic Optimization of Multitool Machining Operations

This paper summarizes the results for one aspect of an ongoing research program concerned with the probabilistic nature of tool life and its affect on tool replacement strategies and optimum machining conditions. Preventive planned, scheduled and failure replacement strategies are considered with single and multiple tool systems. When the coefficient of variation is constant as the cutting conditions change, the optimal cutting conditions using the probabilistic models of tool life are a multiple of the optimum cutting conditions obtained by using the classical deterministic model of tool life. The multiplying factor is a function of the coefficient of variation, tool change policy and the cost parameters of these replacement policies.

Introduction

Tool life variability has long been recognized [1, 2]. The physical and metallurgical aspects of tool life variability have been discussed extensively by Ramalingam [3, 4, 5]. He identified various hazard functions which represent the failure patterns corresponding to different environments in which a tool operates. These hazard functions were used to develop the tool life distribution models and reliability functions. Before this work, Wager [2] and Duncan [6] explored the tool life distributions, and in addition, Duncan [6] developed some mathematical models for determining optimal tool change policies. On the basis of the physics of failure or purely on the basis of statistical analysis of tool failure data, the following models have been suggested to represent tool life: normal [1], log normal [2, 3, 5], Weibull [3, 6, 7], gamma [6], extreme value [8], DDS [9] and exponential [4]. These models can be used to develop optimal tool change intervals for a given tool replacement policy. In the past, attempts were made to develop optimal policies and optimal tool change intervals, primarily using Monte Carlo Simulation techniques [9, 10], other numerical methods, using reliability mathematics, were used by Duncan [6] for various tool replacement policies. Kendall and Sheikh [12] presented a solution for determining the optimal tool replacement interval using a Weibull model for tool reliability and a preventive planned replacement strategy. This work was continued [13] and a number of tool replacement strategies were explored and their optimal solutions were presented for a large number of reliability models. In a recent paper by Sheikh and Kendall [14], the importance of the coefficient of variation as an index of tool life variability was emphasized. Zdeblick and Devor [16] used a normal probability model for tool failure in a systems approach to process planning of the machining operation. A better understanding of the failure probability model and how it can be used to establish to tool replacement interval will help make such systems' studies more precise. This paper will present various tool replacement strategies for single tool and multiple tool production machines and show how the optimal cutting conditions are affected by these tool change policies. In order to develop such relationships, the following must be done:

1 Develop a variable cost model in terms of the tool replacement strategy and the cutting parameters of feed, speed and depth of cut.

2 Treat tool life as a random variable and select, using appropriate statistical tests, a probability model that defines the tool life variations.

3 Introduce this probability model into the cost equation.

4 Find the optimal replacement interval and optimal values for the cutting parameters. The general approach to finding such optimum values has been reported [15, 17].

It has been found that the coefficient of variation is an important parameter for this optimization procedure [14]. A functional relationship between the coefficient of variation and the machining parameters needs to be defined. Presently, the optimization process has been developed for the case where the coefficient of variation is constant with respect to these cutting parameters: feed, depth of cut and speed. The theory is valid for any tool reliability model; however, in this paper, the Weibull model will be used. This work incorporates tool reliability models in a way that utilizes information (data) generated in the laboratory and/or production shop at one cutting condition and then predicts new (optimal) cutting conditions. This approach requires the use of a functional relationship between tool life and the cutting parameters.

In this paper, this appraoch has been used for the following tool change policies;

1 Preventive planned tool change policy.
2 Scheduled tool change policy.
3 Failure replacement policy.

These tool change policies will be developed for single and multiple tool machining systems and is based upon a more detailed develop-

Contributed by the Production Engineering Division for publication in the JOURNAL OF ENGINEERING FOR INDUSTRY. Manuscript received at ASME Headquarters Oct. 1979.

ment found elsewhere [13].

The development in this paper is applicable to the case, where, the volume of production is sufficiently large, so that, the tool replacement and/or failure cycle is repeated many times. This is s common situation in most of automated production.

Optimum Cutting Conditions for a Single Tool Operation

The mean tool life equation for a machining operation (turning, milling, drilling, etc.) can be given as

$$\overline{T} = K_{\overline{T}}/(V^m f^{m_1} d^{m_2}) = A_{\overline{T}}/(V^m f^{m_1}) = (C_{\overline{T}}/V)^m \quad (1)$$

or

$$\overline{T} = D_{\overline{T}}/(N^m f^{m_1} d^{m_2}) = G_{\overline{T}}/(N^m f^{m_1}) = (B_{\overline{T}}/N)^m \quad (2)$$

where $m = 1/n$, $m_1 = 1/n_1$, and $m_2 = 1/n_2$ are exponents of speed, feed, and depth of cut. Constants $K_{\overline{T}}$, $A_{\overline{T}}$ and $C_{\overline{T}}$ are related as follows

$$A_{\overline{T}} = K_{\overline{T}} d^{-m_2} \quad (3)$$

$$C_{\overline{T}} = [K_{\overline{T}}/(f^{m_1} d^{m_2})]^n \quad (4)$$

Equation (2) represents the average tool life in terms of spindle speed, feed and depth of cut, and the relationship between $D_{\overline{T}}$, $G_{\overline{T}}$ and $B_{\overline{T}}$ is as follows:

$$G_{\overline{T}} = D_{\overline{T}} d^{-m_2} \quad (5)$$

and

$$B_{\overline{T}} = [D_{\overline{T}}/(f^{m_1} d^{m_2})]^n \quad (6)$$

Because of the assumption that the coefficient of variation, K, remains independent of V (or N), f and d, equations similar to (1) and (2) will hold true for any tool life fractile t_L (or t_q). These new equations will be obtained by replacing $\overline{T}$ by t_L (or t_q) in equations (1) and (2). Similarly, relationships can be developed for fractiles t_p, t_s, etc. as shown in Table 1. For a discussion on this development, see references [13, 14] where a special case of equations (1) and (2), $V\overline{T}^n = C_{\overline{T}}$, is discussed using a probabilistic approach. This approach is shown in Fig. 1. Fig. 1 also illustrates that when the coefficient of variation is constant with respect to velocity, the exponent n remains constant with respect to velocity for any tool life fractile. Others who have reported variability in both n and C, as the cutting conditions are changed, are basing their conclusions on cutting conditions where K was not necessarily independent of V, f, and/or d [18].

Preventive Planned Tool Replacement Strategy. The tool has a preventive planned replacement interval, t_p, and failure replacement is made if the tool fails during the time interval $(0 - t_p)$. If preventive planned replacement is made, a cost of C_p (\$/cutting edge) occurs, and if a failure replacement is made, a cost of C_f(\$/cutting edge) will occur, where $C_p < C_f$.

In large volume production, where the tool replacement cycle is repeated many times, it is appropriate to establish the tool replacement policy for steady state conditions. The expected value of the variable portion of machining cost per component (\$/component) for the steady state conditions is given by [13]:

$$E[C(t_p)] = x[t_c + E[\theta_r(t_p)]t_c] \quad (7)$$

where

Nomenclature

T, random variable tool life failure (i.e., time to failure of cutting tools)
$f_T(t) = f(t)$, probability density function of random variable T
$R_T(t) = R(t) = \Pr(T > t) = \int_t^\infty f(t)dt$, reliability function of T
$F_T(t) = F(t) = \Pr(T < t) = 1 - R(t) = \int_0^t f(t)dt$, cumulative function of T
$h_T(t) = h(t) = f(t)/R(t)$, hazard function of T
$\overline{T} = \int_0^\infty R(t)dt = \int_0^\infty tf(t)dt$, expected value of T
$\sigma^2 = \int_0^\infty (t - T)^2 f(t)dt$, variance of T
t_q = solution to: $\int_0^{t_q} f(t)dt = q$, tool life corresponding to the probability of failure q, t_q is also known as tool life fractile
K, coefficient of variation of cutting tools, $K = \sigma/\overline{T}$
β, shape parameter of a Weibull reliability function
η, characteristic life of T for a Weibull model
t_L, a generalized symbol to prepresent the characteristic life of T for any reliability model
$\tau = T/t_L$, dimensionless random variable
$f_\tau(\tau) = f(\tau)$, density function of dimensionless random variable τ
$R_\tau(\tau) = R(\tau)$, reliability function of τ
$h_\tau(\tau) = h(\tau)$, hazard function of τ
t_p, planned replacement interval
$\overset{*}{t}_p$, optimum value of planned replacement interval
t_s, scheduled tool replacement interval (also tool life fractile t_s)
$\overset{*}{t}_s$, optimum scheduled tool replacement interval
$H_T(t) = H(t)$, mean number of failures that occurs up to the instant t, or value of the renewal function of T at instant t
$H_T(t_s) = H(t_s)$, mean number of failures (or renewals) up to the instant t_s
$\dot{H}_T(t) = \dot{H}(t)$, renewal density function of T
$H_\tau(\tau) = H(\tau)$, renewal function of nondimensional random variable τ
$\dot{H}_\tau(\tau) = \dot{H}(\tau)$, renewal density function of τ
x, cost of one machine minute, \$/machine minute
C_p, cost of preventive planned replacement of a cutting edge, \$/cutting edge
C_f, cost of failure replacement of a cutting edge, \$/cutting edge
C_s, cost of scheduled replacement of a cutting edge, \$/cutting edge
$\theta_p = C_p/x$, cost of a preventive planned replacement of a cutting edge in time units, machine minutes/cutting edge
$\theta_f = C_f/x$, cost of a failure replacement of a cutting edge in time units, machine minutes/cutting edge
$\theta_s = C_s/x$, cost of a scheduled replacement interval in time units, machine minutes/cutting edge
$\gamma_p = C_p/C_f = \theta_p/\theta_f$, cost ratio in preventive planned replacement policy
$\gamma_s = C_s/C_f = \theta_s/\theta_f$, cost ratio in scheduled replacement policy
$E[C(t)]$, expected value of the variable portion in machining cost per component, \$/component
$E[\theta(t)] = E[C(t)]/x$, expected value of the variable portion of machining cost (in time units) per component, machine minutes/component
$E[\theta_r(t_p)]$, expected tool replacement cost in machine minutes per unit of time
D, diameter of shaft in inches
L, length of cut in inches
d, depth of cut in inches
f, feed in inches/revolution
N, spindle speed, rpm
V, velocity of workpiece (or cutting velocity) in surface feet per minute
$t_c = L/(fN) = DL/(12fV)$, machining time per component in minutes
$\overline{T} = (C_{\overline{T}}/V)^m$, Taylor's tool life model, representing mean tool life, $\overline{T}$, as a function of cutting velocity V
$\overline{T} = (B_{\overline{T}}/N)^m$, Taylor's tool life model representing mean tool life, $\overline{T}$, as a function of spindle speed N
$m = 1/n$, slope of Taylor's line [i.e., $\overline{T} = (C_{\overline{T}}/V)^m = (B_{\overline{T}}/N)^m$] on log × log paper
$C_{\overline{T}}$, Taylor's constant (i.e., cutting velocity in ft/minute) corresponding to one minute of mean tool life
$B_{\overline{T}}$, Taylor's constant (i.e., spindle speed in rpm) corresponding to one minute of mean tool life

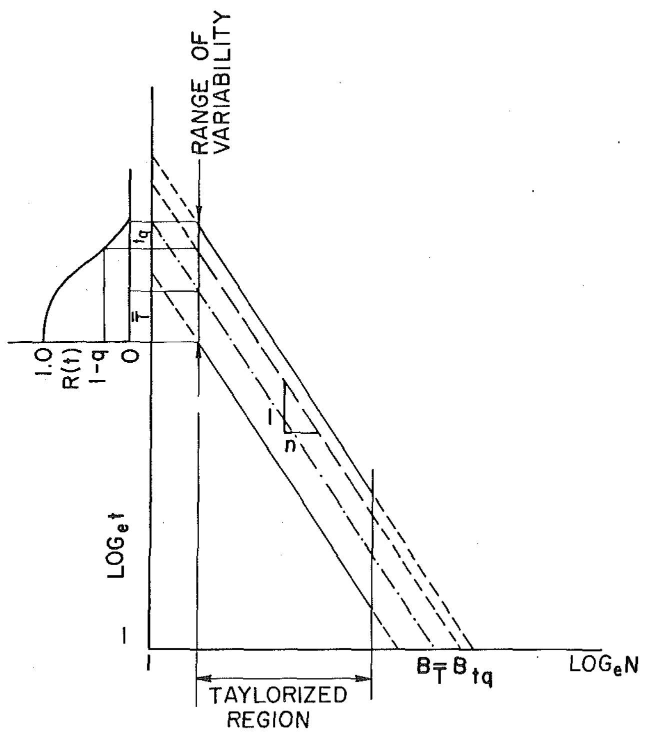

Fig. 1 Sample Function Representing the Causal Relationship, $\bar{T} = (B_{\bar{T}}/N)^m$.

$$E[\theta_r(t_p)] = [\theta_f F_T(t_p) + \theta_p R_T(t_p)] \Big/ \int_0^{t_p} R(t)dt \tag{8}$$

Using the transformation $\tau = T/t_L$ it can be shown [13], that $R_\tau(\tau) = R_T(t)$,

$$F_\tau(\tau) = F_T(t), \quad \text{and } t_L \int_0^{\tau_p} R_\tau(\tau)d\tau = \int_0^{t_p} R_T(t)dt \tag{9}$$

Assuming that the coefficient of variation of cutting tools, K, remains unchanged with respect to cutting parameters (velocity, feed, and depth of cut), it can be shown that the shape parameters of a reliability model (in dimensionless form [13]), and the dimensionless random variable, τ, both remain independent of these cutting parameters; therefore, $F_\tau(\tau_p)$, $R_\tau(\tau_p)$ and $\int_{0_0}^{\tau_p} R_\tau(\tau)d\tau$, all remain independent of these cutting parameters for any known fractile τ_p [13].

Using equations (8) and (9), equation (7) can be rewritten as follows:

$$E[C(t_p)] = x\left[t_c + \left(\frac{\theta_f F_\tau(\tau_p) + \theta_p R_\tau(\tau_p)}{\int_0^{\tau_p} R(\tau)d\tau}\right)\frac{t_c}{t_L}\right] \tag{10}$$

Normally the depth of cut is pre-selected at the time of process selection, and variables to be optimized are $V(\text{or } N), f$, and τ_p. After substituting $t_c = L/fN$ and t_L from Table 1, equation (10) will become

$$E[C(t_p)] = xL[f^{-1}N^{-1} + \psi_3(\tau_p)f^{(m_1-1)}N^{(m-1)}/G_{t_L}] \tag{11}$$

where

$$\psi_3(\tau_p) = [\theta_f F_\tau(\tau_p) + \theta_p R_\tau(\tau_p)] \Big/ \int_0^{\tau_p} R(\tau)d\tau \tag{12}$$

Equation (11), in functional form, can be written as follows:

$$E[C(t_p)] = \psi_1(f)\psi_2(N) + \psi_3(\tau_p)\psi_4(f)\psi_5(N) \tag{13}$$

where

Table 1 Relationship Between Various Tool Life Fractiles and Machining Parameters

$t_L = D_{t_L}/(N^m f^{m_1} d^{m_2})$	$= G_{t_L}/(N^m f^{m_1})$	$= (B_{t_L}/N)^m$
$t_p = D_{t_p}/(N^m f^{m_1} d^{m_2})$	$= G_{t_p}/(N^m f^{m_1})$	$= (B_{t_p}/N)^m$
$t_s = D_{t_s}/(N^m f^{m_1} d^{m_2})$	$= G_{t_s}/(N^m f^{m_1})$	$= (B_{t_s}/N)^m$

$\psi_1(f) = 1/f$, $\psi_2(N) = 1/N$, $\psi_4(f) = f^{m_1-1}$, and $\psi_5(N) = N^{m-1}$, and $\psi_3(\tau_p)$ is given by equation (12). Equation (11) or (13) may be optimized by solving the following.

$$\partial E[C(t_p)]/\partial\tau_p = \partial\psi_3(\tau_p)/\partial\tau_p = 0 \tag{14}$$

to obtain $\overset{*}{\tau}_p$, where $\overset{*}{\tau}_p$ is the optimum replacement fractile (interval) in nondimensional form. Substitution of $\tau_p = \overset{*}{\tau}_p$ into equation (11) or (13) results in

$$E[C(t_p)]|_{\tau_p=\overset{*}{\tau}_p} = \psi_1(f)\psi_2(N) + \psi_3(\overset{*}{\tau}_p)\psi_4(f)\psi_5(N). \tag{15}$$

For obtaining the cutting conditions for minimum expected cost per component, the following conditions should be satisfied:

$$\partial/\partial N[E[C(t_p)]|_{\tau_p=\overset{*}{\tau}_p}] = 0 \tag{16}$$

$$\partial/\partial f[E[C(t_p)]|_{\tau_p=\overset{*}{\tau}_p}] = 0 \tag{17}$$

From the condition expressed in equation (14) and provided $h(\tau)$ is an increasing function of τ, the following result is obtained [13]:

$$h(\overset{*}{\tau}_p)\int_0^{\overset{*}{\tau}_p} R(\tau)d_\tau + R(\overset{*}{\tau}_p) = 1/(1-\gamma_p) \tag{18}$$

where

$$h(\overset{*}{\tau}_p) = f(\overset{*}{\tau}_p)/R(\overset{*}{\tau}_p) \tag{19}$$

$$\gamma_p = C_p/C_f$$

$$f(\tau) = t_L f(t) \tag{20}$$

$$h(\tau) = t_L h(t) \tag{21}$$

Generally, for most total reliability models, a numerical solution of equation (18) is required to obtain $\overset{*}{\tau}_p$. Substituting $\tau_p = \overset{*}{\tau}_p$ into equation (12) and simplifying, the follwing is obtained [13]:

$$\psi_3(\overset{*}{\tau}_p) = [\theta_f - \theta_p]h(\overset{*}{\tau}_p) \tag{22}$$

Satisfaction of equation (16) results in

$$1 = (m-1)[\theta_f(1-\gamma_p)h(\overset{*}{\tau}_p)]N^m f^{m_1}/G_{t_L} \tag{23}$$

and satisfaction of equation (17) results in

$$1 = (m_1-1)[\theta_f(1-\gamma_p)h(\overset{*}{\tau}_p)]N^m f^{m_1}/G_{t_L} \tag{24}$$

Unless $m = m_1$, equations (23) and (24) cannot be solved simultaneously and a unique minimum does not occur. In general, $m \neq m_1$; therefore, one of the variables, N or f, has to be preselected and then the optimal value of the other variable can be determined from either equation (23) or (24). There is substantial evidence [19] that $m_1 < m$; therefore, one approach would be to preselect the feed to be as large as possible to avoid breakage and/or meet a surface roughness specification, then solve equation (23) to obtain the optimal spindle speed. This approach will be used by letting $f = f_o$ in equation (23) to obtain

$$\overset{*}{N} = (G_{t_L}/f_o{}^{m_1})^n[(m-1)\theta_f(1-\gamma_p)h(\overset{*}{\tau}_p)]^{-n} \tag{25}$$

If

$$(G_{t_L}/f_o{}^{m_1})^n = B_{t_L} \tag{26}$$

then

$$\overset{*}{N} = B_{t_L}/[(m-1)\theta_f(1-\gamma_p)h(\overset{*}{\tau}_p)]^{-n} \quad (27)$$

Equation (27) can also be obtained directly from equations (10), (11), or (13) by substituting $t_L = (B_{t_L}/N)^m$ and then optimize the objective function with respect to τ_p and N(or V). In this case the objective function (i.e., expected cost/component) is given by

$$E[C(t_p)] = (xL/f)[N^{-1} + \psi_3(\tau_p)N^{m-1}/B_{t_L}{}^m] \quad (28)$$

Differentiating equation (28) with respect to τ_p will result in equations (18) and (22). With f constant, $\partial/\partial N[E[C(t_p)]|_{\tau=\overset{*}{\tau}_p}] = 0$ gives equation (27).

In cases where the spindle speed is fixed, such as in transfer lines, the only variable which can be optimized is the feed. By substituting $N = N_o$ in equation (24), the following expression for $\overset{*}{f}$ is obtained

$$\overset{*}{f} = (G_{t_L}/N_o{}^m)^{n_1}[(m_1-1)\theta_f(1-\gamma_p)h(\overset{*}{\tau}_p)]^{-n_1} \quad (29)$$

Equation (28) will be used in all further analyses with $\psi_3(\tau_p)$ and/or B_{t_L} being replaced with the appropriate expression for the tool replacement strategy being studied. An optimal value of $\overset{*}{\tau}_p$ and $\overset{*}{N}$ will be obtained for each tool change strategy. Even though equation (28) will be used, the relationships for the optimum spindle speed can be converted to relationships for optimum feed by making the following changes in the equations for $\overset{*}{N}$:

$$\overset{*}{N} \rightarrow \overset{*}{f},\ (1/n = m) \rightarrow (1/n_1 = m_1),\ B_{t_L} = (G_{t_L}/f_o{}^{m_1})^n$$
$$\rightarrow (G_{t_L}/N_o{}^m)^{n_1}, \quad \text{or } B_{\overline{T}} = (G_{\overline{T}}/f_o{}^{m_1})^n \rightarrow (G_{\overline{T}}/N_o{}^m)^{n_1}$$

The validity of this interchange is demonstrated by comparing equations (27) and (29). Since

$$t_L = (C_{t_L}/V)^m = (B_{t_L}/N)^m \quad (30)$$

and

$$\overline{T} = (C_{\overline{T}}/V)^m = (B_{\overline{T}}/N)^m \quad (31)$$

Dividing equation (30) by (31) the following expression is obtained

$$B_{t_L} = B_{\overline{T}}/(\overline{T}/t_L)^n \quad (32)$$

Substituting equation (32) into equation (27), the optimal spindle speed for the preventive planned tool replacement policy is given by

$$\overset{*}{N} = [(\overline{T}/t_L)(1-\gamma_p)h(\overset{*}{\tau}_p)]^{-n}[B_{\overline{T}}/(\theta_f(m-1))^n] \quad (33)$$

For a two parameter Weibull tool reliability model,

$$R(t) = \exp[-(t/\eta)^\beta] \quad (34)$$

where the relationship between β and K is as shown in Fig. 2. If $\eta = t_L$, then [20, 21]

$$\overline{T}/t_L = \overline{T}/\eta = \Gamma(1 + 1/\beta) \quad (35)$$

and

$$h(\overset{*}{\tau}_p) = \beta(\overset{*}{\tau}_p)^{\beta-1} \quad (36)$$

Substitution of equation (25) and (26) into equation (34) results in

$$\overset{*}{N} = [(1-\gamma_p)\beta(\overset{*}{\tau}_p)^{\beta-1}\gamma(1+1/\beta)]^{-n}[B_{\overline{T}}/(\theta_f(m-1))^n] \quad (37)$$

where the quantity

$$B_{\overline{T}}/[(\theta_f(m-1))^n] = \overset{*}{N}_{\text{deterministic}}$$

is equal to the spindle speed calculated on the basis of the deterministic approach [22]. This means that the optimal spindle speed using probabilistic models for tool life is a multiple of the optimum spindle speed calculated from the classic deterministic equation. This multiplying factor depends only upon the coefficient to variation, K, and the cost ratio, γ_p, because

$$\beta = 1/K \qquad \text{for } K \le 0.5$$
$$\beta \simeq 1/K \qquad \text{for } 0.5 \le K \le 1.0 \quad (38)$$

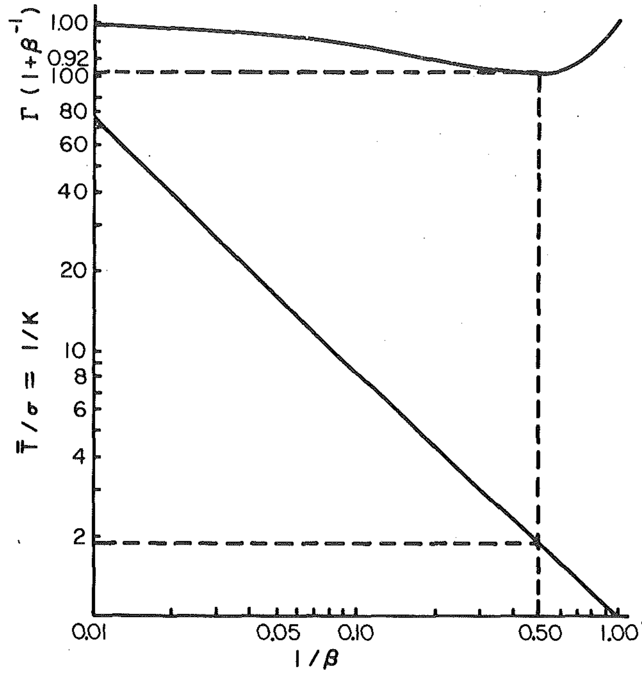

Fig. 2 Graphs (*I*) Between Coefficient of Variation K, and Shape Parameter β, and (*II*) Between $\Gamma(1 + 1/\beta)$ and $1/\beta$.

as shown in Fig. 2 and $\overset{*}{\tau}_p$ depends upon γ_p and β as shown in Fig. 3. Fig. 3 shows solutions for equation (18) for the Weibull model.

Scheduled Tool Replacement Strategy. Scheduled tool replacements at intervals of t_s, $2t_s$, $3t_s$, etc. are used with failure replacement within the intervals and the expected variable cost/component is [13]

$$E[C(t_s)] = x[t_c + [\theta_s + \theta_f H(t_s)]t_c/t_s] \quad (39)$$

Using

$$\tau = T/t_L,\ H(\tau) = H(t), \quad \text{and } \tau_s t_L = t_s, \quad (40)$$

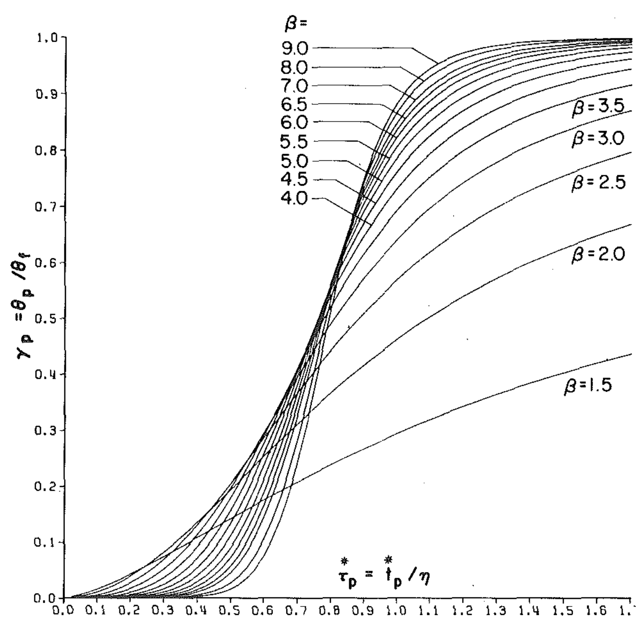

Fig. 3 Optimum Preventive Planned Replacement Interval, $\overset{*}{\tau}_p$, versus Cost Ratio γ_p, for Weibull Model.

then equations (40) can be substituted into equation (39) to obtain the following:

$$E[C(t_s)] = x[t_c + g_3(\tau_c)t_c/t_L] \quad (41)$$

or

$$E[C(t_s)] = (xL/f)[N^{-1} + g_3(\tau_s)N^{m-1}/B_{t_L}{}^m] \quad (42)$$

where

$$g_3(\tau_s) = [\theta_s + \theta_f H(\tau_s)]/\tau_s \quad (43)$$

Equation (42) is of the same form as equation (28) and can be optimized in a similar way to obtain $\overset{*}{\tau}_s$ and N. The differentiation with respect to τ_s results in

$$\gamma_s = \theta_s/\theta_p = \overset{*}{\tau}_s \dot{H}(\overset{*}{\tau}_s) - H(\overset{*}{\tau}_s) \quad (44)$$

where

$$\dot{H}(\tau) = dH(\tau)/d\tau \quad (45)$$

is the renewal density of τ. Substitution of $\overset{*}{\tau}_s$ and differention with respect to N as before results in

$$\overset{*}{N} = [(\overline{T}/t_L)\dot{H}(\overset{*}{\tau}_s)]^{-n}[B_{\overline{T}}/(\theta_f(m-1))^n] \quad (46)$$

where $\dot{H}(\overset{*}{\tau}_s) = [\gamma_s + H(\overset{*}{\tau}_s)]/\overset{*}{\tau}_s$ and is obtained from equation (44). For a Weibull model

$$\overset{*}{N} = [\overset{*}{\tau}_s/\Gamma(1+1/\beta)(\gamma_s + H(\overset{*}{\tau}_s))]^n[B_{\overline{T}}/(\theta_f(m-1))^n] \quad (47)$$

where Fig. 4 shows the relationship between $\overset{*}{\tau}_s$ and γ_s for various values of β. Fig. 4 is obtained by solving equation (42). Reference [20] can be used for obtaining values of $H(\overset{*}{\tau}_s)$. Again, the optimum cutting speed is a multiple of the deterministic optimum spindle speed.

Failure Tool Replacement Strategy. In this case, each tool is replaced upon failure; therefore, $\theta_s = 0$ and $t_s = t$ in equation (39). The expected cost ($/component) becomes

$$E[C(t)] = x[t_c + \theta_f(H(t)/t)t_c] \quad (48)$$

But since $t \to \infty$ for any reliability model, the following is true [19]

$$H(t)/t = 1/\overline{T} = \text{mean numher of failures per unit time} \quad (49)$$

Using relationship (49) in equation (48), the following is obtained

$$E[C] = (xL/f)[N^{-1} + \theta_f N^{m-1}/B_{\overline{T}}{}^m] \quad (50)$$

Optimization of equation (50) results in

$$\overset{*}{N} = B_{\overline{T}}/[\theta_f(m-1)]^n \quad (51)$$

Equation (51) is the same as the conventional deterministic equation for the optimum spindle speed.

Optimum Cutting Conditions for Multi-Tool Machining Operations

Consider M cutting tools operating simultaneously on a workpiece at a spindle speed of N rpm. Let t_{ci} denote the cutting time of the ith tool,[1] and let $t_o = \text{maximum}\{t_{c1}, t_{c2}, t_{c3}, \ldots, t_{cM}\} = L_o/f_oN$.

Preventive Planned Tool Replacement Strategy. The expected cost per component for this case can be developed as an extension of equation (10) for M tools and written as follows:

$$E[C(t_p)] = x\left[t_o + \sum_{i=1}^{M} \psi_{3i}(\tau_{pi})t_{ci}/t_{Li}\right] \quad (52)$$

where

$$\psi_{3i}(\tau_{pi}) = [\theta_{fi}F(\tau_{pi}) + \theta_{pi}R(\tau_{pi})] \Big/ \int_0^{\tau_{pi}} R(\tau_i)d\tau_i.$$

[1] Symbol i is used to identify ith tool, or other parameters or variables defined in the nomenclature, which are related to the ith tool or ith operation.

The optimum value of τ_{pi} is obtained from Fig. 3. Substitution of this optimum value, $\overset{*}{\tau}_{pi}$, in equation (52) results in

$$E[C(t_p)]\,|_{\text{at } \tau_{pi} = \overset{*}{\tau}_{pi}} = x\left[t_o + \sum_{i=1}^{M} (\theta_{fi} - \theta_{pi})h(\overset{*}{\tau}_{pi})\lambda_i t_o/t_{Li}\right] \quad (53)$$

where

$$\lambda_i = t_{ci}/t_o$$

Substituting $t_o = L_o/(f_oN)$, and $t_{Li} = (B_{t_{Li}}/N)^{m_i}$, in equation (53) will result in

$$E[C(t_p)] = xL_o/f_o\left[\frac{1}{N} + \sum_{i=1}^{M} [\theta_{fi} - \theta_{pi}]h(\overset{*}{\tau}_{pi})\lambda_i N^{m_i-1}/B_{t_L}{}^{m_i}\right] \quad (54)$$

Optimization of equation (54) with respect to spindle speed results in

$$1 = \sum_{i=1}^{M} (\theta_{fi} - \theta_{pi})\lambda_i h(\overset{*}{\tau}_{pi})(m_i - 1)(\overset{*}{N}/B_{t_{Li}})^{m_i} \quad (55)$$

For large values of M, equation (55) can be solved using a computer to find $\overset{*}{N}$. For small values of M, equation (55) can easily be solved for $\overset{*}{N}$, using a hand calculator.

A special case of considerable importance is when the tool group consists of M identical cutting tools, operating under identical cutting conditions. In this case

$$m_i = m,\ B_{t_{Li}} = B_{t_L},\ C_{fi} = C_p,\ C_{pi} = C_p,$$

$$\theta_{fi} = \theta_f,\ \theta_{pi} = \theta_p,\ \gamma_{pi} = \gamma_p,\ \lambda_i = \frac{t_o}{t_{ci}} = \frac{t_o}{t_o} = 1,$$

and

$\overset{*}{\tau}_{pi} = \overset{*}{\tau}_p$, for $i = 1, 2, 3, \ldots, M$, and, equation (55) becomes

$$1 = [M\theta_f(1-\gamma_p)h(\overset{*}{\tau}_p)(m-1)\overset{*}{N}^m]/B_{t_L}{}^m \quad (56)$$

Using equation (32), equation (56) can be rewritten as follows:

$$\overset{*}{N} = [(\overline{T}/t_L)(1-\gamma_p)h(\overset{*}{\tau}_p)]^{-n}[B_{\overline{T}}/(M\theta_f(m-1))^n] \quad (57)$$

For the Weibull reliability model, equation (57) becomes

$$\overset{*}{N} = [\Gamma(1+1/\beta)(1-\gamma_p)\beta(\overset{*}{\tau}_p)^{\beta-1}]^{-n}[B_{\overline{T}}/(M\theta_f(m-1))^n] \quad (58)$$

Scheduled Tool Replacement Strategy. The expected cost per component is

$$E[C(t_s)] = x\left[t_o + \sum_{i=1}^{M} g_{3i}(\tau_{si})t_{ci}/t_{Li}\right] \quad (59)$$

where

$$g_{3i}(\tau_{si}) = [\theta_{pi} + \theta_{fi}H(\tau_{si})]/\tau_{si}$$

To find τ_{si} for a Weibull model, use Fig. 4. As in the previous section, expressions for t_o, t_{Li}, and t_{ci} are substituted and an expected cost per component equation using $\tau_{si} = \overset{*}{\tau}_{si}$ is obtained and then optimized with respect to N to obtain

$$1 = \sum_{i=1}^{M} \theta_{fi}[(\gamma_{si} + H(\overset{*}{\tau}_{si}))/\overset{*}{\tau}_{si}][\lambda_i(m_i-1)\overset{*}{N}^{m_i}/B_{t_{Li}}{}^{m_i}] \quad (60)$$

Again, for large values of M, equation (60) may be solved for $\overset{*}{N}$ using a computer and for small values of M, a hand calculator will be sufficient.

Now consider the important case of M identical cutting tools operating under identical cutting conditions. Then, equation (60) will reduce to

$$1 = M\theta_f[(\gamma_s + H(\overset{*}{\tau}_s))/\overset{*}{\tau}_s](m-1)\overset{*}{N}^m/B_{f_L}{}^m \quad (61)$$

Substituting equation (32) into equation (61) and solving for $\overset{*}{N}$ results in

$$\overset{*}{N} = [\overset{*}{\tau}_s/[(\overline{T}/t_L)(\gamma_s + H(\overset{*}{\tau}_s))]]^n[B_{\overline{T}}/(M\theta_f(m-1))^n] \quad (62)$$

For a Weibull model

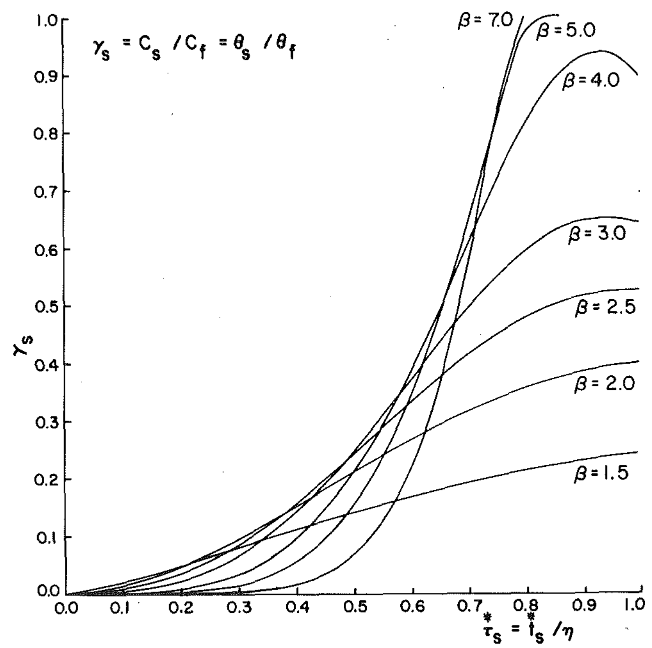

Fig. 4 Optimal Planned Replacement Invervals for Scheduled Replacement Policy for a Weibull Reliability Model.

$$\overset{*}{N} = [\overset{*}{\tau}_s/[\Gamma(1 + 1/\beta)(\gamma_s + H(\overset{*}{\tau}_s))]]^n [B_{\overline{T}}/(M\theta_f(m-1))^n] \quad (63)$$

Failure Tool Replacement Strategy. The expected cost per component is

$$E[C] = x\left[t_o + \sum_{i=1}^{M} (\theta_{fi}/\overline{T}_i)t_{ci}\right] \quad (64)$$

Expressions for t_o, t_{ci}, and $\overline{T}_i = (B_{\overline{T}_i}/N)^{m_i}$ are substituted in equation (64) to obtain

$$E[C] = (xL/f_o)\left[1/N + \sum_{i=1}^{M} \theta_{fi}\lambda_i N^{m_i - 1}/B_{\overline{T}_i}{}^{m_i}\right] \quad (65)$$

Optimization of equation (65) with respect to N results in

$$1 = \sum_{i=1}^{M} \theta_{fi}(m_i - 1)\lambda_i \overset{*}{N}{}^{m_i}/B_{\overline{T}_i}{}^{m_i} \quad (66)$$

Consider the case of M identical cutting tools, cutting under identical cutting conditions. Equation (65) will become

$$\overset{*}{N} = B_{\overline{T}}/(M\theta_f(m-1))^n \quad (67)$$

Group Tool Replacement Strategy. Consider a group of M identical cutting tools (i.e., $\beta_i = \beta$, $m_i = m$, and $C_{\overline{T}_i} = C_{\overline{T}}$ or $B_{\overline{T}_i} = B_{\overline{T}}$, $i = 1, 2, \ldots, M$), operating under identical cutting conditions. Each tool has a Weibull reliability model $R_1(t) = R_2(t) = R_3(t) = \ldots = R_M(t) = R(t) = \exp[-(t/\eta)^\beta]$. Let $R_g(t)$ be the reliability model of the group. If failure of each tool in the group is considered statistically independent, then according to the product law of probabilities:

$$R_g(t) = \prod_{i=1}^{M} R_i(t) = [R(t)]^M = [\exp[-(t/\eta)^\beta]]^M$$

$$= \exp[-M(t/\eta)^\beta] = \exp\left[-\left[\frac{t}{\eta/M^{1/\beta}}\right]^\beta\right] \quad (68)$$

and for this case the hazard function of the group, $h_g(t)$, is the sum of the hazard functions of individual tools [7, 20]:

$$h_g(t) = \sum_{i=1}^{M} h_i(t) = Mh(t) \quad (69)$$

From equation (68) it can be seen that group reliability is also described by a Weibull model with its shape parameter, $\beta_g = \beta$, and characteristic life

$$\eta_g = \eta/M^{1/\beta} \quad (70)$$

Equation (70) can also be obtained directly from equation (69). Substituting $\eta_g = \overline{T}_g/\Gamma(1 + 1/\beta)$, and $\eta = \overline{T}/\Gamma(1 + 1/\beta)$ in equation (70), the following is obtained

$$(1/\overline{T}_g) = M^{1/\beta}(1/\overline{T}) \quad (71)$$

where $\overline{T}_g$ = mean time to failure (MTTF) of the group of cutting tools. Since $\overline{T} = (C_{\overline{T}}/V)^m = (B_{\overline{T}}/N)^m$; therefore, equation (71) will become

$$(1/\overline{T}_g) = M^{1/\beta}(N/B_{\overline{T}})^m \quad (72)$$

Let $\overline{C}_{gf,r}$ equal the average cost of replacement per tool, under the group replacement strategy and $\overline{\theta}_{gf,r} = \overline{C}_{gf,r}/x$. The variable part of the expected machining cost per component (\$/component) is given by [13]

$$E[C] = x[t_o + M\overline{\theta}_{gf,r}t_o/\overline{T}_g] \quad (73)$$

As before, substitute the appropriate expression for t_o and $\overline{T}_g$, using equation (72), into equation (73) to obtain;

$$E[C] = (xL_o/f_o)[1/N + M\theta_{gf,r}M^{1/\beta}N^{m-1}/(B_{\overline{T}})^m] \quad (74)$$

The optimization of equations (74) with respect to N will yield:

$$\overset{*}{N} = B_{\overline{T}}/[M^{1/\beta}(\overline{\theta}_{gf,r}M)(m-1)]^n \quad (75)$$

When $K \to 0$, $\beta \to \infty$, and $1/\beta \to 0$, and equation (75) reduces to

$$\overset{*}{N} = B_{\overline{T}}/[(M\overline{\theta}_{gf,r})(m-1)^n,$$

which is equal to the $\overset{*}{N}$ for the individual tool failure replacement policy which is the same as optimal spindle speed obtained using the deterministic approach.

Example

The drilling of identical holes on a multispindle drilling machine is a common machining operation. This operation will be used to illustrate the use of this theory for all four replacement strategies. This is a simple example but has considerable practical importance. Since identical drills will be used, the subscript, i, will be omitted.

The present operating condition is that eight drills are drilling to a depth of 1.5 inches, at feed rate of .005 ipr, and the present operating speed is 220 rpm. It has been observed that the tool life, $\overline{Q}$, is 400 holes. The coefficient of variation calculated using this tool life data at this operating condition is .333. Typically, the value of n for this type of tool-workpiece combination is 0.12. The estimated costs per cutting edge are: planned replacement (C_p = \$4), failure replacement (C_f = \$7.28), and scheduled replacement (C_s = \$4). The total machine cost is \$0.40 per minute; therefore, $\theta_p = \theta_s = 10$ minutes and $\theta_f = 18.2$ minutes.

Before proceeding to evaluate the replacement strategies, the tool life data is used to establish the appropriate reliability model. Assume that the Weibull model adequately represents the data; therefore, the shape parameter, β, is $1/K$ or 3.0. The characteristic life, η, in number of components, is $\overline{Q}/\Gamma(1 + 1/\beta)$; however, the theory presented uses minutes. The mean tool life in minutes is

$$\overline{T} = \overline{Q}/(L/fN) = 546 \text{ minutes}$$

and the characteristic life is

$$\eta = t_L = \overline{T}/\Gamma(1 + 1/\beta) = 608.4 \text{ minutes}$$

The above data is all that is necessary to find the optimal tool replacement interval at the present operating speed. In order to use the theory to find the optimal spindle speed and the new optimal replacement interval, $B_{\overline{T}}$ and B_{t_L} must be known. Using equation (2) and Table 1, they are as follows:

$$B_{\overline{T}} = N(\overline{T})^n = 468.6$$

$$B_{t_L} = N(t_L)^n = 474.8$$

Preventive Planned Tool Replacement Strategy (PPTRS). The cost ratio, γ_p, is \$4/\$7.28 or .55, and this value is used with Fig. 3 to obtain $\overset{*}{\tau}_p = t_p/\eta = 0.87$. The optimal spindle speed, $\overset{*}{N}$, using equation (58) is 205 rpm. The characteristic life at 205 rpm is

$$\overset{*}{\eta} = \overset{*}{t}_L = (B_{tL}/\overset{*}{N})^m = 1095.5 \text{ minutes}$$

The optimum preventive planned replacement interval in number of components is

$$Q_p = \overset{*}{t}_p/\overset{*}{t}_o = \overset{*}{\eta}\overset{*}{\tau}_p/(L_o/f_o\overset{*}{N}) = 749$$

Using equation (54), the expected variable cost at 205 rpm is $E[C(\overset{*}{t}_p)]$ = \$0.66495/component or \$664.95/1000 components.

Scheduled Tool Replacement Strategy (STRS). The cost ratio, γ_s is 0.55 and this value is used with Fig. 4 to obtain $\overset{*}{\tau}_s = t_s/\eta = 0.77$. The optimal spindle speed, $\overset{*}{N}$, using equation (63) is 201 rpm. The characteristic life at 201 rpm is $\overset{*}{\eta} = \overset{*}{t}_L = 1265.09$ minutes and Q_s equals 847.6 components. Using equation (59), the expected variable cost at 201 rpm is $E[C(\overset{*}{t}_p)]$ = \$0.67843/component or \$678.43/1000 components.

Failure Tool Replacement Strategy (FTRS). Using equation (67), $\overset{*}{N}$ is 203 rpm. It follows that $\overline{T}$ at 203 rpm is 1188.8 minutes and $\overline{Q}$ is 804.4 components. Using equation (64), the expected variable cost at 203 rpm is $E[\overset{*}{C}]$ = \$0.66353/component or \$663.53/1000 components.

Group Tool Replacement Strategy (GTRS). In this case, $C_{gf,r}$ (or $\theta_{gf,r}$) will be slightly less than C_f (or θ_f). This is due to the fact that a single tool failure causes a group failure. The remaining $M - 1$ tools will be removed prior to complete failure; therefore, there will be less regrinding and other replacement costs per cutting edge. In this example, let $\theta_{gf,r} = 16$ machine minutes (i.e., $C_{gf,r} = \$6.4$/cutting edge). Using equation (75), N is 190 rpm and

$$\overline{T} = (B_{\overline{T}}/\overset{*}{N})^m = 1849.5 \text{ minutes}$$

$$\overline{T}_g = \overline{T}/M^{1/\beta} = 925.43 \text{ minutes}$$

$$\overline{Q}_g = \overline{T}_g/\overset{*}{t}_o = 586 \text{ components}$$

Table 2 compares the results for all four replacement strategies. Suppose that by proper regrinding of the tools, using better quality tool material and better control of the overall machining process, the coefficient of variation could be decreased to 0.2. While this is not generally the objective of tool builders and users, reduction in tool life variability (i.e., reduction in coefficient of variation) has been strongly recommended, see discussion by Black and Cohen (4). The new results for $K = 0.2$, and $\gamma_p = \gamma_s = 0.55$, and for $\gamma_p = \gamma_s = .3$, are presented in Tables 3 and 4.

Table 2 Results of Computations when $K = .333$, $\gamma_p = .55$, $\gamma_s = .55$

Tool Replacement Strategy	PPTRS	STRS	FTRS	GTRS
$\overset{*}{N}$, rpm	205	201	203	190
Optimum planned replacement interval or Average Life at $\overset{*}{N}$, (# of Components)	$\overset{*}{Q}_p$ = 290	$\overset{*}{Q}_s$ = 848	$\overline{Q}$ = 804	$\overline{Q}$ = 586
Expected cost/1000 Components, \$	665	678.4	663.53	719

Table 3 Comparison of Various Replacement Strategies when $K = 0.2$ and $\gamma_p = \gamma_s = .55$

Tool Replacement Strategy	PPTRS	STRS	FTRS	GTRS
$\overset{*}{N}$, rpm	208	218	203	196
Optimum planned replacement interval or Average Life at $\overset{*}{N}$, (# of Components)	$\overset{*}{Q}_p$ = 520	$\overset{*}{Q}_p$ = 324	$\overline{Q}$ = 804	$\overline{Q}_g$ = 615
Expected Cost/1000 Components, \$	654.6	677	663.53	695.5

Table 4 Comparison of Various Replacement Strategies when $K = 0.2$, and $\gamma_p = \gamma_s = .3$

Tool Replacement Strategy	PPTRS	STRS	FTRS	GTRS
$\overset{*}{N}$, rpm	207	204	191	†186
Optimum planned replacement interval or Average Life at $\overset{*}{N}$, (# of Components)	$\overset{*}{Q}_p$ = 429	$\overset{*}{Q}_s$ = 439	$\overline{Q}$ = 1127	$\overline{Q}_g$ = 903
Expected Cost/1000 Components, \$	653.7	675.1	713.4	733.7

†Using $\theta_{gf,r}$ = 25 machine minutes

Conclusions

The significance of the coefficient of variation of cutting tools in machining economics is illustrated. The equations developed in this paper require that the coefficient of variation be independent of the cutting parameters. When K is not independent of the cutting parameters, the exponents n, n_1, n_2 in the tool life equation will also vary. Investigations are continuing on how this theory can be modified for the condition when K is a function of the cutting parameters.

Using the Weibull model, the effect of tool life scatter and the cost ratio on the optimum tool replacement interval for preventive and scheduled tool replacement strategies is shown in Figs. 3 and 4. These same relationships have been developed for other reliability models [13].

The optimum spindle speed using probabilistic models of tool life is a multiple of the optimum spindle speed calculated from the classic deterministic equations. This multiplying factor is dependent upon the coefficient of variation; preventive or scheduled replacement and failure replacement cost ratio; and the tool replacement strategy. Tables 2, 3 and 4 show that when the cost ratio (i.e., γ_s or γ_p) and/or coefficient of variation, K decreases, this multiplying factor increases the optimal spindle speed based on a deterministic model to a level that can substantially increase productivity. For smaller values of γ_s or γ_p and/or smaller values of K, the cost effectiveness of preventive and scheduled replacement policy increases as compared to the failure replacement policy.

The most important conclusion concerns how this approach relates to current methods for optimization of machining operations. Simulation has been the primary approach for stochastic optimization. By directly using the probability model with classical optimization techniques, a more direct and efficient way is available to obtain optimal cutting conditions. Additionally it has the potential to be used as an in-process decision making model. Classical optimization approaches using deterministic variables can be enriched by using the probability models and replacement strategies that have been investigated in this research. For these reasons, more effort should be made to expand the use of probability models and replacement strategies in a variety of automatic production systems.

References

1 Taylor, J., "Carbide Cutting Tool Variance and Breakage: Unknown Factors in Machining Economics," *Proceedings of the 8th International MTDR Conference*, 1967, pp. 487–504.

2 Wager, J. G., "The Nature and Significance of the Distribution of High-Speed Tool Life," Ph.D. Thesis, Purdue University, 1967.

3 Ramalingam, S., and Watson, J. D., "Tool-Life Distributions. Part 1: Single Injury Tool-Life Model," ASME, Journal of Engineering for Industry, Vol. 99B, No. 3, 1977, p. 519.

4 Ramalingam, S., and Watson, J. D., "Tool Life Distributions. Part 2: Multiple-Injury Tool-Life Model," ASME, Journal of Engineering for Industry, Vol. 99B, No. 3, 1977, p. 523.

5 Ramalingam, S., Peng, Y. I., and Watson, J. D., "Tool Life Distributions.

Part 3: Mechanism of Single Injury Tool Failure and Tool Life Distribution in Interrupted Cut," ASME Paper No. 77-WA/Prod-39.
6 Duncan, J., "Mathematical and Numerical Methods for Scheduling Cutting Tool Replacements and Applications," Ph.D. Thesis, The University of Michigan, 1969.
7 Sheikh, A. K., "Optimal Tool Replacement Strategies for Multi-Tool Arrangements," M.S. Thesis, Washington State University, 1975.
8 Levi, R., "Extreme-Value Analysis of Twist Drill Performance," Metal Processing and Machine Tools, R. T. M. Vico Canavese, 1970, pp. 59–70.
9 Pandit, S. M., "Data Dependent Systems Approach to Stochastic Tool Life and Reliability," ASME, JOURNAL OF ENGINEERING FOR INDUSTRY, Vol. 100B, 1978, pp. 318–322.
10 Batra, J. L., and Barash, M. M., "Computer-Aided Planning of Optimal Machining Operations for Multi-Tool Setups with Probabilistic Tool Life," Report No. 49, Purdue Laboratory for Applied Industrial Control, Purdue University, Jan. 1972.
11 Keiji Okushima, and Susumu Fujii, "A Contribution to Tool Change Interval," *Bulletin of the Japan Society of Mechanical Engineers,* Vol. 12, No. 52, 1969, pp. 910–917.
12 Kendall, L. A., and Sheikh, A. K., "Tool Replacement Strategy for Multi-Tool Machines Using a Probabilistic Model," *Proceedings Fourth North American Metal-working Research Conference,* 1976, pp. 416–419.
13 Sheikh, A. K., "Tool Life Variability and Its Significance in Automated Production," Ph.D. Thesis, Michigan Technological University, 1978.
14 Kendall, L. A., and Sheikh, A. K., "Coefficient of Variation: An Index of Tool Reliability and Its Significance in Automated Production," NAMRC, VII, 1979, pp. 362–369.
15 Kendall, L. A., and Sheikh, A. K., "Cost Optimization Models for Planned Replacements," Failure Prevention and Reliability Conference, St. Louis, MO, Sept. 9–12, 1979.
16 Zdeblick, W. J., and Devor, R. E., "An Open-loop Adaptive Control Methodology," NAMRC, VII, 1979, pp. 300–306.
17 Pandit, S. M., and Sheikh, A. K., "Reliability and Optimal Replacement via Coefficient of Variation," Failure Prevention and Reliability Conference, St. Louis, MO, Sept. 9–12, 1979, ASME, *Journal of Mechanical Design,* Vol. 102, 1980.
18 Rossetto, S., and Levi, R., ASME, JOURNAL OF ENGINEERING FOR INDUSTRY, Aug. 1975, pp. 945–950.
19 Kronenberg, M., *Machining Science and Application,* Pergamon Press, New York, 1966.
20 Lipson, C., and Sheth, N. J., *Statistical Design and Analysis of Engineering Experiments,* McGraw-Hill Book Company, New York, 1973.
21 Gnedenko, B. V., Belyayev, Yu. K., and Solovyev, A. D., *Mathematical Methods of Reliability Theory,* Academic Press, New York, 1969.
22 Armarego, E. J. A., and Brown, R. H., *The Machining of Metals,* Prentice-Hall, N.J., 1969.