

IMECE2006-15654**SYNCHRONIZATION OF CHAOTIC SYSTEMS USING VARIABLE STRUCTURE CONTROLLERS****Hassan Salarieh**

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ABSTRACT

In this paper a variable structure system based upon sliding mode control with time varying sliding surface and variable boundary layer is introduced to synchronize two different chaotic systems with uncertain parameters. The method is applied to Lur'e-Genesio chaotic systems, as drive-response systems to investigate the effectiveness and robustness of the controlling method. In addition the simulation is repeated with a conventional sliding mode to compare the performance of the proposed sliding mode technique with a simple sliding mode control. The results show the high quality and improved performance of the method presented in the paper for synchronization of different drive-response chaotic systems.

INTRODUCTION

In the last few years, synchronization in chaotic dynamical systems has received a great deal of interests among scientists from various fields [1–2]. The results of chaos synchronization are utilized in biologic synchronization, chemical reaction synchronization, secret communication and cryptography, nonlinear oscillation synchronization and some other nonlinear fields. The first idea of synchronizing two identical chaotic systems with different initial conditions was introduced by Pecora and Carrols [3-5], and the method was realized in electronic circuits. The methods for synchronization of the chaotic systems have been advanced in recent years, and many different methods have been applied theoretically and experimentally to synchronize the chaotic systems [6-8]. A basic configuration for chaos synchronization is the drive-response pattern, where the response chaotic system must track the drive chaotic trajectory. A number of methods based on this configuration have been proposed. More recently some techniques have been investigated for synchronization in hyper-

chaotic systems [9,10], and a generalized method for synchronization of chaotic systems has been proposed [11,12]. Besides, using the active control methods some different synchronization problems have been investigated [13,14]. For example a coupled Lorenz system has been synchronized in [5], and also some active control methods have been applied to Chen and Rossler systems to obtain a synchrony behavior from them [15]. In [16,17], generalized synchronization method with parametric adaptive control has been introduced. Also linear and nonlinear feedback control methods have been studied for chaos synchronization applications. Recently discontinuous control method has been increasingly developed for chaos synchronization [18,19]. In [20,21] an H_∞ robust synchronization methods for chaotic Lur'e systems via static state error feedback and dynamic output error feedback has been proposed. In [22], an adaptive control approach has been presented. The impulsive robust control method has been proposed in [23,24] for the synchronization of uncertain Lur'e systems and the coupled chaotic systems. In [25] a new controller based on delay feedback has been presented for chaos synchronization. Various controlling methods such as variable structure methods [26], parametric adaptive control [27], observer based control [28,29], and so on, have been successfully applied for chaos synchronization. Most of the mentioned works have been applied for two identical chaotic systems. In practice it is difficult to find two exactly identical chaotic systems. Hence, the synchronization of two different chaotic systems plays a significant role in practical applications [30,31]. This problem will be more challenging and difficult if the parameters of two chaotic systems have some uncertainties or the environment apply some random noise to the dynamic systems or measured output. Actually in practice, it hardly occurs that the both master and slave systems have the same

configuration or the measured signals are not influenced by some stochastic noises. In [32] a nonlinear controller based on Lyapunov theorem has been used to design a controller for chaos synchronization of two different systems.

In this paper we use a sliding mode controller with time varying sliding surface and variable boundary layer to synchronize the behavior of two different chaotic systems which have some uncertainties in their parameters. It is assumed that the response system is exercised by some bounded random process. Using the variable boundary layer one can eliminate both the chattering phenomena in the sliding surface and the steady state error that appears due to boundary layer. It causes a smooth behavior in convergence of the tracking error to zero. In addition using time varying sliding surface the rate of convergence can be controlled to obtain a desired convergence performance. The method presented in this paper is applied to a Lur'e like and a Genesio chaotic systems as the drive and the response systems, respectively. The results of the proposed method for synchronization are compared with the ones obtained from a conventional sliding mode controller.

SYNCHRONIZING CONTROL DESIGN

Consider the following system described by:

$$\dot{x}^{(n)} = f(\underline{x}, t) \quad (1)$$

where $\underline{x} = (x, \dot{x}, \dots, x^{(n-1)}) \in \mathcal{R}^n$ is the state vector, and $f: \mathcal{R}^n \times \mathcal{R}^+ \rightarrow \mathcal{R}$ is a nonlinear, and sufficiently smooth function. Equation (1) is considered as a drive system. It is assumed that the function f is not exactly known, and its approximate value is denoted by \hat{f} , and we have

$$|f(\underline{x}, t) - \hat{f}(\underline{x}, t)| < F(\underline{x}, t) \quad (2)$$

where $F(\underline{x}, t)$ is a known bounded function. The controlled response system is given by:

$$\dot{y}^{(n)} = g(\underline{y}, t) + b(\underline{y}, t)u + \delta(t) \quad (3)$$

where $\underline{y} = (y, \dot{y}, \dots, y^{(n-1)}) \in \mathcal{R}^n$ is the state vector, $u \in \mathcal{R}$ is the control variable of the system, $g, b: \mathcal{R}^n \times \mathcal{R}^+ \rightarrow \mathcal{R}$ are sufficiently smooth functions, and $\delta(t)$ denotes the noise disturbing the system. Similarly, the functions g and b have some uncertainties and their nominal values are shown by \hat{g} and \hat{b} . It is also assumed that the function b is a positive definite function which has a strictly positive lower bound b_m :

$$|g(\underline{y}, t) - \hat{g}(\underline{y}, t)| < G(\underline{y}, t) \quad (4)$$

$$b(\underline{y}, t) > b_m(\underline{y}, t) > 0 \quad (5)$$

$$|\delta(t)| < \Delta \quad (6)$$

The functions f and g are not the same.

The synchronization problem is to design a controller u which synchronizes the states of both the drive and the response systems.

By subtracting (1) from (3) it is obtained that:

$$\dot{e}^{(n)} = g(\underline{y}, t) - f(\underline{x}, t) + b(\underline{y}, t)u \quad (7)$$

where $\underline{e} = \underline{y} - \underline{x}$. The aim of synchronization is:

$$\lim_{t \rightarrow \infty} \|\underline{e}(t)\| = 0 \quad (8)$$

Let a time varying sliding surface be:

$$S(t) = \left(\frac{d}{dt} + \lambda(t) \right)^{n-1} e(t) \quad (9)$$

The goal is to design a controller that makes the system reach to the sliding surface as smoothly as possible. Eventually it is shown that in the sliding surface the system trajectories approach to the origin. To this end define a Lyapunov function as $V = \frac{1}{2} S(t)^2$, where V is a positive definite function. Now the derivative of the Lyapunov function along the error trajectories are obtained as:

$$\dot{V} = S(t)\dot{S}(t) = S(t) \left[\sum_{m=0}^{n-1} C_m^{n-1} e^{(m+1)} \lambda(t)^{n-1-m} + \sum_{m=0}^{n-1} C_m^{n-1} (n-1-m) e^{(m)} \dot{\lambda}(t) \lambda(t)^{n-m-2} \right] \quad (10)$$

Now using the equation (7) it is obtained that:

$$\dot{V} = S(t) \left[\sum_{m=0}^{n-2} C_m^{n-1} e^{(m+1)} \lambda(t)^{n-1-m} + \sum_{m=0}^{n-1} C_m^{n-1} (n-1-m) e^{(m)} \dot{\lambda}(t) \lambda(t)^{n-m-2} + g - f - bu - \delta(t) \right] \quad (11)$$

The controller u must be designed such that the trajectories of the error dynamics approach to the sliding surface in a finite time. In the sliding surface the dynamics of the system is degenerated to:

$$S(t) = \sum_{m=0}^{n-1} C_m^{n-1} e^{(m)} \lambda(t)^{n-1-m} = 0 \quad (12)$$

So in the sliding surface we have a time-varying linear system. $\lambda(t)$ must be chosen in such a way that the dynamic system on the sliding surface be stable. So one may define a dynamics for $\lambda(t)$ as:

$$\dot{\lambda}(t) = \begin{cases} \eta \lambda(t) & t < T_0 \\ 0 & \text{otherwise} \end{cases}, \quad \eta > 0, \quad \lambda(t=0) = \lambda_0 \quad (13)$$

where T_0 is an arbitrary constant time. Equation (13) implies that the rate of convergence in the sliding surface increases as time goes on. To obtain the negative definiteness condition of \dot{V} , one can define the controller u as:

$$u = \frac{-1}{b_m} \left[\hat{f} - \hat{g} - \sum_{m=0}^{n-2} C_m^{n-1} e^{(m+1)} \lambda(t)^{n-1-m} - \sum_{m=0}^{n-1} C_m^{n-1} (n-1-m) \dot{\lambda}(t) \lambda(t)^{n-m-2} - K \text{sign}(S(t)) \right] \quad (14)$$

Substituting equation (14) in (11), results in:

$$\begin{aligned} \dot{V} = S(t) \left\{ \frac{b}{b_m} (\hat{f} - \hat{g}) + g - f - \delta(t) + \left(1 - \frac{b}{b_m}\right) \left[\sum_{m=0}^{n-2} C_m^{n-1} e^{(m+1)} \lambda(t)^{n-1-m} \right. \right. \\ \left. \left. + \sum_{m=0}^{n-1} C_m^{n-1} (n-1-m) e^{(m)} \dot{\lambda}(t) \lambda(t)^{n-m-2} \right] - \frac{b}{b_m} K \text{sign}(S(t)) \right\} \end{aligned} \quad (15)$$

So setting:

$$\begin{aligned} K \geq \frac{b_m}{b} \left\{ F(\underline{x}, t) + G(\underline{y}, t) + \left(\frac{b}{b_m} - 1\right) [\hat{f}(\underline{x}, t) + \hat{g}(\underline{y}, t) \right. \\ \left. + \sum_{m=0}^{n-2} C_m^{n-1} \left| e^{(m+1)} \right| \lambda(t)^{n-1-m} \right. \\ \left. + \sum_{m=0}^{n-1} C_m^{n-1} (n-1-m) \left| e^{(m)} \right| \dot{\lambda}(t) \lambda(t)^{n-m-2} \right] + \Delta + \theta \right\}, \quad \theta > 0 \end{aligned} \quad (16)$$

where θ is an arbitrary positive number, it is easily seen that:

$$\dot{V} \leq -\theta |S(t)| \quad (17)$$

So the sliding surface attracts the system trajectories in a finite time, and the trajectories of the system approach to the sliding surface globally.

$$u = \begin{cases} \frac{-1}{b_m} [\hat{f} - \hat{g} - \sum_{m=0}^{n-2} C_m^{n-1} e^{(m+1)} \lambda(t)^{n-1-m} - \sum_{m=0}^{n-1} C_m^{n-1} (n-1-m) e^{(m)} \dot{\lambda}(t) \lambda(t)^{n-m-2} - K \text{sign}(S(t)) \cdot 1(\text{dist}(\underline{e}, S(t)) - \varepsilon)] & \text{if } \text{dist}(\underline{e}, S(t)) > \varepsilon \\ \frac{-1}{b_m} [\hat{f} - \hat{g} - \sum_{m=0}^{n-2} C_m^{n-1} e^{(m+1)} \lambda(t)^{n-1-m} - \sum_{m=0}^{n-1} C_m^{n-1} (n-1-m) e^{(m)} \dot{\lambda}(t) \lambda(t)^{n-m-2} - K \text{sign}(S(t)) \cdot \text{dist}(\underline{e}, S(t)) / \varepsilon] & \text{if } \text{dist}(\underline{e}, S(t)) \leq \varepsilon \end{cases} \quad (20)$$

where

$$1(\xi) = \begin{cases} 1 & \xi > 0 \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

ε is the thickness of the boundary layer around the sliding surface. To investigate the attraction of the boundary layer we substitute the controller law of equation (20) in equation (15). When $\text{dist}(\underline{e}, S(t)) > \varepsilon$,

$$\begin{aligned} \dot{V} = S(t) \left\{ \frac{b}{b_m} (\hat{f} - \hat{g}) + g - f + \delta(t) + \left(1 - \frac{b}{b_m}\right) \left[\sum_{m=0}^{n-2} C_m^{n-1} e^{(m+1)} \lambda(t)^{n-1-m} \right. \right. \\ \left. \left. + \sum_{m=0}^{n-1} C_m^{n-1} (n-1-m) e^{(m)} \dot{\lambda}(t) \lambda(t)^{n-m-2} \right] - \frac{b}{b_m} K \text{sign}(S(t)) \right\} \end{aligned} \quad (22)$$

So for $\text{dist}(\underline{e}, S(t)) > \varepsilon$ we have:

$$\dot{V} \leq -\theta |S(t)| \quad (23)$$

It follows that the sliding surface is attracting, consequently the boundary layer will become attracting, because the sliding surface is a subset of boundary layer.

The designed controller is not continuous due to using sign function in definition of u . The discontinuity of the controller results in undesired nonlinear phenomena, chattering, which may cause some difficulties in numerical or practical applications of the controller. One of the natural solutions to eliminate this problem is to define a boundary layer for sliding surface. Here we define a time varying boundary layer for the sliding surface. The thickness of boundary layer is denoted by $\varepsilon(t)$ that is time varying, and it is defined as:

$$B = \{z \in \mathbb{R}^n, \text{dist}(z, S(t)) \leq \varepsilon\} \quad (18)$$

where,

$$\text{dist}(\underline{z}, S(t)) = \inf \{\|z - p\|, p \in S(t)\} \quad (19)$$

The controller law is defined in such a way that the boundary layer becomes an attracting set,

Definition of a boundary layer for sliding surface results in some steady state errors, because the controller law within the boundary layer does not necessarily make the trajectories attract to the sliding surface. If we define a contracting dynamics for the boundary layer thickness, the steady state error can approach to zero, also due to existence of the boundary layer the discontinues behavior of the controlling signal is eliminated. Here a simple contracting dynamics for the thickness of the boundary layer is proposed as:

$$\dot{\varepsilon} = -\mu \varepsilon, \quad \varepsilon(t=0) = \varepsilon_0 \quad (24)$$

It must be noted that similar to a conventional sliding mode control method, one can define the integral of the state error as a new state to get a better performance for synchronization. To this end, the sliding surface of the system turns into:

$$S(t) = \left(\frac{d}{dt} + \lambda(t) \right)^n \int_0^t e(\tau) d\tau \quad (25)$$

The controller law can be rewritten as:

$$u = \begin{cases} \frac{1}{b_m} [\hat{f} - \hat{g} - \sum_{m=0}^{n-1} C_m^{n-1} e^{(m)} \lambda(t)^{n-1-m} - \sum_{m=0}^n C_m^{n-1} (n-1-m) \int_0^t e^{(m)} d\tau \dot{\lambda}(t) \lambda(t)^{n-m-2} - K \text{sign}(S(t)) \cdot 1(\text{dist}(\int_0^t e(\tau) d\tau, \underline{e}, S(t)) - \varepsilon)] & \text{if } \text{dist}(\underline{e}, S(t)) > \varepsilon \\ \frac{1}{b_m} [\hat{f} - \hat{g} - \sum_{m=0}^{n-1} C_m^{n-1} e^{(m+1)} \lambda(t)^{n-1-m} - \sum_{m=0}^n C_m^{n-1} (n-1-m) \int_0^t e^{(m)} d\tau \dot{\lambda}(t) \lambda(t)^{n-m-2} - K \text{sign}(S(t)) \cdot \text{dist}(\int_0^t e(\tau) d\tau, \underline{e}, S(t)) / \varepsilon] & \text{if } \text{dist}(\underline{e}, S(t)) \leq \varepsilon \end{cases} \quad (26)$$

Remark 1: The variable boundary layer in a sliding mode control increases the system dimension, and this implies that some extra analysis is necessary to be accomplished about the system stability. Generally if the boundary layer thickness is constant, the control laws in equation (19) or equation (26) make the boundary layer be an attracting set. On the other hand it must be noted that the variations of boundary layer structure due to equation (24) does not affect on the sliding surface and its corresponding Lyapunov function or its derivative. In other words the attraction of the boundary layer is independent of its structural variations due to equation (24). So the stability of the closed loop system with variable boundary layer is achieved.

Remark 2: To achieve properly the objective of chattering reduction in addition to steady state error decrease, when the variable boundary layer is used, one must carefully adjust the coefficient μ in equation (24). It must be noted that the rate of decreasing the boundary layer thickness must be less than the rate of convergence of the system trajectories to the boundary layer or the sliding surface.

SYNCHRONIZATION OF TWO DIFFERENT CHAOTIC SYSTEMS

We use the proposed method to synchronize two different chaotic systems. One is the Lur'e-like system considered as the drive system. The other is the chaotic Genesis system considered as the controlled response system. Our aim is to design a controller and make the controlled response system track the trajectories of the drive system.

Lur'e-like system, as the drive system, is considered as follows:

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= a_1 x_1 + a_2 x_2 + a_3 x_3 + 12f(x_1)\end{aligned}\quad (27)$$

where

$$f(x_1) = \begin{cases} kx_1, & |x_1| < \frac{1}{k} \\ \text{sign}(x_1), & \text{otherwise} \end{cases}\quad (28)$$

For $a_1 = -7.4$, $a_2 = -4.1$, $a_3 = -1$, and $k = 3.6$, the system shows chaotic response. The chaotic Genesis system is described by the set of three order differential equations:

$$\begin{aligned}\dot{y}_1 &= y_2 \\ \dot{y}_2 &= y_3\end{aligned}\quad (29)$$

$$\dot{y}_3 = -b_1 y_1 - b_2 y_2 - b_3 y_3 + y_1^2 + u$$

where $b_1 = 5.6$, $b_2 = 2.92$, and $b_3 = 1.2$ are the parameters of the system, and u is the control action. The behavior of both systems for $a_1 = -7.4$, $a_2 = -4.1$, $a_3 = -1$, $b_1 = 5.6$, $b_2 = 2.74$ and $b_3 = 1.1$, and $u = 0$ are shown in the phase space in figures 1, and 2.

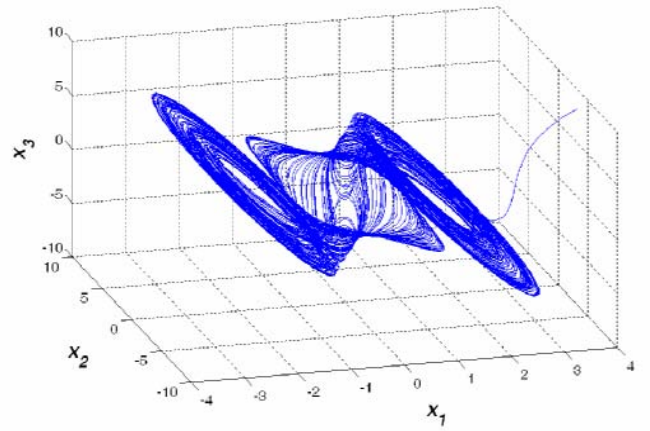


Figure 1. The chaotic attractor of the Lur'e system with $a_1 = -7.4$, $a_2 = -4.1$, $a_3 = -1$, and $k = 3.6$

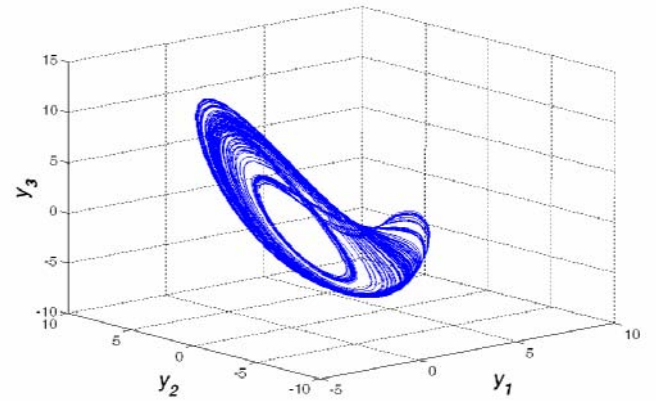


Figure 2. The chaotic attractor of the Genesis system with $b_1 = 5.6$, $b_2 = 2.74$ and $b_3 = 1.1$

We subtract (27) from (29) and get the error equation as follows:

$$\begin{aligned}\dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ \dot{e}_3 &= -b_1 y_1 - b_2 y_2 - b_3 y_3 + y_1^2 - a_1 x_1 - a_2 x_2 - a_3 x_3 \\ &\quad - 12f(x_1) + u\end{aligned}\quad (30)$$

where $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, and $e_3 = y_3 - x_3$. Suppose that there are some uncertainties for constants and functions of the Genesis and Lur'e like equations. These uncertainties are modeled as some continuous random variables which are added to the nominal values of the system parameters and functions, e.g.

$$\begin{aligned}a_i &= \hat{a}_i [1 + \rho_i n_i^1(t)] \quad , i = 1, 2, 3 \\ b_i &= \hat{b}_i [1 + \nu_i n_i^2(t)] \quad , i = 1, 2, 3 \\ f(x_1) &= \hat{f}(x_1) [1 + \vartheta_i n_i^3(t)] \quad , i = 1, 2, 3\end{aligned}\quad (31)$$

where $n_i^j(t)$ are uniform random variables between -1 and +1, and ρ_i , ν_i , and g_i are constant weight coefficients. Besides, the system dynamics of the Genesio system is excited with a random input, $\delta(t)$ which has a uniform probability density function in the interval $(-\Delta, \Delta)$. The bounds of the coefficients are

$$\begin{aligned} & \left| [a_1x_1 + a_2x_2 + a_3x_3 + 12f(x_1)] - [\hat{a}_1x_1 + \hat{a}_2x_2 + \hat{a}_3x_3 + 12\hat{f}(x_1)] \right| \\ & \leq |\hat{a}_1\rho_1x_1| + |\hat{a}_2\rho_2x_2| + |\hat{a}_3\rho_3x_3| + 12|g_i\hat{f}(x_1)| \end{aligned} \quad (32)$$

$$\begin{aligned} & \left| [b_1y_1 + b_2y_2 + b_3y_3 - y_1^2] - [\hat{b}_1y_1 + \hat{b}_2y_2 + \hat{b}_3y_3 - y_1^2] \right| \\ & \leq |\hat{b}_1\nu_1y_1| + |\hat{b}_2\nu_2y_2| + |\hat{b}_3\nu_3y_3| \end{aligned} \quad (33)$$

Let:

$$F(\underline{x}, t) = |\hat{a}_1\rho_1x_1| + |\hat{a}_2\rho_2x_2| + |\hat{a}_3\rho_3x_3| + 12|g_i\hat{f}(x_1)| \quad (34)$$

$$G(\underline{y}, t) = |\hat{b}_1\nu_1y_1| + |\hat{b}_2\nu_2y_2| + |\hat{b}_3\nu_3y_3| \quad (35)$$

$$u = \begin{cases} -[\hat{a}_1x + \hat{a}_2x + \hat{a}_3x_3 + 12\hat{f}(x_1) + \hat{b}_1y_1 + \hat{b}_2y_2 + \hat{b}_3y_3 - y_1^2 - (3\lambda(t)e_2 + 3\lambda(t)^2e_1 + \lambda(t)^3\int_0^t e(\tau)d\tau + 3\lambda(t)e_2 + 6\lambda(t)\dot{\lambda}(t)e_1 + 3\lambda(t)^2\dot{\lambda}(t)\int_0^t e_1(\tau)d\tau \\ - K\text{sign}(S(t)).1(\text{dist}(\underline{e}, S(t)) - \varepsilon)] & \text{if } \text{dist}(\underline{e}, S(t)) > \varepsilon \\ -[\hat{a}_1x + \hat{a}_2x + \hat{a}_3x_3 + 12\hat{f}(x_1) + \hat{b}_1y_1 + \hat{b}_2y_2 + \hat{b}_3y_3 - y_1^2 - (3\lambda(t)e_2 + 3\lambda(t)^2e_1 + \lambda(t)^3\int_0^t e(\tau)d\tau + 3\lambda(t)e_2 + 6\lambda(t)\dot{\lambda}(t)e_1 + 3\lambda(t)^2\dot{\lambda}(t)\int_0^t e_1(\tau)d\tau \\ - K\text{sign}(S(t)).\text{dist}(\underline{e}, S(t))/\varepsilon] & \text{if } \text{dist}(\underline{e}, S(t)) \leq \varepsilon \end{cases} \quad (38)$$

where

$$\begin{aligned} K \geq & |\hat{a}_1\rho_1x_1| + |\hat{a}_2\rho_2x_2| + |\hat{a}_3\rho_3x_3| + 12|g_i\hat{f}(x_1)| \\ & + |\hat{b}_1\nu_1y_1| + |\hat{b}_2\nu_2y_2| + |\hat{b}_3\nu_3y_3| + \Delta + \theta, \quad \theta > 0 \end{aligned} \quad (39)$$

So

$$\dot{V}(t) \leq -\theta|S(t)| \quad (40)$$

The derivation of the Lyaounov function along the error trajectory is negative definite, so it implies that the error trajectories converge to the boundary layer in a finite time. On the other hand, by defining a dynamics for the boundary layer in the form of:

$$\dot{\varepsilon} = -\mu\varepsilon, \quad \varepsilon(t=0) = \varepsilon_0, \quad \mu > 0 \quad (41)$$

the boundary layer will approach to the sliding surface, hence the trajectories of the error dynamics must move toward the sliding surface. Assuming that the distance between the error trajectory at time t , and the sliding surface is denoted by χ and manipulating some calculations it is obtained that:

$$\chi = \text{dist}(\underline{e}, S(t)) = \frac{\left| e_3 + 3\lambda(t)e_2 + 3\lambda(t)^2e_1 + \lambda(t)^3\int_0^t e(\tau)d\tau \right|}{\sqrt{1 + 9\lambda(t)^2 + 9\lambda(t)^4 + \lambda(t)^6}} \quad (42)$$

Inside the boundary layer we have $\chi \leq \varepsilon(t)$, and due to attraction of boundary layer and the dynamics of the boundary layer thickness, χ must approach to zero, as a function of time.

It must be noticed that because the coefficients $\lambda(t)$ increase, and the degree of the numerator and the denominator of χ , in

The time varying sliding surface of the error dynamics is written as:

$$S(t) = e_3 + 3\lambda(t)e_2 + 3\lambda(t)^2e_1 + \lambda(t)^3\int_0^t e_1(\tau)d\tau \quad (36)$$

Let a Lyapunov function, $V = \frac{1}{2}S(t)^2$, and using the variable boundary layer method explained in the previous section, the controlling law is designed such that the derivative of the Lyapunov function before reaching to the boundary layer is negative definite.

$$\begin{aligned} \dot{V} = & S(t)[3\lambda(t)e_3 + 3\lambda(t)^2e_2 + \lambda(t)^3e_1 + 3\dot{\lambda}(t)e_2 + 6\lambda(t)\dot{\lambda}(t)e_1 \\ & + 3\lambda(t)^2\dot{\lambda}(t)\int_0^t e(\tau)d\tau - b_1y_1 - b_2y_2 - b_3y_3 - y_1^2 - a_1x_1 - a_2x_2 - a_3x_3 \\ & - 12f(x_1) - u - \delta(t)] \end{aligned} \quad (37)$$

The control action is obtained as:

equation (42), are equal, and $\chi \rightarrow 0$ as $t \rightarrow \infty$, so we must have,

$$\lim_{t \rightarrow \infty} \chi \sqrt{1 + 9\lambda(t)^2 + 9\lambda(t)^4 + \lambda(t)^6} = 0 \quad (43)$$

Near the sliding surface, the error dynamics has the below dynamics,

$$e_3 + 3\lambda(t)e_2 + 3\lambda(t)^2e_1 + \lambda(t)^3\int_0^t e_1(\tau)d\tau = \tilde{\chi} \quad (44)$$

where $\tilde{\chi}$ has the property of,

$$|\tilde{\chi}| = \chi \sqrt{1 + 9\lambda(t)^2 + 9\lambda(t)^4 + \lambda(t)^6} \quad (45)$$

Equation (44) can be rewritten in the form of state space.

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\lambda(t)^3 & -3\lambda(t)^2 & -3\lambda(t) \end{bmatrix} \begin{bmatrix} \int_0^t e_1(\tau)d\tau \\ e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \tilde{\chi} \end{bmatrix} \quad (46)$$

Using the linear control theory, due to asymptotically stability of the system we have $\underline{e} = (\int_0^t e_1d\tau, e_1, e_2) \rightarrow (0,0,0)$, because $\tilde{\chi} \rightarrow 0$. Hence the synchronization of two different chaotic systems is completely achieved.

For two different chaotic systems, which have different structures and parameter mismatches, the proposed controller can synchronize the states of the drive and the response systems.

Remark: The system parameters used for simulation are chosen in such a way that both systems have chaotic response. For the

Lur'e and the Genesion dynamical system, the selected parameters are conventional and well-known [21], and for these conventional parameters the chaotic attractor of both systems in the state space occupy approximately the same region but their behaviors are completely different. It must be noted that the synchronizing algorithm is not dependent on the chaotic systems used for simulation and for synchronization, it is enough that the condition in equation (17) is satisfied.

SIMULATION RESULTS

In this section, numerical simulations are given to examine the effectiveness of the proposed method. In these numerical simulations, the fourth order Runge-Kutta with step size 0.001 is used. The parameters are selected as follows: $a_1 = -7.4$, $a_2 = -4.1$, $a_3 = -1$, $b_1 = 5.6$, $b_2 = 2.74$, $b_3 = 1.1$, $\rho_i = \nu_i = g_i = 0.1$, with initial values $x_1(0) = 4$, $x_2(0) = -3$, $x_3(0) = 8$, $y_1(0) = 2.3$, $y_2(0) = 0.7$, $y_3(0) = 0.2$, $\varepsilon(0) = 0.4$, and $\lambda(0) = 1$. The simulation results are illustrated in figures 3 to 5. Figure 5, shows the synchronization results of applying a conventional sliding mode control with a constant boundary layer and time-invariant sliding surface. It is seen that the performance of the system is improved in the case of variable boundary layer and time-varying sliding surface.

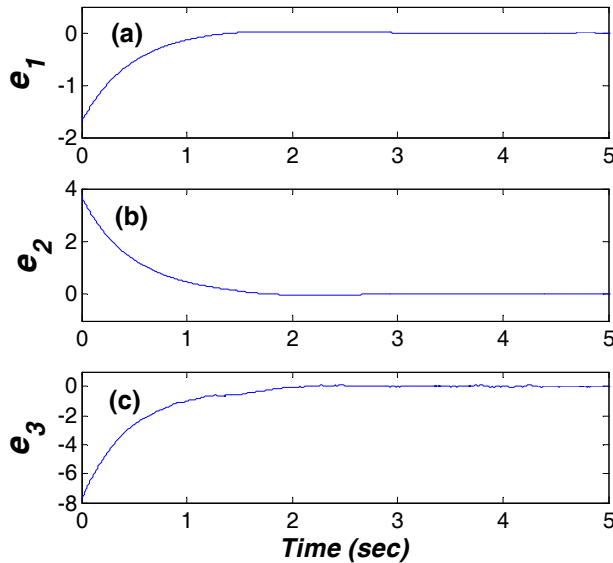


Figure 3. Dynamics of synchronization error for two different chaotic systems with random type mismatch, using time-varying sliding surface and boundary layer, (a) $e_1 = y_1 - x_1$, (b) $e_2 = y_2 - x_2$, and (c) $e_3 = y_3 - x_3$

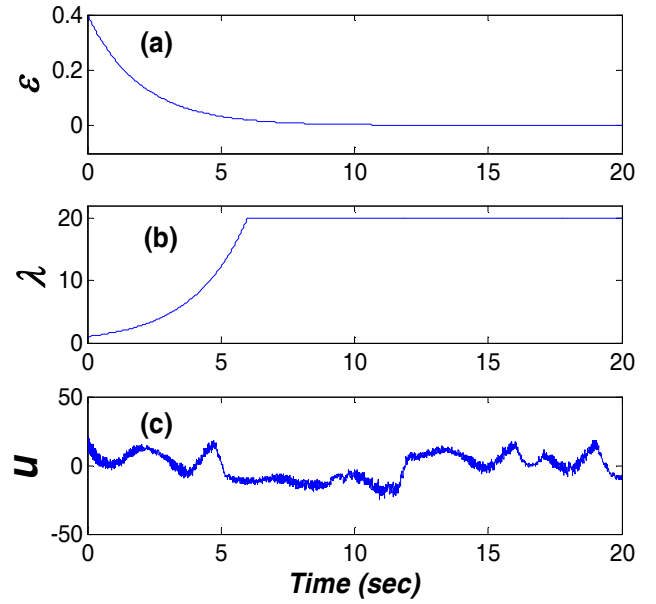


Figure 4. Variations of the controller parameters in synchronization of two different chaotic systems with random type mismatch, using time-varying sliding surface and boundary layer, (a) boundary layer thickness: $\varepsilon(t)$, (b) sliding surface eigen-values: $\lambda(t)$, and (c) controller, u .

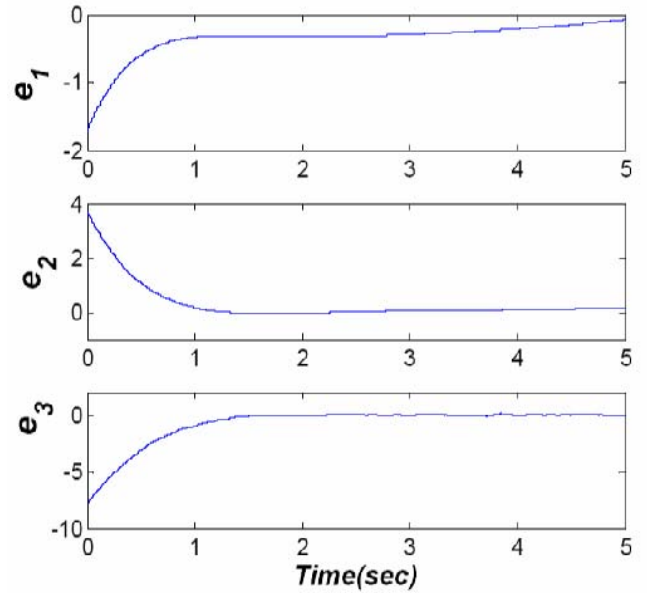


Figure 5. Dynamics of synchronization error for two different chaotic systems with random type mismatch, using constant sliding surface and constant boundary layer, (a) $e_1 = y_1 - x_1$, (b) $e_2 = y_2 - x_2$, and (c) $e_3 = y_3 - x_3$

CONCLUSION

We have provided a nonlinear control method based on variable structure systems to synchronize two different

dynamic systems with random uncertainties. The variable structure control method introduced in this paper is the sliding mode control with variable sliding surface and variable boundary layer. The advantages of the proposed method in compare to the conventional sliding mode are that the variable sliding surface and variable boundary layer adjust and increase the rate of convergence to the sliding surface, decrease the steady state error that is generated in conventional sliding mode with constant boundary layer and raise the robustness of system against uncertainties.

Simulation results show the high performance of the proposed method in synchronization of two different chaotic systems which have random uncertainties in their parameters. The drive and the response systems used for simulation are the Lur'e and the Genesio chaotic systems. Furthermore the simulation results of applying a conventional sliding mode control show the improvement of controller performance when the presented sliding mode method is exercised to the systems in compare to a conventional sliding mode control.

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