

Contents lists available at [SciVerse ScienceDirect](http://SciVerse.Sciencedirect.com)

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Supply chain outsourcing risk using an integrated stochastic-fuzzy optimization approach

Dexiang Wu^{a,*}, Desheng Dash Wu^{b,c,*}, Yidong Zhang^d, David L. Olson^{e,1}^a Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario, Canada M5S 3G8^b RiskLab, University of Toronto, Toronto, Ontario, Canada M5S 3G8^c School of Management, University of Science and Technology of China, 96 Jinzhai, Hefei, Anhui 230026, PR China^d SUNY, Buffalo, Department of Industrial and System Engineering, USA^e Department of Management, University of Nebraska, Lincoln, NE 68588-0491, USA

ARTICLE INFO

Article history:

Received 8 October 2010

Received in revised form 22 January 2012

Accepted 6 February 2013

Available online 13 February 2013

Keywords:

Supplier selection

Supply chain risk management

Fuzzy multi-objective programming

Utility

ABSTRACT

A stochastic fuzzy multi-objective programming model is developed for supply chain outsourcing risk management in presence of both random uncertainty and fuzzy uncertainty. Utility theory is proposed to treat stochastic data and fuzzy set theory is used to handle fuzzy data. An algorithm is designed to solve the proposed integrated model. The new model is solved using the proposed algorithm for a three stage supply chain example. Computation suggests an analysis of risk averse and procurement behavior, which indicates that a more risk-averse customer prefers to order less under uncertainty and risk. Trade-off game analysis yields supported points on the trade-off curve, which can help decision makers to identify proper weighting scheme where Pareto optimum is achieved to select preferred suppliers.

© 2013 Elsevier Inc. All rights reserved.

1. Introduction

Supply chain (SC) risk management has attracted considerable and increasing attention in both industry and academia, as their activities and requirements become increasingly complex [25,68]. This is especially true when the companies are expanding the geographical scope of their sourcing activities into areas where they have little experience as low cost country sourcing, e.g., China [81].

Outsourcing is a recent trend, usually adopted to gain lower production costs, but also can be used to reduce core organizational risk. In a global market, supply management offshore-sourcing strategies can include manufacturers at low cost locations such as China, India, or Vietnam, assemblers at high-tech operations in Taiwan and Korea, and distributors where customers reside all over the globe. They can also include e-business operations such as Amazon.com. The selection of suppliers in a global market is often considered as a problem involving complex systems. This has been severe under the supply chain management framework, because the factors such as default risk from other SC members and the effects to SC partners need to be considered. On the other hand, external stakeholders such as rating agencies affect the selection of appropriate suppliers by assigning independent, objective and non-binding opinions (not recommendations) on the financial strength of outsourcing candidates in the form of a globally consistent rating scale. The credit rating process historically includes business and financial risks of an organization in addition to indicators of macroeconomic conditions [3]. External credit ratings are thus particularly important in financial services because a higher credit rating establishes and maintains market

* Corresponding author.

E-mail addresses: dextre.wu@utoronto.ca (D. Wu), dash@risklab.ca (D.D. Wu), Dolson3@unl.edu (D.L. Olson).¹ Tel.: +1 402 472 4521; fax: +1 402 472 5855.

confidence. All these indicate that factors causing SC risks, their relation and the possible effects of these risks can be very complex, making practical supply risk management in industry a difficult undertaking [36].

Since supplier selection activities under supply chain management framework are complex, a supplier selection approach must be able to take this complexity into account. Many models are available to support supplier selection and outsourcing. Ref. [42] modeled supplier risk attitude with respect to risk aversion. Some studies [2,10,37] have recognized this imprecision through methods accommodating fuzzy data. But these methods fail to consider uncertainty and risk factors in an integrated model. Probability distributions from historical data are widely recognized by researchers [13,15,24,28,29,41,65,69] to model SC uncertainty (e.g., uncertain demand) in a decision model. However, because one single criteria such as the minimization of expected cost or maximization of expected profit is used, these decision models may result in sub-optimal solutions. A practical decision of selecting SC partners and sourcing arrangements usually exhibits as a multi-objective decision making problem [49], where multi-objective programming models have been presented [76,77]. But existing multi-objective programming seldom simultaneously considers multiple objective and uncertainty and risk.

In this paper, a stochastic fuzzy multi-objective programming model (SFMOP) vendor selection model is developed for supply chains outsourcing risk management. We recognize that data regarding the expected performance of suppliers in a global market are necessarily imprecise. Moreover, selecting an ideal supplier is much riskier than its domestic counterpart due to a number of exogenous risk factors influencing offshore sourcing. Therefore, both quantitative and qualitative supplier selection risk factors are examined. Quantitative risk factors include cost, quality and logistics, each expressed with stochastic data with some probability distribution. Qualitative risk factors include economic environmental factors and vendor ratings, which are of a fuzzy nature and can be quantified by a degree of belief (e.g. membership function).

We model a SC consisting of three levels and use an example with simulated data extracted from our previous study. We conduct various analyses, to include sensitivity analysis on certain confidence of level (α -cut-level), simulation on weight, trade-off game analysis and two-way comparison between the proposed model and the model with three-objective case (see Section 3 for both three-objective and five-objective cases).

The rest of the paper is organized as follows. Section 2 presents literature review. Section 3 presents stochastic fuzzy multi-objective programming models. Section 4 discusses our solution approach. Section 5 gives numerical illustration analysis, and Section 6 concludes the paper.

2. Literature review

We review three streams of literature that are relevant for this paper. The first stream is the widely studied research on supplier selection or outsourcing, where research can be dated back to the early 1960s [20]. Supply management in SCs seeks the participation of good suppliers providing low cost and high quality. Selection of SC partners is an important decision involving many important factors. Supplier selection by its nature involves the need to trade off multiple criteria, to include both tangible and intangible information. [20] identified 23 distinct criteria in various supplier selection problems. [76] found multiple criteria in 47 of the 76 supplier selection articles that they reviewed. [12] applied 4 methods to evaluate potential suppliers based on competitive strategy. We conduct a search via ProQuest, and ScienceDirect and collected 30 journal articles on the topic of supplier selection and outsourcing. Table 1 presents traditional vendor selection criteria that consistently appear the most in academic studies. Key indices such as total purchase (ing) amount, number of items late and number of items reject are usually selected to characterize the outsourcing performance related to cost, quality and logistics. Emphasis was given to cost in the Literature between the late 1970s and early 1980s. Time factor and customer responsiveness were added to the performance metrics system in the early 1990s. In the late 1990s, researchers began to care about the importance of flexibility service. In recent years, a major concern is economic environmental safety among the industrialized nations, as one of the major reasons to drive the development of supply chain risk management [32].

Outsourcing and offshore-sourcing has been popular recently. During the outsourcing process to geographically expand their business, companies are considering supplier selection decision as a more complicated multi-criteria decision making problem. They are facing not only classical choices such as cost or quality selection, but also on various risk and socio-economic factors there they may have little experience. Therefore, supply selection has to be considered from the systematic point of view under the framework of supply chain risk management.

The second stream of literature is the emerging area of research related to SC risk management. Many SC risks have been identified. [53] analyzed supplier investment risks, and how each could be managed. [81] classified a broader set of SC risks as internal and external, as well as by the level of controllability. [70] investigated the supply chain risks of the automotive industry in German, and gave an empirical analysis. Supply chain risks can have serious negative impacts on the affected firms [17,31]. These risks may be caused by internal factors inside their own company such as faulty planning and coordination procedures, in the SC such as loss or downgrading of supplier or by external factors such as natural catastrophes, economic environment. SC risks can affect the whole range of SC performance indicators such as product quality, operational cost and cost of assets, delivery reliability and delivery lead time, and flexibility in production [61]. Supply chain risk management are also motivated by different national and international laws and regulations from financial and accounting sectors, e.g., the Sarbanes–Oxley Act in the United States in 2002, the Basel II Capital Accord in Europe in 2006, or the German Corporate Governance Code in 2007. Actually, SC risk management has been developed as a new multiple disciplinary embedded in the emerging area of enterprise risk management [54] and service risk management [79]. Under these circumstances, outsourcing has also evolved as an emerging area requiring handling of multiple disciplinary silos.

Table 1
Vendor selection criteria.

Criteria (number of references)	References
Price/cost (29)	[8,9,14,18–20,26,27,39,40,43,45,49–52,58–60,63–66,73,75–77,80,81]
Acceptance/quality (29)	The same as above
On-time response/logistics (29)	The same as above
R&D in technology/innovation/design (10)	[8,10,14,19,27,50,58,64,76,77]
Production facilities/assets (10)	[1,6,8,14,19,27,45,52,59,66]
Flexibility/agility (7)	[8,10,19,32,43,60,76]
Service (6)	[9,10,14,19,26,45]
Management and organization (5)	[8,26,45,53,60]
Reliability/risk (5)	[9,32,38,42,65]

The third stream of literature has been heavily populated by studies of is fuzzy programming, possibilistic programming, chance-constrained programming [5,67,84] in dealing with risk issues in SC management. These applications mainly include:

- Uncertain SC production planning and control: [71] developed a multi-objective possibilistic programming to formulate a supply chain master planning problem that integrated procurement, production and distribution planning in a multi-echelon, multi-product and multi-period supply chain network. Ref. [82] studied the intrinsic evolutionary mechanism of the vendor-managed inventory SCs by applying the evolutionary game theories. Ref. [74] models SC uncertainties by fuzzy sets and develops a possibilistic SC configuration model for new products in a SC network.
- SC risk-based partner selection: [50] developed a fuzzy AHP model to partnership selection in the formation of virtual enterprises. [51] employed fuzzy logic and neural-fuzzy approaches to rank the performance of suppliers in new product development. Ref. [39] applied the fuzzy goal programming to solve the fuzzy vendor selection problem, and developed a fuzzy goal programming approach for a vendor selection problem when minimizing multiple objectives including net cost, net rejections, and net late deliveries.
- SC inventory management under uncertainty: Fuzzy logic and possibilistic programming have employed in modeling risk factors such as demand or lead time uncertainty in inventory systems. Ref. [35] constructed a single-period inventory model for the cases of fuzzy demands; Using fuzzy inventory costs and demand parameters, [72] develop an EOQ model, which is extended in [47] to a multi-item fuzzy EOQ system. Ref. [10] assumes triangular fuzzy variable of production quantity and derives a membership function of the total cost and EOQ in a production-inventory problem. Besides demand and lead time, other risk factors such as various inventory costs are also tackled using fuzzy set and possibility theories (e.g., [55,56]). In a broader setting, inventory management strategies are presented using a risk-attitude parameter (e.g., three levels of risk attitude–pessimistic, neutral, and optimistic) under uncertain SC environment [74]. Ref. [4] implemented 3 algorithms for maximum covering location problems where the factory faces a fuzzy demand.

In a practical supply chain outsourcing risk management problem, lots of uncertainties exist. Random uncertainty of financial indices such as total purchasing amount, number of items late and number of items reject are usually modeled by use of probability theory [20]. Vague uncertainty of intangible criteria such as economic environment and vendor rate has been treated using fuzzy set theory [78]. When multiple uncertainties are presented, integration of both theories is desirable. This has motivated our current study of developing integrated optimization model using both stochastic, fuzzy and possibilistic programming to treat risks and uncertainties in supply chain outsourcing problems.

3. Stochastic fuzzy multi-objective programming models

Before the development of our stochastic fuzzy multi-objective programming models, we define various notations as follows:

Indices

i customers
 j suppliers

Parameters:

n_i the number of candidate suppliers desired by the i th customer
 c_{ij} per unit purchase cost from supplier j by the i th customer
 λ_{ij} percentage of items late from supplier j to the i th customer
 β_{ij} percentage of rejected units from supplier j
 D_i demand for item over planning period from the i th customer

u_{ij}^u	maximum amount of business for item to be given to supplier j by the i th customer
u_{ij}^l	minimum amount of business for item to be given to supplier j by the i th customer
w_{ij}^u	maximum order quantity from supplier j by the i th customer
w_{ij}^l	minimum order quantity from supplier j by the i th customer
Functions	
$U(\cdot)$	utility functions
Variables:	
x_{ij}	quantity purchased by the i th customer from supplier j
z_{ij}	decision variables = $\begin{cases} 1, & \text{if supplier } j \text{ is selected by the } i\text{th customer,} \\ 0, & \text{otherwise.} \end{cases}$

Objectives and constraints are defined as follows.

- Objective 1: Maximize the total utility of purchase cost.
- Objective 2: Maximize the utility of the number of rejected items.
- Objective 3: Maximize the utility of the number of late deliveries.
- Objective 4 and 5 in Model SFMOP (2): Minimize the risk factors. This equals to minimize the negative effect of economic environment and vendor service rating.
- Constraint 4: Ensures that the quantity demand is met.
- Constraint 5: Ensures that the vendor's capacity is not exceeded.
- Constraint 6: Ensures that the customer's proposed business to the vendor is not exceeded.
- Constraint 7: Establishes minimum order quantities the vendors supply.
- Constraint 8: Establishes minimum business for selected vendors.
- Constraint 9: Ensures that there are no negative orders.
- Constraint 10: Establishes binary nature of vendor selection decision.

We now present a deterministic multi-objective programming supplier selection model as follows. This model is based on many traditional models but differs from prior models (e.g. [27,75,77]) due to the consideration of various demand risk from many different customers. Note that in such a traditional model there are no constraints related to other indices, besides purchasing amount, demand for items, and the relation between vendors selected and customers.

LMOP (1)

$$\min f_1(x_{ij}) = \sum_{i=1}^m \sum_{j=1}^{n_i} c_{ij}x_{ij} \quad \{\text{total cost}\} \tag{1.1}$$

$$\min f_2(x_{ij}) = \sum_{i=1}^m \sum_{j=1}^{n_i} \beta_{ij}x_{ij} \quad \{\#\text{ rejected}\} \tag{1.2}$$

$$\min f_3(x_{ij}) = \sum_{i=1}^m \sum_{j=1}^{n_i} \lambda_{ij}x_{ij} \quad \{\#\text{ late}\} \tag{1.3}$$

$$\text{subject to : } \sum_j x_{ij} \geq D_i, \quad i = 1, \dots, m \tag{1.4}$$

$$x_{ij} \geq z_{ij}u_{ij}^l, \quad \forall i, j \tag{1.5}$$

$$x_{ij} \leq z_{ij}u_{ij}^u \quad \{\text{lower and upper business bound set for the purchased amount}\}, \quad \forall i, j \tag{1.6}$$

$$x_{ij} \geq z_{ij}w_{ij}^l, \quad \forall i, j \tag{1.7}$$

$$x_{ij} \leq z_{ij}w_{ij}^u \quad \{\text{lower and upper order bound for the purchased amount}\}, \quad \forall i, j \tag{1.8}$$

$$x_{ij} \geq 0 \tag{1.9}$$

$$z_{ij} = \{0, 1\}, \quad \forall i, j \tag{1.10}$$

where $j = 1, \dots, n_i$, represents the possible vendors selected for the i th customer. The model **LMOP (1)** simultaneously minimizes purchase cost ($f_1(x_{ij})$), percentage of items delivered late ($f_2(x_{ij})$) and percentage of items rejected ($f_3(x_{ij})$), while meeting various constraints with respect to minimum and maximum order quantities. These goals were also used by Narasimhan et al. in a multicriteria mathematical programming supply chain model. We have lower and upper bounds for x_{ij} from both the vendor and the customer's point of view, as expressed in Constraints (1.5)–(1.7) and (1.8). A “minimum business” constraint is employed in (1.8) to guarantee a non-zero procurement solution in the model. This is adopted from existing work of [49]. Readers can refer to [23] for other techniques to cope with fuzzy constraints in the model. We note that minimizing cost as a real measure of budget spent, might not fit real sense in business in the first objective. Other measures such as revenue

or profit maximization could be a good alternative. We also recognize that in a great deal of literature minimizing cost is used (e.g., [18]). For clarity of explosion, we leave this for further research.

In reality, rather than being exposed to pure exact and complete information, both tangible and intangible information are usually available to decision makers related to decision criteria and constraints. Financial data such as cost and late delivery can exhibit a great deal of fluctuation. As a result, they are stochastic data rather than crisp values. Qualitative variables such as the economic environment and vendor service rating evaluated by customers do not behave crisply and they are typically fuzzy in nature. To take both data into consideration, we develop an integrated multiobjective model to accommodate both stochastic and fuzzy data, which we call stochastic-fuzzy multiobjective model (SFMOP). Here, utility theory is employed to treat stochastic data and fuzzy set theory is used to handle fuzzy data. Note that although Fuzzy Randomness might be used to characterize such data, we prefer not to use such complicated concepts in our models [48]. We also note that fuzzy data can be associated not only with qualitative information but also with quantitative information when it is impossible to construct convincing stochastic estimates (“short samples”) but it is possible to construct fuzzy estimates. Before presenting the fuzzy model, we briefly discuss some definitions and properties on fuzzy set theory as follows. Readers can refer to [44,83] for details of these definitions.

Definition 1 (Fuzzy Set). Let X be a space of points, with a generic element of X denoted by x . Hence $X = \{x\}$.

A **Fuzzy Set** is a class of objects with a continuum of grades of membership. It defined as $A = \{(x, \mu_A(x)) | x \in X\}$ where $\mu_A(x)$ measures the degree to which element x belongs to set A , i.e., $\mu_A: X \rightarrow [0, 1]$.

Definition 2 (Membership Function). A **membership function** $\mu_A(x)$ is the membership function of x in A and a curve indicating how each point in the universe of discourse is mapped to a value between 0 and 1.

Definition 3 (Convex fuzzy variables). A fuzzy variable \tilde{x} correspond to his membership function is **non-convex** based on:

$$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, x_1, x_2 \in X, \lambda \in [0, 1]$$

Definition 4 (α -cut). For every $\alpha \in [0, 1]$, a given fuzzy set A yields a crisp set $A_\alpha = \{x \in X | A(x) \geq \alpha\}$ which is called an α -cut of A .

Property. Denote by $\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n$ n convex fuzzy variables, and $(\tilde{x}_i)_{\alpha}^L, (\tilde{x}_i)_{\alpha}^U$ be the α -cut level lower and upper bounds of \tilde{x}_i , for any given possibility levels α_1, α_2 and $\alpha_3 (0 < \alpha_1, \alpha_2, \alpha_3 < 1)$, the following properties hold.

- (i) $P(\tilde{x}_1 + \dots + \tilde{x}_n \leq b) \geq \alpha_1$ if and only if $(\tilde{x}_1)_{\alpha_1}^L + \dots + (\tilde{x}_n)_{\alpha_1}^L \leq b$.
- (ii) $P(\tilde{x}_1 + \dots + \tilde{x}_n \geq b) \geq \alpha_2$ if and only if $(\tilde{x}_1)_{\alpha_2}^U + \dots + (\tilde{x}_n)_{\alpha_2}^U \leq b$. Moreover, if $\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n$ are trapezoidal fuzzy numbers, (i) and (ii) are reduced to (iii) and (iv) as follow:
- (iii) $P(\tilde{x}_1 + \dots + \tilde{x}_n \leq b) \geq \alpha_1$ if and only if $(1 - \alpha_1) \left[(\tilde{x}_1)_{\alpha_1}^L + \dots + (\tilde{x}_n)_{\alpha_1}^L \right] + \alpha_1 \left[(\tilde{x}_1)_{\alpha_1}^U + \dots + (\tilde{x}_n)_{\alpha_1}^U \right] \leq b$.
- (iv) $P(\tilde{x}_1 + \dots + \tilde{x}_n \geq b) \geq \alpha_2$ if and only if $(1 - \alpha_2) \left[(\tilde{x}_1)_{\alpha_2}^L + \dots + (\tilde{x}_n)_{\alpha_2}^L \right] + \alpha_2 \left[(\tilde{x}_1)_{\alpha_2}^U + \dots + (\tilde{x}_n)_{\alpha_2}^U \right] \leq b$.

Now we present our stochastic-fuzzy multiobjective model as SFMOP (2).

SFMOP (2)

$$\begin{aligned} \max \hat{f}_1(x_{ij}) &= \sum_{i=1}^m \sum_{j=1}^{n_i} U(\hat{c}_{ij}x_{ij}) \quad \{\text{total purchase amount}\} \\ \max \hat{f}_2(x_{ij}) &= \sum_{i=1}^m \sum_{j=1}^{n_i} U(\hat{\lambda}_{ij}x_{ij}) \quad \{\text{number of items late}\} \\ \max \hat{f}_3(x_{ij}) &= \sum_{i=1}^m \sum_{j=1}^{n_i} U(\hat{\beta}_{ij}x_{ij}) \quad \{\text{number of items rejected}\} \\ \min \tilde{f}_4(x_{ij}) &= \sum_{i=1}^m \sum_{j=1}^{n_i} \tilde{\phi}_{ij}x_{ij} \quad \{\text{economic environment}\} \\ \min \tilde{f}_5(x_{ij}) &= \sum_{i=1}^m \sum_{j=1}^{n_i} \tilde{\epsilon}_{ij}x_{ij} \quad \{\text{vendor rate}\} \end{aligned}$$

subject to : $\sum_j x_{ij} \geq \tilde{D}_i, \quad i = 1, \dots, n_i \quad \{\text{the purchased amount from each customer must satisfy demand}\}$

$$\begin{aligned} x_{ij} &\leq z_{ij} \min \left\{ \tilde{u}_{ij}^u, \tilde{w}_{ij}^u \right\} \quad \forall i, j \\ x_{ij} &\geq z_{ij} \max \left\{ \tilde{u}_{ij}^l, \tilde{w}_{ij}^l \right\} \quad \forall i, j \quad \{\text{max and min amount per vendor demanded}\} \\ x_{ij} &\geq 0, \quad \forall i, j \\ z_{ij} &\in \{0, 1\}, \quad \forall i, j \end{aligned}$$

In Model **SFMOP (2)**, variables with the sign “ \sim ” and “ $\hat{\sim}$ ” denote stochastic and fuzzy variables respectively. For example, \hat{c}_{ij} denotes the stochastic per unit purchase cost from supplier j by the i th customer; $\hat{\phi}_{ij}$ denotes the fuzzy influence on economic environment from supplier j by the i th customer; $\hat{\varepsilon}_{ij}$ denotes the fuzzy vendor rate from supplier j by the i th customer and $U(\hat{c}_{ij}x_{ij})$ denotes the utility of the corresponding stochastic term $\hat{c}_{ij}x_{ij}$. We consider in our outsourcing problem two risk factors: economic environment and vendor service rating. Thus, two extra objectives are added to the objective functions to minimize the negative effect of economic environment and vendor service rating.

4. Solution approach

SFMOP (2) is not easy to be solved. But using a specific property of fuzzy set and the mean–variance utility function, we transform SFMOP (2) to QNMOP (5), which could be finally solved by regular solution approaches.

First we discuss the simplification of stochastic variable and fuzzy variable respectively. We use an assumption of normal distribution and constant absolute risk aversion to simplify the stochastically distributed utility function, which will lead to a mean–variance utility function.

When the central planner (manufacturer) is risk averse, we denote by ρ_{ij} (Arrow–Pratt measure of absolute risk aversion) the degree of risk aversion from supplier j by the i th customer. When the utility function of the retailer has constant absolute risk aversion, i.e., $U(\hat{c}_{ij}x_{ij}) = -\exp(-\rho_{ij}\hat{c}_{ij}x_{ij})$, maximizing the expected utility is equivalent to maximizing the following certainty equivalent income:

$$EU(\hat{c}_{ij}x_{ij}) = \bar{c}_{ij}x_{ij} - \frac{1}{2}\rho_{ij}x_{ij}^2V(\hat{c}_{ij}), \tag{i}$$

where $V(\hat{c}_{ij})$ denotes the variance of stochastic data \hat{c}_{ij} . The constant ρ_{ij} measures the degree of risk aversion: the larger ρ_{ij} is, the more risk averse the central planner is.

Generally, when the income is normally distributed with mean m and variance v , the term $m - \frac{1}{2}\rho v$ is the certainty equivalent income of a risk averse player, where $\frac{1}{2}\rho v$ is the risk premium. A risk premium is defined as the minimum difference a person is willing to take an uncertain bet, between the expected value of the bet and the certain value that he is indifferent to. The utility of the central planner decreases with the mean of his consumption and decreases with the variance. The rate of decrease with the variance is larger the more risk averse the central planner is. Readers can refer to, for example, [64,65] for further information on risk aversion.

For fuzzy variable terms, we introduce chance-constrained programming (CCP) by [11] as a way to make transformation so that the resulted model is a possibility programming. In that sense, we actually propose a possibility approach to solve the SFMOP (2). CCP deals with uncertainty by specifying the desired levels of confidence with which the constraints hold. Using the concepts of CCP and possibility of fuzzy events (see Property of fuzzy set) and the mean–variance utility function, the SFMOP model becomes the following possibility MOP model with quadratic objectives (**QPMOP**):

QPMOP (3)

$$\begin{aligned} & \min \{-\bar{U}_1, -\bar{U}_2, -\bar{U}_3, \bar{f}_4, \bar{f}_5\} \\ \text{subject to : } & \pi\left(\sum_{i=1}^m \sum_{j=1}^{n_i} \hat{\phi}_{ij}x_{ij} \leq \bar{f}_4\right) \leq \alpha_4 \quad \{\text{economic environment}\} \\ & \pi\left(\sum_{i=1}^m \sum_{j=1}^{n_i} \hat{\varepsilon}_{ij}x_{ij} \leq \bar{f}_5\right) \leq \alpha_5 \quad \{\text{vendor rate}\} \\ & \pi\left(\sum_{j=1}^n x_{ij} \geq \bar{D}_i\right) \leq \alpha_6 \quad \{\text{the purchased amount must satisfy demand}\} \\ & \pi(x_{ij} \leq z_{ij}\hat{u}_{ij}^u) \leq \alpha_7 \\ & \pi(x_{ij} \leq z_{ij}\hat{w}_{ij}^u) \leq \alpha_8 \\ & \pi(x_{ij} \geq z_{ij}\hat{u}_{ij}^l) \leq \alpha_9 \\ & \pi(x_{ij} \geq z_{ij}\hat{w}_{ij}^l) \leq \alpha_{10} \\ & x_{ij} \geq 0 \quad \forall i, j \\ & z_{ij} \in \{0, 1\} \quad \forall i, j \end{aligned}$$

where

$$\begin{aligned} \bar{U}_1 &= \sum_{i=1}^m \sum_{j=1}^{n_i} u(\hat{c}_{ij}x_{ij}) = \sum_{i=1}^m \sum_{j=1}^{n_i} \left(\bar{c}_{ij}x_{ij} - \frac{1}{2}\rho_{ij}x_{ij}^2V(\hat{c}_{ij})\right), \\ \bar{U}_2 &= \sum_{i=1}^m \sum_{j=1}^{n_i} u(\hat{\lambda}_{ij}x_{ij}) = \sum_{i=1}^m \sum_{j=1}^{n_i} \left(\bar{\lambda}_{ij}x_{ij} - \frac{1}{2}\rho_{ij}x_{ij}^2V(\hat{\lambda}_{ij})\right), \\ \bar{U}_3 &= \sum_{i=1}^m \sum_{j=1}^{n_i} u(\hat{\beta}_{ij}x_{ij}) = \sum_{i=1}^m \sum_{j=1}^{n_i} \left(\bar{\beta}_{ij}x_{ij} - \frac{1}{2}\rho_{ij}x_{ij}^2V(\hat{\beta}_{ij})\right), \end{aligned}$$

$\pi(\cdot)$ denote the possibility of a certain event, \bar{f}_1 denote the upper bound of the related fuzzy variable, and α_k ($k = 1, \dots, 10$) is the acceptable risk level technique. Note that in the computation we transform all benefit-type variables into cost-type variables to keep consistent with all objectives in the QPMOP (3) model. The stochastic-fuzzy MOP model can not be solved like a crisp model. We adopt a possibility approach and acceptable risk level technique, i.e., α -cut technique to convert the stochastic-fuzzy MOP model to the standard deterministic problem. Given that parameters in the model QPMOP (3) are normal and convex, we can convert QPMOP (3) into QPMOP (4) as follows.

QPMOP (4)

$$\begin{aligned} & \min \{-\bar{U}_1, -\bar{U}_2, -\bar{U}_3, \bar{f}_4, \bar{f}_5\} \\ \text{subject to: } & \sum_{i=1}^m \sum_{j=1}^{n_i} (\tilde{\phi}_{ij})_{\alpha_4}^L x_{ij} \leq \bar{f}_4 \text{ \{economic environment\}} \\ & \sum_{i=1}^m \sum_{j=1}^{n_i} (\tilde{\epsilon}_{ij})_{\alpha_5}^L x_{ij} \leq \bar{f}_5 \text{ \{vendor rate\}} \\ & \sum_{j=1}^{n_i} x_{ij} \geq (\bar{D}_i)_{\alpha_6}^L \text{ \{the purchased amount must satisfy demand\}} \\ & x_{ij} \leq z_{ij} \min \left\{ \left(\tilde{u}_{ij}^u \right)_{\alpha_7}^U, \left(\tilde{w}_{ij}^u \right)_{\alpha_8}^U \right\}, \quad \forall i, j \\ & x_{ij} \geq z_{ij} \max \left\{ \left(\tilde{u}_{ij}^l \right)_{\alpha_9}^U, \left(\tilde{w}_{ij}^l \right)_{\alpha_{10}}^U \right\}, \quad \forall i, j \text{ \{max and min amount per vendor demanded\}} \\ & x_{ij} \geq 0 \quad \forall i, j \\ & z_{ij} \in \{0, 1\} \quad \forall i, j \end{aligned}$$

where α_1 to α_{10} are all acceptable risk level technique, i.e., α -cut levels used to convert associated fuzzy number to a crisp one, $(\cdot)_{\alpha_k}^L$ and $(\cdot)_{\alpha_k}^U$ ($k = 1, \dots, 10$) denote the lower and upper bounds of the α_k -level set of associated fuzzy variable. In this model, fuzzy numbers are converted to an interval number with upper and lower bounds by use of α -cut level techniques. For example, $(\bar{D}_i)_{\alpha_6}^L$ refers to the lower bound of fuzzy number \bar{D}_i at the α_6 -cut level.

Constraints corresponding to fuzzy data in the QPMOP (4) model may take linear or non-linear forms depending upon the membership functions of fuzzy parameters in the model. We consider trapezoidal fuzzy numbers in the model. A trapezoidal membership function is confined by four parameters $\{a_1, a_2, a_3, a_4\}$. For example, a trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ can be transformed into an interval number $a = [\alpha * a_1 + (1 - \alpha) * a_2, \alpha * a_3 + (1 - \alpha) * a_4]$ by use of the α -cut technique [21]. Therefore, using the trapezoidal fuzzy numbers, model QPMOP (4) is reduced to the following non-linear deterministic programming model with quadratic objectives (QNMOP).

QNMOP (5)

$$\begin{aligned} & \min \{-\bar{U}_1, -\bar{U}_2, -\bar{U}_3, \bar{f}_4, \bar{f}_5\} \\ \text{subject to: } & (1 - \alpha_4) \sum_{i=1}^m \sum_{j=1}^{n_i} (\tilde{\phi}_{ij})_1^L x_{ij} + \alpha_4 \sum_{i=1}^m \sum_{j=1}^{n_i} (\tilde{\phi}_{ij})_2^L x_{ij} \leq \bar{f}_4 \text{ \{economic environment\}} \\ & (1 - \alpha_5) \sum_{i=1}^m \sum_{j=1}^{n_i} (\tilde{\epsilon}_{ij})_1^L x_{ij} + \alpha_5 \sum_{i=1}^m \sum_{j=1}^{n_i} (\tilde{\epsilon}_{ij})_2^L x_{ij} \leq \bar{f}_5 \text{ \{vendor rate\}} \\ & \sum_j x_{ij} \geq (1 - \alpha_6) (\bar{D}_i)_1^L + \alpha_6 (\bar{D}_i)_2^L \text{ \{the purchased amount must satisfy demand\}} \\ & x_{ij} \leq (1 - \alpha_7) z_{ij} \left(\tilde{u}_{ij}^u \right)_4^U + \alpha_7 z_{ij} \left(\tilde{u}_{ij}^u \right)_3^U \quad \forall i, j \\ & x_{ij} \leq (1 - \alpha_8) z_{ij} \left(\tilde{w}_{ij}^u \right)_4^U + \alpha_8 z_{ij} \left(\tilde{w}_{ij}^u \right)_3^U \quad \forall i, j \\ & x_{ij} \geq (1 - \alpha_9) z_{ij} \left(\tilde{u}_{ij}^l \right)_4^U + \alpha_9 z_{ij} \left(\tilde{u}_{ij}^l \right)_3^U \quad \forall i, j \\ & x_{ij} \geq (1 - \alpha_{10}) z_{ij} \left(\tilde{w}_{ij}^l \right)_4^U + \alpha_{10} z_{ij} \left(\tilde{w}_{ij}^l \right)_3^U \quad \forall i, j \text{ \{max and min amount per vendor demanded\}} \\ & x_{ij} \geq 0, \quad \forall i, j \\ & z_{ij} \in \{0, 1\} \quad \forall i, j \end{aligned}$$

The deterministic programming model QNMOP (5) can be solved using regular solution approaches to a MOP (e.g. [34,49]). It has been recognized that various techniques may be used to treat various objectives [62]. Typical means to handle various objectives includes (I) sequential optimization where the objective functions are minimized sequentially (II) the weighted sum of various objectives approach or goal programming approach where weights are introduced to convert MOP into a single criterion search problem and (III) minimax criteria [34,49]. The sequential optimization and weighted

sum approach usually are sensitive to the weights attached to different objectives and also computationally expensive. Therefore, we use the minimax criteria to solve MOP, where the worst-case value of a set of multi-objective functions will be minimized. This is generally referred to as the minimax problem. We use Matlab fminimax module in our computation, where a sequential quadratic programming (SQP) method [7] is coded. Modifications are made to the line search and Hessian. In the line search an exact merit function [46] is used together with the merit function proposed by [30,57]. The line search is terminated whenever the merit function shows improvement.

Denote by $X = \begin{pmatrix} x_{11} \\ x_{12} \\ x_{13} \\ \dots \\ x_{mn} \end{pmatrix}$, $LB = \begin{pmatrix} lb_{11} \\ lb_{12} \\ lb_{13} \\ \dots \\ lb_{mn} \end{pmatrix}$ and $UB = \begin{pmatrix} ub_{11} \\ ub_{12} \\ ub_{13} \\ \dots \\ ub_{mn} \end{pmatrix}$ the order quantity, the lower bound, and the upper bound of X

respectively. Solving Model QPMOP (4) is equivalent to solving the following minimax problem:

$$\begin{aligned} & \min_x \max_{\{f_i\}} \{\bar{f}_i(x)\} \quad i = 1, \dots, 8 \\ & A \cdot x \leq b \\ & lb \leq x \leq ub \end{aligned}$$

where the first constraint corresponds to the first six constraints in Model QPMOP (4) and the second constraint corresponds to the last three constraints in Model QPMOP (4). Note that the lower bound LB imposed to the order quantity cannot be set before the preferred suppliers are selected.

5. Numerical illustration

5.1. Supply chain model

In this section we use the proposed model to evaluate a supply chain consisting of three levels: a set of ten suppliers, a core level represents the organizing, decision-making retail system and ten customers at the third level. Fig. 1 shows a diagram about the connections among suppliers, distributions, and customers. Each customer represents a demand assumed to be normal for a given period. The performance of each supplier is characterized with three quantitative variables: expected costs, quality acceptance levels, and on-time delivery distributions, and two qualitative variables: economic environment and vendor rating. The example is extracted from our previous study [81], where intangible variables are not considered. Again, we use a simulated data based on distributions empirically derived for demonstration purpose because we are lack of sufficient real historical data. Distributions of costs are assumed normal, distributions of acceptance failure are assumed exponential and distributions of late delivery are assumed lognormal. These represent assumptions that could be replaced by distributions empirically derived. The retailer must anticipate demand and order quantities of the modeled good to be delivered to arrive on time at each demand destination.

This supply chain system takes both internal operating risk such as demand and supply risks and external risk into consideration. Both demand and supply risks are modeled by using probability distributions in data. External risk, denoted by two economic environment and vendor ratings, is expressed in fuzzy data. The vendor ratings data can be collected from various rating agencies such as Standard and Poors, where risk quality definitions have been classified into four categories, i.e., “excellent”, “strong”, “adequate”, and “weak” [33]. Profit is gained from sales made for goods successfully delivered to each demand. Costs are probabilistic as outlined above, but total cost of goods sold has a mean given for each source supplier. Goods not passing quality acceptance level are not paid for. Goods delivered late are paid for at a reduced rate, and are carried forward at an inventory cost.

Table 2 provides data for vendor selection. Costs and product late delivery rate are crisp values as outlined in Table 2, but risk factors and supplier’s service performance have fuzzy data for each source supplier. Data given is means (standard deviation); Unit costs normally distributed, Accept rates are exponentially distributed, on-time rate lognormally distributed, maximum and minimum order quantities from supplier j by the i th customer normally distributed.

The central retail system faces ten customers, each has a demand and seeks one common product. Product price is \$2 per item, time period assumed is a week. Different conditions could be modeled with little difficulty other than scale. Table 3 presents demand data, where Demand, maximum and minimum amount of business for item to be given to supplier j by the i th customer are normally distributed.

5.2. Implementation

Now we implement the method from the prior section to data in Table 2. Qualitative data in both Tables 2 and 3 are fuzzified using Trapezoidal fuzzy data and results are shown in Tables 4 and 5. For parameters such as cost, vendor rating, demand, and rejection rate, the u_{ij}^u , u_{ij}^l , w_{ij}^u , and w_{ij}^l follow the normal distribution.

We use the following membership function for late rate since this variable follows lognormal distribution:

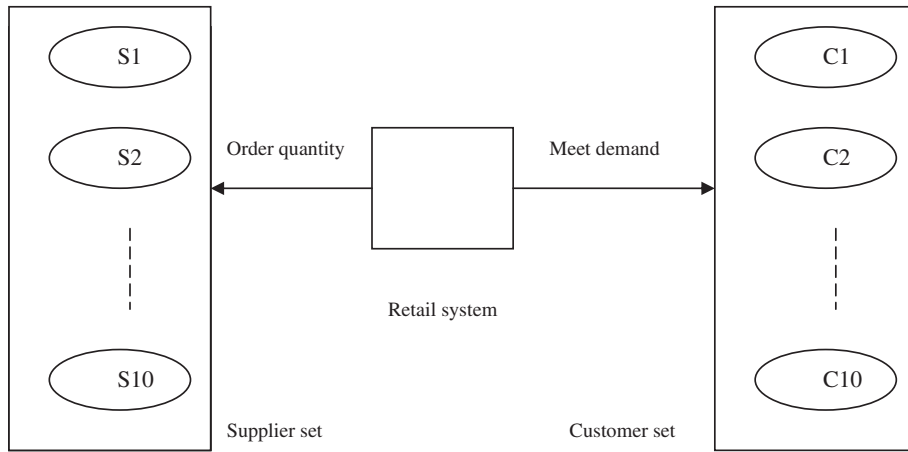


Fig. 1. the connections among suppliers, distributions, and customers.

Table 2
Data for supplier selection.

Variable	Vendor									
	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
Unit cost	1.02 (0.01)	0.95 (0.02)	1.00 (0.03)	1.03 (0.01)	1.07 (0.05)	1.10 (0.03)	0.77 (0.05)	0.59 (0.06)	0.53 (0.07)	0.50 (0.08)
Accept rate	0.99 (0.002)	0.99 (0.002)	0.99 (0.002)	0.98 (0.020)	0.97 (0.003)	0.95 (0.01)	0.98 (0.020)	0.95 (0.020)	0.90 (0.030)	0.8 (0.1)
On-time rate	0.97 (0.003)	0.96 (0.004)	0.96 (0.004)	0.97 (0.003)	0.95 (0.005)	0.96 (0.004)	0.91 (0.009)	0.89 (0.0011)	0.87 (0.013)	0.86 (0.014)
Economic environment	Very bad	Bad	Bad	Very bad	Very bad	Very bad	Good	Good	Very good	Very good
Vendor rating	Low	Low	Low	Very Low	Very Low	Very Low	High	High	Very High	Very High
Maximum	10,000 (1000)	10,000 (1000)	9000 (900)	8000 (800)	8000 (800)	7000 (700)	10,000 (1000)	8000 (800)	6000 (600)	4000 (400)
Minimum	1000(100)	1000(100)	1000(100)	1000(100)	500(50)	500(50)	500(50)	500(50)	400(40)	300(20)

Table 3
Demand data.

Customer	Parameter		
	Demand	Maximum	Minimum
C1	2725 (200)	50,000(1000)	3(0.30)
C2	2525 (200)	46,000(800)	3(0.30)
C3	1983 (300)	34,000(700)	2(0.20)
C4	1583 (300)	26,000(700)	1(0.10)
C5	1283 (300)	25,000(600)	2(0.20)
C6	1082 (300)	21,000(600)	1(0.10)
C7	1020 (250)	19,000(600)	4(0.40)
C8	1020 (250)	19,000(800)	4(0.40)
C9	1020 (250)	20,000(750)	2(0.20)
C10	920 (250)	19,000(650)	2(0.20)

$$\mu(x) = \begin{cases} 0(x < x_1) \\ f(x)(x_1 < x < x_2) \\ 1(x_2 < x < x_3) \\ f(x)(x_3 < x < x_4) \\ 0(x > x_4) \end{cases}$$

where $f(x)$ is the probability distribution function of x .

Table 4
Risk premium and trapezoidal fuzzy data for supplier selection.

Vendor variable	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	
Unit cost ($\times E-05$)	5.00	8.00	12	5.0	25	15	12.5	15	7.00	8.0	
Rejected rate ($\times E-05$)	1.00	40	8.0	1.0	1.5	5.0	0.5	5.0	3.0	10	
Late rate ($\times E-05$)	1.50	1.60	1.60	1.5	2.5	2.0	2.25	2.75	1.3	1.4	
Economic environment	[0.09, 0.092, 0.108, 0.11]	[0.2, 0.212, 0.228, 0.24]	[0.17, 0.192, 0.208, 0.23]	[0.06, 0.062, 0.078, 0.08]	[0.01, 0.052, 0.068, 0.11]	[0.01, 0.032, 0.048, 0.07]	[0.01, 0.032, 0.048, 0.07]	[0.55, 0.592, 0.608, 0.65]	[0.692, 0.694, 0.706, 0.708]	[0.73, 0.792, 0.808, 0.87]	[0.77, 0.842, 0.858, 0.93]
Vendor rate	[0.19, 0.192, 0.208, 0.21]	[0.28, 0.292, 0.308, 0.32]	[0.32, 0.342, 0.358, 0.38]	[0.17, 0.172, 0.188, 0.19]	[0.13, 0.142, 0.158, 0.17]	[0.07, 0.092, 0.108, 0.13]	[0.07, 0.092, 0.108, 0.13]	[0.7, 0.742, 0.758, 0.8]	[0.74, 0.792, 0.808, 0.86]	[0.78, 0.842, 0.858, 0.92]	[0.82, 0.892, 0.908, 0.98]
Maximum	[9000, 9500, 10,500, 11,000]	[9000, 9500, 10,500, 11,000]	[8100, 8550, 9450, 9900]	[7200, 7600, 8400, 8800]	[7200, 7600, 8400, 8800]	[6300, 6650, 7350, 7700]	[9000, 9500, 10,500, 11,000]	[7200, 7600, 8400, 8800]	[5400, 5700, 6300, 6600]	[3600, 3800, 4200, 4400]	[3600, 3800, 4200, 4400]
Minimum	[900, 950, 1050, 1100]	[900, 950, 1050, 1100]	[900, 950, 1050, 1100]	[900, 950, 1050, 1100]	[450, 475, 525, 550]	[450, 475, 525, 550]	[450, 475, 525, 550]	[450, 475, 525, 550]	[360, 380, 420, 440]	[280, 290, 310, 320]	

Table 5
Trapezoidal demand data.

Customer	Parameter		
	Demand	Maximum	Minimum
C1	[2525, 2625, 2775, 2925]	[49,000, 49,500, 50,500, 51,000]	[2.7, 2.85, 3.15, 3.3]
C2	[2325, 2425, 2625, 2725] [2410]	[45,200, 45,600, 46,400, 46,800]	[2.7, 2.85, 3.15, 3.3]
C3	[1682.5, 1832.5, 2132.5, 2282.5]	[33,300, 33,650, 34,350, 34,700]	[1.8, 1.9, 2.1, 2.2]
C4	[1282.5, 1432.5, 1732.5, 1882.5]	[25,300, 25,650, 26,350, 26,700]	[0.9, 0.95, 1.05, 1.1]
C5	[982.5, 1132.5, 1432.5, 1582.5]	[24,400, 24,700, 25,300, 25,600]	[1.8, 1.9, 2.1, 2.2]
C6	[782.5, 932.5, 1232.5, 1382.5]	[20,400, 20,700, 21,300, 21,600]	[0.9, 0.95, 1.05, 1.1]
C7	[768.75, 893.75, 1143.75, 1268.75]	[18,400, 18,700, 19,300, 19,600]	[3.6, 3.8, 4.2, 4.4]
C8	[768.75, 893.75, 1143.75, 1268.75]	[18,200, 18,600, 19,400, 19,800]	[3.6, 3.8, 4.2, 4.4]
C9	[768.75, 893.75, 1143.75, 1268.75]	[19,250, 19,625, 20,375, 20,750]	[1.8, 1.9, 2.1, 2.2]
C10	[668.75, 793.75, 1043.75, 1268.75]	[18,350, 18,675, 19,325, 19,650]	[1.8, 1.9, 2.1, 2.2]

A simple data-preprocessing technique is applied to Unit cost data by dividing a constant value of 10 in order to yield a similar unit cost values to late delivery and reject rate data. Trapezoidal fuzzy data are yielded in Tables 4 and 5 from Tables 2 and 3, where Trapezoidal data are given by: [minimum (value 0), left (value 1), right (value 1), maximum (value 0)]. Table 4 also presents risk premium values for suppliers. Risk premium values are yielded using Formula (i) for three stochastic variables: Unit Cost, Rejected Rate and Late Rate.

Trapezoidal data can be transformed to an interval number by employing the acceptable risk level technique, i.e., α -cut technique. For example, if we use the value of 0.5 for α , the interval data in Tables 6 and 7 are obtained. We present a sensitivity analysis of optimal solutions with respect to α in Section 5.2.1 and depict the result in Fig. 2.

5.2.1. Optimization

Optimization model QNMOP (5) is solved by use of the proposed algorithm and minimax criteria ([16]). Codes are written in MATLAB language, where “fminmax” function of MATLAB’s Optimization Toolbox is used. The subroutine “fminmax” basically uses the sequential quadratic programming algorithm proposed in [7].

Table 6
Interval data.

Vendor variable	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
Economic environment	[0.0917, 0.1083]	[0.2102, 0.2298]	[0.1887, 0.2113]	[0.0617, 0.0783]	[0.0457, 0.0743]	[0.0287, 0.0513]	[0.5857, 0.6143]	[0.6923, 0.7077]	[0.7827, 0.8173]	[0.8312, 0.8688]
Vendor Rate	[0.1917, 0.2083]	[0.2902, 0.3098]	[0.3387, 0.3613]	[0.1717, 0.1883]	[0.1402, 0.1598]	[0.0887, 0.1113]	[0.7357, 0.7643]	[0.7842, 0.8158]	[0.8327, 0.8673]	[0.8812, 0.9188]
Maximum	[9425, 10,750]	[9425, 10,750]	[8482.5, 9675]	[7540, 8600]	[7540, 8600]	[6597.5, 7525]	[9425, 10,750]	[7540, 8600]	[5655, 6450]	[3770, 4300]
Minimum	[942.5, 1057.5]	[942.5, 1057.5]	[942.5, 1057.5]	[942.5, 1057.5]	[471.25, 528.75]	[471.25, 528.75]	[471.25, 528.75]	[471.25, 528.75]	[377, 423]	[288.5, 311.5]

Table 7
Interval demand data.

Customer	Parameter		
	Demand	Maximum	Minimum
C1	[2610, 2840]	[49,425, 50,575]	[2,775, 3,1725]
C2	[2410, 2640]	[45,540, 46,460]	[2,775, 3,1725]
C3	[1810, 2140]	[33597.5, 34402.5]	[1.85, 2,115]
C4	[1410, 1740]	[25597.5, 26402.5]	[0.925, 1,0575]
C5	[1110, 1440]	[24,655, 25,345]	[1.85, 2,115]
C6	[910, 1240]	[20,655, 21,345]	[0.925, 1,0575]
C7	[875, 1162.5]	[18,655, 19,345]	[3.7, 4,23]
C8	[875, 1162.5]	[18,540, 19,460]	[3.7, 4,23]
C9	[875, 1162.5]	[19568.75, 20431.25]	[1.85, 2,115]
C10	[775, 1062.5]	[18626.25, 19373.75]	[1.85, 2,115]

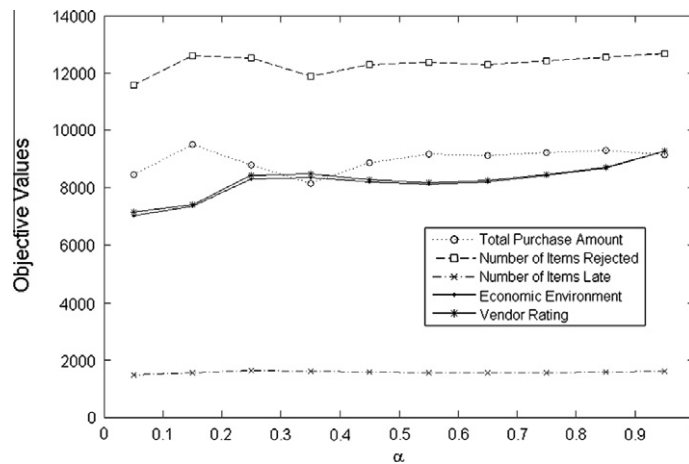


Fig. 2. The trend of five objective values with changes in α -cut level.

Our optimization model has a couple of parameters, which can complicate our analysis. We need to find appropriate parameter values to facilitate our result analysis. First we check the effect of changing two parameters of acceptable risk levels and weights over the optimal objective solution. Fig. 2 depicts the trend of five optimal objective values with the change of α -cut level. As can be seen from Fig. 2, five optimal objectives values do not change much with respect to α -cut level when α is no less than 0.5. Thus, we set the α -cut level value to 0.5 in our hereby computation.

We then ran the model twice in two deterministic cases using $\alpha = 0.5$ and different weights. Table 8 reports the order quantity for each customer using equal weights over five criteria from each supplier, using a α -cut level value of 0.5. Computation suggests service outsourcing to four suppliers, i.e., S6, S8, S9 and S10. Eight customers selected at least two suppliers while the rest two prefer only S9 or S10. S1–S5 are never selected, which means that no customer can distinguish S1–S5.

The order quantity using unequal weights: $(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) = (0.304, 0.182, 0.007, 0.328, 0.178)$ are presented in Table 9. Given unequal weights to different objectives, S6 was never selected by any customer. Only C4 selected a single supplier. S1–S7 are never selected, which means that no customer can distinguish S1–S7. From both Tables 8 and 9 we know that weights seriously affect the final decision and a simulation analysis over weights is important as we will do in next section.

Table 8
The order quantity using equal weight.

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
C1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2625.000
C2	0.000	0.000	0.000	0.000	0.000	462.500	0.000	0.000	1962.489	0.000
C3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	462.500	542.500	820.000
C4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1424.857	0.000
C5	0.000	0.000	0.000	0.000	0.000	462.500	0.000	462.500	370.000	0.000
C6	0.000	0.000	0.000	0.000	0.000	462.500	0.000	462.500	0.000	0.000
C7	0.000	0.000	0.000	0.000	0.000	462.500	0.000	0.000	370.000	285.000
C8	0.000	0.000	0.000	0.000	0.000	462.500	0.000	0.000	370.000	285.000
C9	0.000	0.000	0.000	0.000	0.000	462.500	0.000	0.000	370.000	285.000
C10	0.000	0.000	0.000	0.000	0.000	462.500	0.000	0.000	370.000	0.000

Table 9
The order quantity using unequal weight.

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
C1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1378.041	1231.959
C2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1309.959	1100.041
C3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1810.000	0.000	0.000
C4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	592.000	534.000	284.000
C5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	460.000	368.000	284.000
C6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	626.000	284.000
C7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	591.000	284.000
C8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	591.000	284.000
C9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	591.000	284.000
C10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	491.000	284.000

5.3. Simulation

Simulation is a popular tool used for performance optimization, safety engineering, testing, training and education. Simulation is used to demonstrate the eventual “real” effects of alternative conditions and courses of action. Simulation models in multiple criteria decision making (MCDM) has been discussed a lot [80,81]. The simple multiattribute rating theory in MCDM bases supplier selection on the rank order of the product of criteria weights and alternative scores over these criteria [22]. In this section, we conduct MCDM optimization simulation with random distributed weights by assuming a set of weights with ranges and order pre-specified. Weights are assumed to be independently uniformly distributed between 0 and 1. 500 simulation runs are conducted in Matlab. Results are presented in Table 10, where probabilities of each alternative being preferred by each customer and total optimal order quantity are reported. Other than four suppliers selected in Table 8, Supplier S2 was selected. Simulation generates very consistent result to Tables 8 and 9 in terms of identifying most preferred suppliers. When employing the mean of probability scores to rank the rest suppliers, Supplier S9 is the most preferred candidate with the largest amount of order quantity. Five suppliers are never selected. It can be seen from Table 10 that

$$S9 \succ S10 \succ S8 \succ S2 \succ S6,$$

where the symbol “ \succ ” denotes “is superior to”.

An interesting observation is that although S8 was more preferred to S2, the total order quantity from S8 was less than that from S2. This is mainly because that S2 allows larger amounts of order quantity with the lower bound value being 1000 than S8 with a value of 500. Moreover, all customers seem to be concerned about economic environment and vendor ratings.

Another very interesting observation is the risk averse and procurement behavior shown in Table 11, where risk averse parameter values and mean of order quantities for different customers are given. The risk averse parameter ρ measures the degree of risk aversion: the larger ρ is, the more risk averse the central planner is. From Table 11 we can see that Customers C1–C6 are more risk averse than C7–C10 because they are using larger risk averse parameters. Table 11 indicates a more risk averse customer prefers to order less. For example, C1 is more risk averse than C7–C10 and only order the lowest quantities 2610 which is allowed by Lower Bound of Demand. A less averse customer such as C10 will order 852.2 which is much higher than Lower Bound of Demand value of 775.

There are practical situations where company managers may prefer to put more emphasis on one criteria over the other. In such a situation, understanding how the trade-off results are affected by manager’s preference is important. To model such a situation, we draw trade-offs games among different optimal objective values in Figs. 3–5, where X-axis value denotes weight values assigned to the left Y-axis variable. Fig. 3 depict the optimal objective trade-off between *Total Purchase Amount*

Table 10
Simulation estimates of probability of selection and order quantity with five criteria.

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
C1	0.0	0.4	0.0	0.0	0.0	0.0	0.0	0.3	0.9	0.9
C2	0.0	0.4	0.0	0.0	0.0	0.1	0.0	0.3	0.9	0.9
C3	0.0	0.4	0.0	0.0	0.0	0.0	0.0	0.8	0.9	0.7
C4	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.7	1.0	0.7
C5	0.0	0.2	0.0	0.0	0.0	0.1	0.0	0.7	0.9	0.9
C6	0.0	0.3	0.0	0.0	0.0	0.1	0.0	0.1	0.6	0.7
C7	0.0	0.4	0.0	0.0	0.0	0.1	0.0	0.1	0.6	0.7
C8	0.0	0.3	0.0	0.0	0.0	0.1	0.0	0.1	0.8	0.8
C9	0.0	0.4	0.0	0.0	0.0	0.1	0.0	0.1	0.7	0.7
C10	0.0	0.3	0.0	0.0	0.0	0.1	0.0	0.2	0.7	0.7
Mean	0.00	0.33	0.00	0.00	0.00	0.07	0.00	0.34	0.80	0.77
Order Q	0	310705.3	0	0	0	3237.5	0	221612.2	472719.8	392,820

Table 11
Risk averse and procurement behavior.

Parameter	Customer									
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
ρ	0.010	0.008	0.008	0.010	0.010	0.010	0.005	0.005	0.002	0.002
Lower bound of demand	2610	2410	1810	1410	1110	910	875	875	875	775
Mean of order quantities	2610	2410	1810	1426.2	1120.6	913	893	912.2	916.7	852.2

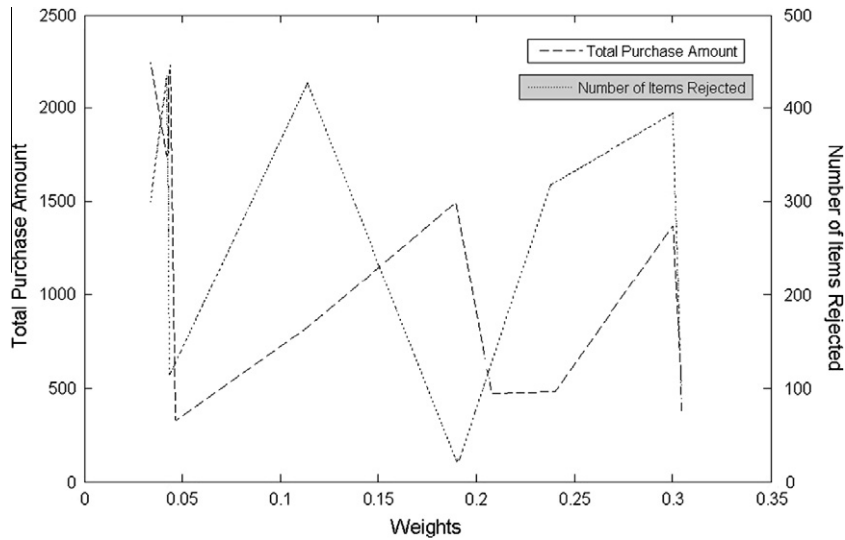


Fig. 3. trade-off game between Total Purchase Amount and Number of Items Rejected.

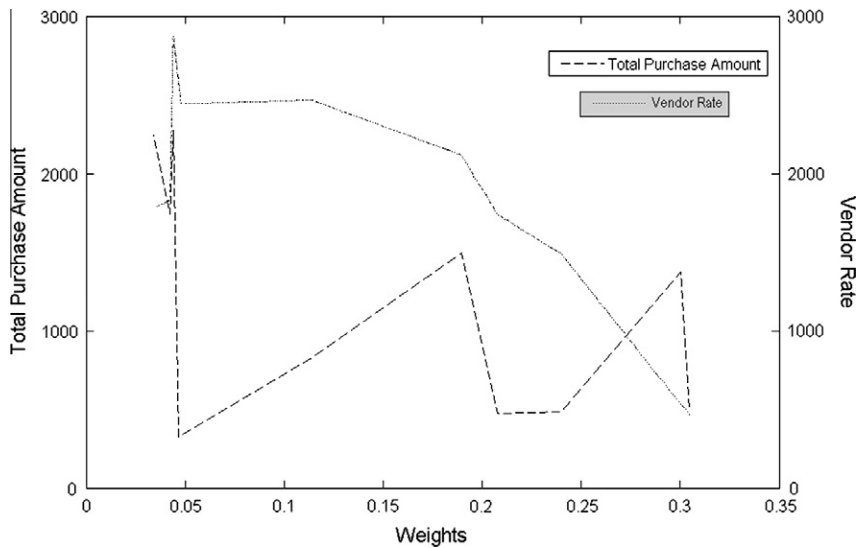


Fig. 4. trade-off game between Total Purchase Amount and Vendor Rate.

and Number of Items Rejected. From Fig. 3, we see that when the weight is either very small or large there exists consistent trend between two objectives while an obvious conflicting trend exists when the weight falls in the middle area. The worst Total Purchase Amount and the best Number of Items Rejected are obtained when the weight is smaller than 0.2.

It can be observed from Fig. 4 that the Total Purchase Amount and Vendor Rate hold conflict changing trend almost everywhere in the weight area. The worst Total Purchase Amount and the best Vendor Rate are achieved a weight value of Total

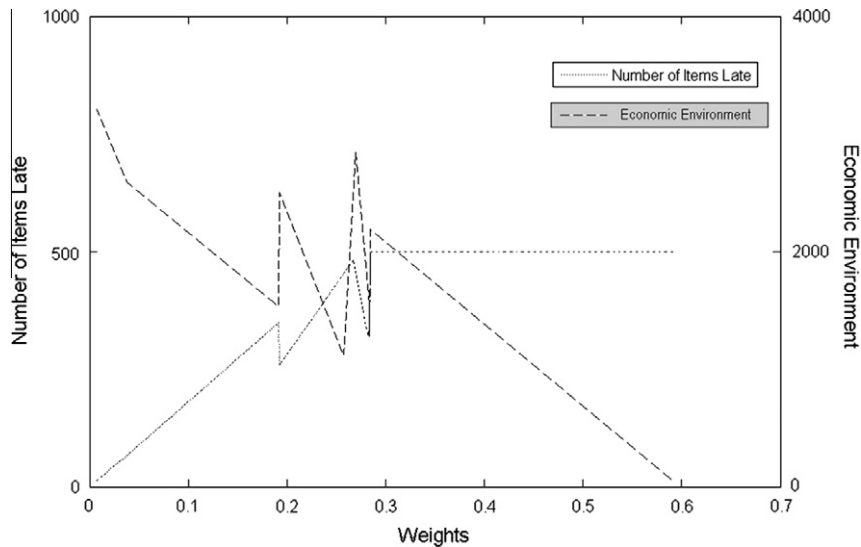


Fig. 5. trade-off game between Number of Item Late and Economic Environment.

Purchase Amount smaller than 0.05. Fig. 5 gives the trade-off game between optimal Number of Item Late and Economic Environment. Fig. 5 indicates that these two indices hold conflict changing trend over anywhere in the weight area of being less than 0.29. Optimal Number of Item Late does not change when the weight attached to it is larger than 0.29. Such trade-off game information including supported points on the trade-off curve is generally very useful for decision makers to identify proper weighting scheme where Pareto optimum is achieved to yield preferred suppliers.

5.4. Comparison analysis

This section conducts a comparison analysis between previous five-objective case and three-objective case by excluding external risk factors of economic environment and vendor rating. This comparison allows us to check the effect of external risk factors such as economic environment and vendor rating over the vendor selection decision process. The stochastic MOP models are solved by using only three quantitative variables: cost, delivery and acceptance. Simulation estimates of probability of selection and order quantity are presented in Table 12 using these three criteria.

Table 12 indicates Supplier S6 is never selected but S1 and S4 are sometimes selected. Should the mean of probability scores to rank the rest suppliers, Supplier S10 is the most preferred candidate but S9 yields the largest amount of order quantity. The explanation here preserves the same to that for the five-objective case. Again, this is because S9 has a larger lower bound order quantity than S10. From Table 12, we can see that using the mean of probability scores to rank suppliers lead to the rank order:

$$S10 \succ S9 \succ S8 \succ S2 \succ S1 \succ S4$$

The rank order of S2 and S8 is the same to that in the five-objective case. The other suppliers change orders. This means external risk factors such as economic environment rate and vendor rate do change selection decision of appropriate suppliers.

Table 12
Simulation estimates of probability of selection and order quantity with three criteria.

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
C1	0.20	0.20	0.00	0.10	0.00	0.00	0.00	0.30	0.80	0.70
C2	0.20	0.20	0.00	0.10	0.00	0.00	0.00	0.40	0.70	0.70
C3	0.10	0.20	0.00	0.00	0.00	0.00	0.00	0.90	0.40	0.50
C4	0.10	0.20	0.00	0.00	0.00	0.00	0.00	0.80	0.60	0.60
C5	0.10	0.20	0.00	0.00	0.00	0.00	0.00	0.80	0.80	0.80
C6	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.80	0.80
C7	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.70	0.80
C8	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.10	0.70	0.80
C9	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.80	0.80
C10	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.10	0.50	0.80
Mean	0.12	0.15	0.00	0.02	0.00	0.00	0.00	0.34	0.68	0.73
Order Q	11040.0	13800.0	0.0	1840.0	0.0	0.0	0.0	23591.0	51840.0	37104.0

A comparison between the three-objective case and five-objective case from Tables 11 and 12 indicates that the probability value of selecting S9, S10 and S2 in the five-objective case is in general larger than that in the three-objective case. However, the probability of selecting S1, S4 and S8 in the five-objective case is in general smaller than that in the three-objective case. This indicates most customers prefer to order more when from suppliers providing better economic environment and vendor rate, which is actually a risk-averse behavior.

6. Conclusions and further consideration

We have developed a stochastic fuzzy multi-objective programming model for supply chain outsourcing risk management in presence of both random uncertainty and fuzzy uncertainty. Utility theory is proposed to treat stochastic data and fuzzy set theory is used to handle fuzzy data. An algorithm is designed to solve the proposed integrated model. We apply this new approach to model a supply chain consisting of three levels: a set of ten suppliers, a core distribution level and ten customers at the third level. Example is extracted from existing study to demonstrate implementation of the proposed models.

This approach allows specification of different risk aversion and various levels of uncertainty by use of the predefined fuzzy number with the certain confidence of level (α -cut-level). Several interesting managerial insights are yielded from scenario analysis of computation results. Risk averse and procurement behavior indicates that a more risk averse customer prefers to order less under uncertainty and risk. Trade-off game analysis yields supported points on the trade-off curve, which is very useful for decision makers to identify proper weighting scheme where Pareto optimum is achieved to yield preferred suppliers. It is known that the uncertainty of information generates the uncertainty decision regions. Therefore, in a further research we will show how the results permit one to generate a set of alternative solutions which cannot be distinguished within the framework of the developed model.

Acknowledgement

This paper is supported by One Hundred Person Project of The Chinese Academy of Sciences, the National Natural Science Foundation of China (No. 70671039; No. 71073177; Grant No. 71110107024).

References

- [1] M. Akarte, N.V. Surendra, B. Ravi, N. Rangaraj, Web based casting supplier evaluation using analytical hierarchy process, *Journal of the Operational Research Society* 52 (5) (2001) 511–522.
- [2] R. Aliev, W. Pedrycz, B. Fazlollahi, O.H. Huseynov, A.V. Alizadeh, B.G. Guirimov, Fuzzy logic-based generalized decision theory with imperfect information, *Information Sciences* 189 (2012) 18–42.
- [3] J.D. Amato, C.H. Furfine, Are credit ratings procyclical?, *Journal of Banking & Finance* 28 (11) (2004) 2641–2677
- [4] V. Batanovic, D. Petrovic, R. Petrovic, Fuzzy logic based algorithms for maximum covering location problems, *Information Sciences* 179 (2009) 120–129.
- [5] R.E. Bellman, L.A. Zadeh, Decision-making in a fuzzy environment, *Management Science* 17B (1970) 141–164.
- [6] E. Bottani, A. Rizzi, An adapted multi-criteria approach to suppliers and products selection – an application oriented to lead-time reduction, *International Journal Production Economics* 111 (2) (2008) 763–781.
- [7] R.K. Brayton, S.W. Director, G.D. Hachtel, L. Vidigal, A new algorithm for statistical circuit design based on quasi-Newton methods and function splitting, *IEEE Trans. Circuits and Systems* 26 (9) (1979) 784–794.
- [8] F.T.S. Chan, Interactive selection model for supplier selection process: an analytical hierarchy process approach, *International Journal Production Research* 41 (15) (2003) 3549–3579.
- [9] F.T.S. Chan, N. Kumar, Global supplier development considering risk factors using fuzzy extended AHP-based approach, *OMEGA – International Journal of Management Science* 35 (4) (2007) 417–431.
- [10] H.C. Chang, L.Y. Ouyang, K.S. Wu, C.H. Ho, Integrated vendor-buyer cooperative inventory models with controllable lead time and ordering cost reduction, *European Journal of Operational Research* 170 (2006) 481–495.
- [11] A. Charnes, W.W. Cooper, Programming with linear fractional functionals, *Naval Research Logistics Quarterly* 9 (3–4) (1962) 181–186.
- [12] Y.J. Chen, Structured methodology for supplier selection and evaluation in a supply chain, *Information Sciences* 181 (2011) 1651–1670.
- [13] I.J. Chen, A. Paulraj, Understanding supply chain management: critical research and a theoretical framework, *International Journal of Production Research* 42 (1) (2004) 131–163.
- [14] K.L. Choy, W.B. Lee, V. Lo, Development of a case based intelligent customer – supplier relationship management system, *Expert Systems with Applications* 23 (3) (2002) 281–297.
- [15] M.A. Cohen, H.L. Lee, Strategic analysis of integrated production – distribution systems: models and methods, *Operations Research* 36 (2) (1988) 216–228.
- [16] T. Coleman, M.A. Branch, A. Grace, *Optimization Toolbox User's Guide (Version 2)*, third ed., The MathWorks, Inc., Natick, MA, 1999.
- [17] C.W. Craighead, J. Blackhurst, M.J. Rungtusanatham, R.B. Handfield, The severity of supply chain disruptions: design characteristics and mitigation capabilities, *Decision Sciences* 38 (2007) 131–156.
- [18] Z. Degraeve, E. Labro, F. Roodhooft, An evaluation of supplier selection methods from a total cost of ownership perspective, *European Journal of Operational Research* 125 (1) (2000) 34–58.
- [19] E.A. Demirtas, Ö. Üstün, Analytic network process and multi-period goal programming integration in purchasing decisions, *Computer and Industrial Engineering* 56 (2) (2009) 677–690.
- [20] G.W. Dickson, An analysis of vendor selection systems and decisions, *Journal of Purchasing* 2 (1) (1966) 5–17.
- [21] D. Dubois, H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, Inc., New York, 1980.
- [22] W. Edwards, F.H. Barron, Smarts and smarter: improved simple methods for multi attribute utility measurement, *Organizational Behavior and Human Decision Processes* 60 (1994) 306–325.
- [23] P. Ekel, W. Pedrycz, R. Schinzing, A general approach to solving a wide class of fuzzy optimization problems, *Fuzzy Sets and Systems* 97 (1998) 49–66.

- [24] M. Fasli, Y. Kovalchuk, Learning approaches for developing successful seller strategies in dynamic supply chain management, *Information Sciences* 181 (2011) 3411–3426.
- [25] P. Finch, Supply chain risk management, *Supply Chain Management: An International Journal* 9 (2) (2004) 183–196.
- [26] R. Florez-Lopez, Strategic supplier selection in the added-value perspective: a CI approach, *Information Sciences* 177 (5) (2007) 1169–1179.
- [27] S.H. Ghodspour, C. O'Brien, The total cost of logistic in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint, *International Journal of Production Economics* 73 (1) (2001) 15–27.
- [28] M. Goetschalckx, C.J. Vidal, K. Dogan, Modeling and design of global logistics systems: a review of integrated strategic and tactical models and design algorithms, *European Journal of Operational Research* 143 (1) (2002) 1–18.
- [29] S.C. Graves, S.P. Willems, Optimizing strategic safety stock placement in supply chains, *Manufacturing & Service Operations Management* 2 (1) (2000) 68–83.
- [30] S.P. Han, A globally convergent method for nonlinear programming, *Journal of Optimization Theory and Applications* 22 (3) (1977) 297–309.
- [31] K.B. Hendricks, V.R. Singhal, An empirical analysis of the effect of supply chain disruptions on long-run stock price performance and equity risk of the firm, *Production and Operations Management* 14 (2005) 35–52.
- [32] S.H. Huang, H. Keskar, Comprehensive and configurable metrics for supplier selection, *International Journal of Production Economics* 105 (2) (2007) 510–523.
- [33] D.N. Ingram, Standard & Poor's enterprise risk management evaluation of insurers risk management newsletter of society of actuaries, Schaumburg (2006) 14–17.
- [34] P.A. Jensen, J.F. Bard, *Operations Research: Models and Methods*, John Wiley & Sons, New York, 2003.
- [35] C. Kao, W. Hsu, A single-period inventory model with fuzzy demand, *Computers & Mathematics with Applications* 43 (6–7) (2002) 841–848.
- [36] T. Kremic, O.I. Tukul, W.O. Rom, Outsourcing decision support: a survey of benefits, risks, and decision factors, *Supply Chain Management: An International Journal* 11 (6) (2006) 467–482.
- [37] O. Kulak, C. Kahraman, Fuzzy multi-attribute selection among transportation companies using axiomatic design and analytic hierarchy process, *Information Sciences* 170 (2–4) (2005) 191–210.
- [38] T.J. Kull, S. Talluri, A supply-risk reduction model using integrated multi-criteria decision making, *IEEE Transactions on Engineering Management* 55 (3) (2008) 409–419.
- [39] M. Kumar, V.R. Shankar, A fuzzy goal programming approach for vendor selection problem in a supply chain, *Computer & Industrial Engineering* 46 (1) (2004) 273–285.
- [40] M. Kumar, P. Vrat, R. Shankar, A fuzzy goal programming approach for vendor selection problem in a supply chain, *Computers & Industrial Engineering* 101 (2) (2006) 69–85.
- [41] H.L. Lee, C. Billington, Material management in decentralized supply chain, *Operations Research* 41 (5) (1993) 835–847.
- [42] Q. Li, Risk, risk aversion and the optimal time to produce, *IIE Transactions* 39 (2) (2007) 145–158.
- [43] Z. Liao, J. Rittscher, A multi-objective supplier selection model under stochastic demand conditions, *International Journal of Production Economics* 105 (1) (2007) 150–159.
- [44] B. Liu, *Uncertain Programming*, A Wiley-Interscience Publication, New York, 1999.
- [45] F.H.F. Liu, H.L. Hai, The voting analytic hierarchy process method for selecting supplier, *International Journal of Production Economics* 97 (3) (2005) 308–317.
- [46] K. Madsen, H. Schjaer-Jacobsen, Algorithms for worst case tolerance optimization, *IEEE Transactions of Circuits and Systems* 26 (9) (1979) 775–783.
- [47] L. Mikhailov, Fuzzy analytical approach to partnership selection in formation of virtual enterprises, *OMEGA – International Journal of Management Science* 30 (5) (2002) 393–401.
- [48] B. Möller, M. Beer, *Fuzzy Randomness-Uncertainty in Civil Engineering and Computational Mechanics*, Springer, Berlin Heidelberg New York, 2004.
- [49] S. Mondal, M. Maiti, Multi-item fuzzy EOQ models using genetic algorithm, *Computers & Industrial Engineering* 44 (2002) 105–117.
- [50] R. Narasimhan, S. Talluri, S.K. Mahapatra, Multiproduct, multicriteria model for supplier selection with product life-cycle considerations, *Decision Sciences* 37 (4) (2006) 577–603.
- [51] G. Nassimbeni, F. Battain, Evaluation of supplier contribution to product development: fuzzy and neuro-fuzzy based approaches, *International Journal of Production Research* 41 (13) (2003) 2933–2956.
- [52] W.L. Ng, An efficient and simple model for multiple criteria supplier selection problem, *European Journal of Operational Research* 186 (3) (2008) 1059–1067.
- [53] M. Ojala, J. Hallikas, Investment decision-making in supplier networks: management of risk, *International Journal of Production Economics* 104 (1) (2006) 201–213.
- [54] D.L. Olson, D. Wu, *Enterprise Risk Management*, World Scientific Press, Singapore, 2008.
- [55] P.F. Pai, M.M. Hsu, Continuous review reorder point problems in a fuzzy environment, *The International Journal of Advanced Manufacturing Technology* 22 (2003) 436–440.
- [56] D. Petrovic, R. Petrovic, M. Vujosevic, Fuzzy models for the newsboy problem, *International Journal Production Economics* 45 (2001) 435–441.
- [57] M.J.D. Powell, A fast algorithm for nonlinearly constrained optimization calculations, in: G.A. Watson (Ed.), *Numerical Analysis, Lecture Notes in Mathematics*, 630, Springer Verlag, 1978.
- [58] C.P. Roa, G.E. Kiser, Educational buyer's perception of vendor attributes, *Journal of Purchasing Material Management* 16 (1980) 25–30.
- [59] R.F. Saen, Supplier selection in the presence of both cardinal and ordinal data, *European Journal of Operational Research* 183 (2) (2007) 741–747.
- [60] J. Sarkis, S. Talluri, A model for strategic supplier selection, *Journal of Supply Chain Management* 38 (1) (2002) 18–28.
- [61] P. Schönsleben, *Integral Logistics Management Operations and Supply Chain Management in Comprehensive Value-added Networks*, third ed., Auerbach Publications, Boca Raton, 2007.
- [62] R.E. Steuer, *Multiple Criteria Optimization: Theory, Computation, and Application*, John Wiley & Sons, New York, 1986.
- [63] S. Talluri, A buyer-seller game model for selection and negotiation of purchasing bids, *European Journal of Operational Research* 143 (1) (2002) 171–180.
- [64] S. Talluri, R. Narasimhan, Vendor evaluation with performance variability: a max-min approach, *European Journal of Operational Research* 146 (3) (2003) 543–552.
- [65] S. Talluri, R. Narasimhan, A. Nair, Vendor performance with supply risk: a chance-constrained DEA approach, *International Journal of Production Economics* 100 (2) (2006) 212–222.
- [66] S. Talluri, J. Sarkis, A model for performance monitoring of suppliers, *International Journal of Production Research* 40 (16) (2002) 4257–4269.
- [67] H. Tanaka, T. Okuda, K. Asai, On fuzzy mathematical programming, *Journal of Cybernetics* 3 (1974) 37–46.
- [68] C.S. Tang, Perspectives in supply chain risk management, *International Journal of Production Economics* 103 (2006) 451–488.
- [69] D.J. Thomas, P.M. Griffin, Coordinated supply chain management, *European Journal of Operational Research* 94 (1) (1996) 1–15.
- [70] J.H. Thun, D. Hoenig, An empirical analysis of supply chain risk management in the German automotive industry, *International Journal of Production Economics* 131 (2011) 242–249.
- [71] S.A. Torabi, E. Hassini, An interactive possibilistic programming approach for multiple objective supply chain master planning, *Fuzzy Sets and Systems* 159 (2) (2008) 193–214.
- [72] M. Vujosevic, D. Petrovic, R. Petrovic, EOQ formula when inventory cost is fuzzy, *International Journal Production Economics* 45 (1996) 499–504.
- [73] V. Wadhwa, A.R. Ravindran, Vendor selection in outsourcing, *Computers and Operations Research* 34 (12) (2007) 3725–3737.
- [74] J. Wang, Y. Shu, A possibilistic decision model for new product supply chain design, *European Journal of Operational Research* 177 (2) (2007) 1044–1061.

- [75] C.A. Weber, J.R. Current, A multiobjective approach to vendor selection, *European Journal of Operational Research* 68 (2) (1993) 173–184.
- [76] C.A. Weber, J.R. Current, W.C. Benton, Vendor selection criteria and methods, *European Journal of Operational Research* 50 (1991) 2–18.
- [77] C.A. Weber, L.M. Ellram, Supplier selection using multi-objective programming: a decision support system approach, *International Journal of Physical Distribution & Logistics Management* 23 (2) (1992) 3–14.
- [78] D. Wu, Performance evaluation: an integrated method using data envelopment analysis and fuzzy preference relations, *European Journal of Operational Research* 194 (1) (2009) 227–235.
- [79] D. Wu, D.L. Olson, Enterprise risk management: coping with model risk in a large bank, *Journal of the Operational Research Society* 61 (2) (2010) 179–190.
- [80] D. Wu, D.L. Olson, A comparison of stochastic dominance and stochastic DEA for vendor evaluation, *International Journal of Production Research* 46 (8) (2008) 2313–2327.
- [81] D. Wu, D.L. Olson, Supply chain risk, simulation, and vendor selection, *International Journal of Production Economics* 114 (2) (2008) 646–655.
- [82] H. Yu, A.Z. Zeng, L. Zhao, Analyzing the evolutionary stability of the vendor-managed inventory supply chains, *Computers & Industrial Engineering* 56 (1) (2009) 274–282.
- [83] L.A. Zadeh, Fuzzy sets, *Information and Control* 8 (1965) 338–353.
- [84] H.-J. Zimmermann, Description and optimization of fuzzy systems, *International Journal General System* 2 (1976) 209–215.