Theory and Methodology

On the single-assignment p-hub center problem ☆

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Abstract

We study the computational aspects of the single-assignment p-hub center problem on the basis of a basic model and a new model. The new model's performance is substantially better in CPU time than different linearizations of the basic model. We also prove the NP-Hardness of the problem. © 2000 Elsevier Science B.V. All rights reserved.

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Hub location problems arise when it is desirable to consolidate and disseminate flows at certain centralized locations called hubs. Typical applications arise in airline passenger travel (Toh and Higgins, 1985), cargo delivery (Kuby and Gray, 1993), and message delivery in computer communication networks (Klincewicz, 1998).

The existing studies in the literature on hub location have almost exclusively focused on the *p-hub median problem* which involves the minimization of the *total cost*. The case with $p = 1, 2$ is posed by O'Kelly (1986) and the case for general p is formulated as a quadratic binary program by O'Kelly (1987). Different linearizations of the basic

model of O'Kelly (1987) are investigated by Aykin (1995), Campbell (1994), Campbell (1996), Ernst and Krishnamoorthy (1996, 1998), Skorin-Kapov et al. (1996).

Our focus in this paper is on the minimax criterion which is essentially unstudied in the literature. The minimax criterion is traditionally used in location applications to minimize the adverse effects of worst case scenarios in providing emergency service. In hub location, even though emergency service protection does not seem to be an issue, the minimax criterion is still important from the viewpoint of minimizing the maximum dissatisfaction of passengers in air travel and minimizing the worst case delivery time in cargo delivery systems. The latter case is particularly important for delivery of perishable or time sensitive items.

The literature on hub location under the minimax criterion is restricted to two papers. The

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initial motivation for the minimax criterion is given by O'Kelly and Miller (1987) in the context of cargo delivery. In the same paper, the special case with $p = 1$ is shown to be equivalent to the well known 1-center location problem in which a single facility is to be located to minimize the maximum distance to the users of the facility. The second paper that deals with minimax criterion in hub location is Campbell (1994) in which he formulates the p-hub center location problem. This problem involves locating a fixed number, p, of hubs to minimize the maximum travel time between origin destination pairs. Campbell (1994) gives a quadratic binary program for the p-hub center problem which we refer to as the basic model in the sequel. Campbell also gives a linearization for the basic model, but he does not report any computational results.

In this paper, our focus is on the single-assignment p-hub center location problem. We first give a combinatorial formulation of this problem and prove that it is NP-Hard. We then focus on different linearizations of the basic model as well as a different model of the problem and study their computational performances. We first study the computational performance of Campbell's original linearization. Then we adapt a linearization of Skorin-Kapov et al. (1996), developed initially for the total cost criterion, to the p-hub center problem and study its computational performance. Even though this linearization gives somewhat improved performance, we obtain an even better performance from a new linearization that we propose in the paper. However, a dramatic computational improvement is obtained from a new formulation of the problem. This shows that it is sometimes more important to devise a new model for a given problem than to focus solely on improvements that come from different linearizations of the basic model.

The rest of the paper is organized as follows. In Section 1 we provide a combinatorial formulation of the p-hub center problem and prove that the problem is NP-Hard for $p < n - 1$. We present the basic model proposed by Campbell in Section 2. We provide three linearizations of the basic model, including Campbell's original linearization, in the same section. We also report the computational

results on these linearizations. In Section 3, we propose a new model of the p-hub center problem and provide computational results on the new model. The paper ends with concluding remarks in Section 4.

1. Complexity

In this section we give a combinatorial formulation of the p-hub center problem for the single-assignment case and prove its NP-Hardness. Even though hub location problems are customarily defined based on a complete graph whose arc costs satisfy the triangle inequality, we deviate from this tradition and define the problem on a physical transportation network which is assumed to be connected but not necessarily complete. This way of defining the problem permits reducing the dominating set problem to the p-hub center problem, thereby proving its NP-Hardness.

Let $G = (N, E)$ be a connected undirected transportation network with node set $N = \{1, \ldots, n\}$ and arc set E. We may think of the arcs in E as 'physically existing' links of the transportation network, e.g. the arcs correspond to nonstop flight segments in air transport whereas they correspond to physical roads in surface transportation. Associated with each arc $(i, j) \in E$ is a weight $\tau_{ij} > 0$ which represents the length of that arc. We may interpret τ_{ij} as the time to traverse the arc (i, j). For each pair of nodes $i, j \in N$, define t_{ij} to be the length of a shortest path in G connecting i and j. Note that $0 \leqslant t_{ij} < \infty$ $\forall i, j \in N$ due to the connectedness assumption, $t_{ij} \leqslant \tau_{ij}$ $\forall (i, j) \in E$, $t_{ij} = 0$ iff $i = j$, $t_{ij} = t_{ji}$ and $t_{ij} + t_{jk} \geqslant t_{ik}$ $\forall i, j, k$. Let $H \subset N$ be a set of nodes that specify the locations of hubs and denote by $a(i) \in H$ the hub to which node i is assigned. Let $a = (a(1), \ldots, a(n))$ and let H^n be the n-fold Cartesian product of H with itself. Let $\alpha(0 \leqslant \alpha \leqslant 1)$ be the discount factor for hub-to-hub transportation. Given a positive integer p $(1 \leqslant p < n)$ the p-hub center problem is:

$$\min_{\substack{H \subset N \\ |H| = p}}, \min_{a \in H^n} \max_{\substack{i, j \in N \\ i < j}} \left(t_{i\,a(i)} + \alpha t_{a(i)\,a(j)} + t_{a(j)\,j} \right).$$

We remark that every node is assigned to exactly one hub in the above formulation (single-assignment). There is also a multi-assignment version of the problem in which a node may be allocated to more than one hub meaning that the travel from a given node i to different destinations j may be routed through different hubs each of which is assigned to node i. In this paper we do not consider the multi-assignment problem and omit the term 'single-assignment' in the rest of the paper.

We now state the recognition form of the p-hub center problem: Given $G = (N, E)$ with edge lengths $\tau_{ij} > 0, (i, j) \in E$, a rational α in the unit interval, a positive rational β, and a positive integer p $(1 \leqslant p \leqslant n - 1)$, does there exist a subset H of N consisting of at most p nodes and an assignment vector $a = (a(1), \ldots, a(n)) \in H^n$ such that $t_{i\,a(i)} + \alpha t_{a(i)\,a(j)} + t_{a(j)\,j} \leqslant \beta$ for $1 \leqslant i < j \leqslant n$?

Theorem. *The recognition form of the p-hub center problem for $p < n - 1$ is NP-Complete even if $\alpha = 0$ and $G = (V, E)$ is a planar graph with unit arc lengths and maximum degree three.*

Proof. The theorem will be proved by reduction from the dominating set problem.

Dominating set problem: Given a connected graph $G' = (N', E')$ and a positive integer $k \leqslant |N'|$, does there exists a subset X of N' with $|X| \leqslant k$ such that every node not in X is adjacent to at least one node in X, i.e. $\forall u \in N' \setminus X \, \exists v \in X$ for which $(u, v) \in E'$?

We note that the dominating set problem is NP-Complete even if G' is planar with maximum degree 3 (Garey and Johnson, 1979).

Clearly, the recognition form of the p-hub center problem is in class NP. Consider an instance of the dominating set problem. We reduce it to the p-hub center problem as follows: Take

$$N = N', \ \ E = E', \ \ \tau_{ij} = 1 \ \ \forall (i, j) \in E, \ \ p = k,$$

$$\alpha = 0, \ \ \beta = 2.$$

We first prove that if X solves the dominating set problem, then X also solves the created instance of the p-hub center problem. To prove the claim, take $H = X$ and construct an assignment vector $a = (a(1), \ldots, a(n))$ where, for each $i \in N$,

$a(i)$ is a closest node in H to i. The constructed solution (H, a) satisfies $|H| \leqslant k = p$ and $t_{i\,a(i)} + \alpha t_{a(i)\,a(j)} + t_{a(j)\,j} \leqslant 2$ since $\alpha = 0$ and H is a dominating set so that $t_{i\,a(i)} \leqslant 1 \ \forall i \in N$. Conversely, if (H, a) solves the created instance of the p-hub center problem, then $X = H$ solves the dominating set problem. To prove the claim, suppose there is a node i which is not adjacent to any $h \in H$. Then, the distance of node i to a closest member of H is at least 2. Since $p < n - 1$ there is at least one other node $j \notin H$, $j \neq i$, so that $t_{i\,a(i)} + \alpha t_{a(i)\,a(j)} + t_{a(j)\,j} \geqslant 2 + 0 + 1 = 3$ contradicting that (H, a) is a feasible solution to the created instance of the p-hub center problem. Note also that $|H| \leqslant p = k$.

Hence, the dominating set problem has a YES answer if and only if the corresponding instance of the p-hub center problem has a YES answer. $\square$

Since the recognition form of the p-hub center problem is NP-Complete, we might say that the optimization form for $p < n - 1$ is NP-Hard.

2. Basic model and its linearizations

In this section we first give the original integer programming (IP) formulation of Campbell (1994). In Campbell's formulation the objective function consists of the maximum of quadratic terms in binary variables.

Let X_{ik} be a binary variable which takes on the value 1 if node i is allocated to a hub at node k and the value 0 otherwise. Note that $X_{kk} = 1$ iff there is a hub at node k. The p-hub center problem, p-HC1, is:

$$\min \max_{i, j, k, m} X_{ik} X_{jm} (t_{ik} + \alpha t_{km} + t_{jm})$$

s.t

$$\sum_k X_{ik} = 1 \quad \forall i, \tag{1}$$

$$X_{ik} \leqslant X_{kk} \quad \forall i, k, \tag{2}$$

$$\sum_k X_{kk} = p, \tag{3}$$

$$X_{ik} \in \{0, 1\} \quad \forall i, k. \tag{4}$$

Constraints (1) and (4) ensure that every node is assigned to exactly one hub while constraint (2)

ensures that such an assignment cannot be made unless there is a hub at node k. Constraint (3) limits the number of hubs to p. The above quadratic binary program has n^2 binary variables and $n^2 + n + 1$ constraints.

We now give a linearization of p-HC1 proposed by Campbell (1994). Let X_{ijkm} be a binary variable which takes on the value 1 if the path from origin i to destination j is via hubs k and m ($i \rightarrow k \rightarrow m \rightarrow j$). The linearization proposed by Campbell, LIN1, is:

min Z

s.t

$$Z \geqslant X_{ijkm}(t_{ik} + \alpha t_{km} + t_{jm}) \quad \forall i, j, k, m, \tag{5}$$

$$\sum_k \sum_m X_{ijkm} = 1 \quad \forall i, j, \tag{6}$$

$$\sum_j \sum_m (w_{ij} X_{ijkm} + w_{ji} X_{jimk})$$
$$= \sum_j (w_{ij} + w_{ji}) X_{ik} \quad \forall i, k, \tag{7}$$

$$X_{ijkm} \in \{0, 1\} \quad \forall i, j, k, m, \tag{8}$$

and constraints (2)–(4),

where $w_{ij} \geqslant 0$ is the flow from origin i to destination j. Constraints (6) and (8) ensure that there is exactly one pair of hubs (k, m) which are, respectively, the first and last hubs on the path from origin i to destination j ($k = m$ is possible). Constraint (7) is the constraint that correctly relates the path variables X_{ijkm} to the allocation variables X_{ik}. The right hand side of (7) is the total flow originating and ending at node i provided that i is allocated to a hub at node k. When $X_{ik} = 1$, the left side of (7) achieves the same total flow by summing all the incoming and outgoing flows on all paths each of which includes a shortest path between i and k as a subpath. Note also that when $X_{ik} = 0$, such path variables are forced to take on the value zero. We refer to the above formulation as LIN1.

In linearizing the problem, it is desired that $X_{ijkm} = 1$ if and only if $X_{ik} = X_{jm} = 1$. This is accomplished by constraint (7) in the above linearization. The same thing can be achieved by using the constraints:

$$\sum_m X_{ijkm} = X_{ik} \quad \forall i, j, k, \tag{9}$$

$$\sum_k X_{ijkm} = X_{jm} \quad \forall i, j, m, \tag{10}$$

as was done previously by Skorin-Kapov et al. (1996) for the p-hub median problem. Imposing the constraints (9) and (10) together with the zero/one requirement on the variables X_{ijkm} makes constraints (6) and (7) redundant. We refer to the linearization obtained from LIN1 by including (1) and replacing (6) and (7) with (9) and (10) as LIN2.

We now propose a third linearization, called LIN3, which we obtain from LIN2 by replacing (9) and (10) with constraint (11) below and by replacing the zero/one requirement on the variables.

$$X_{ijkm} \text{ by } X_{ijkm} \geqslant 0 \quad \forall i, j, k, m$$
$$X_{ijkm} \geqslant X_{ik} + X_{jm} - 1 \quad \forall i, j, k, m. \tag{11}$$

Note that, integrality on X_{ijkm} variables is not necessary in LIN3, because the objective function and constraints (5) and (11) force X_{ijkm} variables to take on their lowest possible values which is either one or zero.

In all these linearizations, the objective function and the constraints (2)–(4), and (5) are common. Additionally, (8) is common in LIN1 and LIN2 and (1) is common in LIN2 and LIN3. In LIN1 and LIN2, there are $n^4 + n^2$ binary variables and, in LIN3, there are n^2 binary, n^4 real variables, while there are $n^4 + 3n^2 + 1$ constraints in LIN1, $n^4 + 2n^3 + n^2 + n + 1$ constraints in LIN2, and $2n^4 + n^2 + n + 1$ constraints in LIN3.

We test these linearizations with the CAB data by using CPLEX 5.0 on a 8 CPU, 50 Mhz super Sparc station with 16B memory. The CAB Data set is generated from the Civil Aurenatics Board Survey of 1970 passenger travel data in the U.S. It provides the passenger flows and distances between 25 cities. Following the standard practice that has been customarily utilized in computational p-hub median research, we generate a total of $4 * 3 * 5 = 60$ instances corresponding to $n \in \{10, 15, 20, 25\}$, $p \in \{2, 3, 4\}$, and $\alpha \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$. The four problem sizes corresponding to different n utilize the distance data (and flow data for LIN1) for the first n cities in the CAB

Data. An upper limit of 15 hours is imposed on the CPU time.

LIN1 has a poor computational performance as it has not been able to solve any of the 60 instances within the 15 hour limit. LIN2 has limited success as it has been able to solve, within the 15 hour limit, only 10 of the 60 instances corresponding to all values of α for $n = 10$ and $p = 2, 3$. The maximum CPU time of LIN2 for the solved 10 instances is 14.45 hours. LIN3 has a better performance. It has been able to solve the 10 instances that have also been solved by LIN2 within a maximum time of 40.3 minutes, thus achieving about a 20-fold reduction in CPU time. In addition, it has been able to solve the 5 instances corresponding to $n = 10$ and $p = 4$ within 1.1 hours. The largest problem size that can be solved by LIN3 is $n = 15$ for $p = 2$ (the cases $p = 3, 4$ are not solved within the 15 hour limit). All the 5 instances corresponding to $(n, p) = (15, 2)$ has been solved by LIN3 within the 15 hour limit where the maximum CPU time is 13.6 hours.

As can be seen from the reported results, LIN3 has the best performance among the three linearizations but with limited success. The largest problem size it can handle is $n = 15$ with $p = 2$ while none of the instances with larger n can be solved by LIN3 regardless of p. In Section 3 we reformulate the p-hub center problem from a different perspective. The resulting model solves, for example, the $(n, p) = (15, 2)$ combination in the order of a few minutes while LIN3 spends almost 13.5 hours to solve the same combination. Substantial improvement has also been obtained from the new model for larger sized problems.

3. New model

In the new model, t_{ij} is interpreted to be the shortest travel time between nodes i and j. Define now a real variable T_{ij} which stands for the travel time from node i to node j via the two hubs to which i and j are assigned. Let $T_{ij} = S_{ir} + t_{rj}$ where S_{ir} is another real variable which stands for the travel time from origin i to node r under the assumption that node j is assigned to a hub at node r. In order to ensure that the real variables T_{ij}'s and

S_{ir}'s take on the correct values we impose the constraints

$$S_{ir} = \sum_{k} (t_{ik} + \alpha t_{rk}) X_{ik}, \tag{12}$$

$$T_{ij} = \sum_{r} (S_{ir} + t_{rj}) X_{jr}, \tag{13}$$

where X_{ij} is a binary variable which takes on the value 1 if node i is allocated to a hub at node j and value 0 otherwise. With the single assignment constraint (1), there is exactly one k for which $X_{ik} = 1$ and exactly one r for which $X_{jr} = 1$ so that (12) and (13) supply the correct values for S_{ir} and T_{ij}.

The new model, which we call p-HC2′, is as follows:

$$\min Z$$

s.t.

$$Z \geqslant T_{ij} \quad \forall i, j, \tag{14}$$

(1)–(4), (12), (13).

p-HC2′ is a nonlinear mixed integer program with $2n^2 + 1$ real variables and n^2 binary variables.

We may eliminate the real variables T_{ij} and S_{ir} from p-HC2′ to obtain a simplified model which retains the binary variables and the real variable Z. Observe that, because of the single assignment constraint, the summation operator in (13) can be replaced by the maximum operator. With this and using the right side of (12) for S_{ir}, we have

$$T_{ij} = \max_{r} \left\{ \left(\sum_{k} (t_{ik} + \alpha t_{kr}) X_{ik} + t_{rj} \right) X_{jr} \right\}. \tag{15}$$

Using (15), it is direct to replace (14) by

$$Z \geqslant \left\{ \sum_{k} [(t_{ik} + \alpha t_{kr}) X_{ik}] + t_{rj} \right\} X_{jr} \; \forall r \; \text{ and } \forall i, j. \tag{16}$$

The simplified model which we refer to as p-HC2 is

$$\min Z$$

s.t. $(16), (1)$–(4)

p-HC2 is a nonlinear mixed integer program with one real and n^2 binary variables and $n^3 + n^2 +$

$n + 1$ constraints. In what follows we linearize p-HC2.

Lemma.

$$Z \geqslant \sum_k [(t_{ik} + \alpha t_{kr})X_{ik}] + t_{jr}X_{jr} \ \forall i, j, r \qquad (17)$$

correctly linearizes the constraint (16).

Proof. There are 2 cases to consider depending on the value of X_{jr}. Let s be the index for which $X_{is} = 1$. Then $\sum_k (t_{ik} + \alpha t_{kr})X_{ik} = t_{is} + \alpha t_{sr}$ both in (16) and (17).

- Case 1: $X_{jr} = 1$. Then $Z \geqslant t_{is} + \alpha t_{sr} + t_{jr}$ which is the time of journey between nodes i and j when i is assigned to a hub at node s and j is assigned to a hub at node r. Hence, the right sides of (16) and (17) are identical for the pair i, j in this case.

Table 1
CPU times for the p-HC2LIN

n	α	p				
		2	3	4	5	
CPU in seconds						
	0.2	8.0	**8.1**	6.1	7.4	
	0.4	6.4	4.0	2.6	2.9	
10	0.6	4.5	5.9	2.5	3.7	
	0.8	2.4	5.5	4.0	1.2	
	1.0	1.8	4.4	3.3	1.4	
	Avg.	4.6	5.6	3.7	3.3	4.3
	Max.	8.0	8.1	6.0	7.0	8.1
CPU in seconds						
	0.2	211.8	**313.2**	311.8	238.2	
	0.4	124.3	180.6	137.3	62.2	
15	0.6	16.3	25.9	77.4	77.4	
	0.8	20.0	20.5	17.8	35.1	
	1.0	23.7	15.3	13.4	6 .3	
	Avg.	79.2	111.1	111.5	83.8	96.4
	Max.	211.8	313.2	311.8	238.2	313.2
CPU in minutes						
	0.2	43.4	62.2	**69.2**	45.4	
	0.4	35.5	55.6	56.6	34.4	
20	0.6	23.3	36.1	21.8	15.4	
	0.8	13.0	21.4	11.6	27.6	
	1.0	2.4	0.9	6.8	1.9	
	Avg.	23.5	35.2	33.2	24.9	29.2
	Max.	43.4	62.2	69.2	45.4	69.2
CPU in hours						
	0.2	3.8	8.1	10.2	7.1	
	0.4	4.0	7.5	**11.3**	8.5	
25	0.6	3.0	8.2	7.9	8.2	
	0.8	1.9	4.2	4.4	5.4	
	1.0	0.8	0.5	1.9	1.8	
	Avg.	2.8	5.7	7.2	6.2	5.4
	Max.	4.0	8.2	11.3	8.5	11.3

- Case 2: $X_{jr} = 0$. This case gives $Z \geqslant t_{is} + \alpha t_{sr}$ in (17) while it gives $Z \geqslant 0$ in (16). Even though $X_{jr} = 0$, there exists another j' such that $X_{j'r} = 1$. For the pair i, j', we have $X_{is} = 1$, $X_{j'r} = 1$ so that $Z \geqslant t_{is} + \alpha t_{sr} + t_{rj'}$. Hence, $Z \geqslant t_{is} + \alpha t_{sr}$ is ineffective since $t_{rj'} \geqslant 0$. $\quad \square$

The linearized version of p-HC2, referred to as p-HC2LIN, is as follows:

min Z

s.t. $(17), (1)–(4).$

Note that p-HC2LIN requires n^2 zero/one variables and $n^3 + n^2 + n + 1$ constraints.

We test the computational performance of p-HC2LIN by using 80 instances generated from the CAB Data set corresponding to the same combinations of (n, p, α) described in Section 2 with the additional parameter setting $p = 5$ which was not included in the experimental design of Section 2. In Table 1, we present the CPU times reported by CPLEX 5.0 for each of the 80 instances. In addition, for each (n, p) combination we report the average and maximum CPU times of the 5 settings of α. The last column of the same table provides the averages and the maxima over p for each setting of n. In addition, the maximum reported CPU time for each setting of n is highlighted in bold.

As can be seen from Table 1, in comparison to LIN3 which solves $(n, p) = (15, 2)$ in a maximum CPU time of 13.6 hours, p-HC2LIN solves the same combination in a maximum CPU time of 3.5 minutes. This shows that the computational performance of the new model is significantly better than all three linearizations of the basic model.

This significant improvement is also detected in the larger problem sizes. For example, while the linearizations of the basic model cannot solve the problems with $n = 15$, $p \geqslant 3$ within the 15 hour limit, the linearization of the new model solves these instances in a matter of about 5 minutes. Additionally, the 15 hour limit has not been encountered by the new model for the large problem instances $n = 20$ and 25. For $n = 20$, the maximum CPU time of the linearization of the new model is

a little over 1 hour while the average time is about half an hour. For $n = 25$, the average and maximum times go up to 5.4 and 11.3 hours, respectively. This shows that the exponential behavior of the solution time becomes pronounced after $n \geqslant 20$.

4. Conclusion

In this paper we focused on the p-hub center problem which is essentially unstudied in the literature. A combinatorial formulation is provided and its NP-Completeness is established. A computational study based on 80 instances generated from the traditionally used CAB Data is carried out to test the computational performance of three linearizations of the basic model provided by Campbell (1994) and a linearized new model proposed in this paper. The computational tests indicate that the linearization of the new model's performance is far more superior to the linearizations of the basic model. We also note that the linearization of the new model results in a binary program with n^2 binary variables while the linearizations of the basic model involve $n^4 + n^2$ binary variables for LIN1 and LIN2, n^2 binary and n^4 real variables for LIN3. This also shows that there are substantial reductions in core storage requirements in favor of the new model.

References

Aykin, T.F., 1995. The hub location and the routing problem. European Journal of Operational Research 83 (1), 200–219.

Campbell, J.F., 1994. Integer programming formulations of discrete hub location problems. European Journal of Operational Research 72, 387–405.

Campbell, J.F., 1996. Hub location and the p-hub median problem. Operations Research 44, 923–935.

Ernst, A.T., Krishnamoorthy, M., 1996. Efficient algorithms for the uncapacitated single allocation p-hub median problem. Location Science 4 (3), 139–154.

Ernst, A.T., Krishnamoorthy, M., 1998. Exact and heuristic algorithms for the uncapacitated multiple allocation p-hub median problem. European Journal of Operational Research 104, 100–112.

Garey, M.R., Johnson, D.S., 1979. Computers and Intractability. A Guide to the Theory of NP-Completeness, Freeman, New York.

Klincewicz, J.G., 1998. Hub location in backbone/tributary network design: A review, Unpublished manuscript (to appear in Location Sci.), AT&T Labs, Holmdel, NJ.

Kuby, M.J., Gray, R.G., 1993. The hub network design problem with stopover and feeders The case of Federal Express. Transportation Research 27A(1), 1–12.

O'Kelly, M.E., 1986. The location of interacting hub facilities. Transportation Science 20 (2), 92–105.

O'Kelly, M.E., 1987. A quadratic integer program for the location of interacting hub facilities. European Journal of Operational Research 32, 393–404.

O'Kelly, M.E., Miller, H.J., 1987. Solution strategies for the single facility minimax hub location problem. Papers in Regional Science 70, 393–404.

Skorin-Kapov, D., Skorin-Kapov, J., O'Kelly, M.E., 1996. Tight linear programming relaxations of uncapacitated p-hub median problems. European Journal of Operational Research 94, 582–593.

Toh, R.S., Higgins, R.C., 1985. The impact of hub and spoke network centralization and route monopoly on domestic airline profitability. Transportation Journal 24, 16–27.