Proceedings of IDETC/CIE 2005
ASME 2005 International Design Engineering Technical Conferences
& Computers and Information in Engineering Conference
September 24-28, 2005, Long Beach, California USA

DETC2005-85158

SCALABLE HPC SIMULATIONS OF FLEXIBLE MULTIBODY INDEX-3 DYNAMIC SYSTEMS

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ABSTRACT

In this paper a highly scalable parallel formulation of the primal-dual technique is presented for index-3 constrained flexible multi-body dynamics system. The key features of the primal-dual approach are constraint preservation, preserving the original order of accuracy of time integration operators that are employed, and faster convergence rates of nonlinear iterations for the solution of flexible multi-body dynamical systems. In addition, this technique not only preserves the underlying properties of time integration operators for ordinary differential equations, but also eliminates the need for index reduction, constraint stabilization and regularization approaches. The key features of the parallel formulation of rigid and flexible modeling and simulation technology are capabilities such as adaptive high/low fidelity modeling that is useful from the initial design concept stage to the intermediate and to the final design stages in a single seamless simulation environment. The examples considered illustrate the capabilities and scalability of the proposed high performance computing (HPC) approach for large-scale simulations.

Keywords: Flexible multibody dynamics; Differential-algebraic equations; Index-3 systems; Constrained systems; Index reduction; Order reduction; Order preserving; Constraint stabilization; Primal-Dual technique, Parallel formulation.

INTRODUCTION

Traditionally, dynamic analyses of a multi-body system employing high performance computing (HPC) platforms are performed considering the components of the system as rigid. For example, see [1] for HPC computations of vehicle system dynamics for providing an understanding and controlling of the gross motion of the system such as the roll angle, pitch angle, steering angle for ride quality, etc. The assumption that the bodies are rigid is reasonable as the traditional systems are big and heavy. To consider the components of lightweight modern multi-body systems, operating at high speeds, to be rigid could render the prediction of system performance erroneous. The study of dynamics of flexible multi-body systems has become increasingly important in recent years due to the challenges posed in the design of light weight terrain vehicles, and in other applications such as robotic manipulators, large scale mobile lightweight robots, space stations, space crafts and machine components subjected to dynamic high speed operating conditions.

The rigid and flexible modeling environment has traditionally been one in which independently developed and validated codes were used to analyze different sub-systems. This is accomplished by coupling two different codes, namely, rigid-body dynamic analysis capabilities with finite element analysis software. It is increasingly being recognized that there is no shortage of multi-disciplinary software codes available especially on serial computing environments to perform various simulations. However, what is desired is fewer codes that can solve more problems for the design engineer in a HPC computing environment. More importantly, a unified and seamless coupled approach is required to address the issues of parallel scalability in addition to avoiding any errors in interfacing multi-physics codes, as there is no unique way of coupling them [2]. These errors in the uncoupled approach are highlighted in the literature and few of them are citied here. For example: 1) the deformation of single bodies cannot be computed accurately by means of an uncoupled rigid multiple body system, using the computed inertial forces of the multiple body system [3], 2) accurate stress computations requires special treatment. For example, proper design of the so-called quasi-comparison functions (combination of eigenfunctions and static deformation modes to represent body deformation in the small deformation regime) has shown to improve stress representation in the flexible bodies [4], 3) standard approaches in commercial software for 3D multiple body systems that use component mode synthesis in order to reduce the number of degrees of freedom shows a lack in the modeling of contact and material non-linearities, 4) they are not readily suitable for bodies undergoing large deformations [5]. These issues need to be addressed in modern flexible multi-body dynamic systems because the design of these systems can consist of many sub-system designs, and an integrated modeling and testing environment may suffer from these inaccuracies. A typical illustration of a complex system is described in Fig. 1.

In addition to the above, computer modeling and simulation of such flexible multi-body dynamical systems requires accurate, efficient and robust time integration schemes to handle complexities of high index differential-algebraic equations (DAE). The concept of the index of a DAE system is discussed in [6]. In the generalized co-ordinates of the system, the equations of motion of the constrained dynamic systems are inherently index-3 differential algebraic equations (DAE). It is critical to readily use existing finite element software for simulating flexible-rigid multi-body dynamics, and also to address the challenging issues pertaining to parallel scalability [7]. However, the difficulties associated with the application of ordinary differential equation (ODE) methods to the solution of such index-3 DAEs are: (i) they are prone to numerical instability for simplectic integrators [8, 9] in the presence of multiple roots on the unit circle at infinite sampling frequencies which leads to unbounded solutions if the high frequencies are not resolved, (ii) induce constraint violation leading to computationally expensive constraint stabilization methods [10, 11] to preserve it or the need of various regularization approaches such as [12-15], etc., on the reduced index system such as index 2 or 1 to stabilize and preserve the constraint of the original system (index 3), and (iii) leads to order reductions for stiff integrators [16-18, 6]. It is particularly difficult to obtain an accurate solution for the algebraic variable, namely the Lagrange multipliers. In addition, often, the velocity and accelerations suffer from an order reduction. Recently, in contrast to symplectic integrators, it was shown that algorithms designed to simultaneously preserve the total momentum and the energy of the system are shown to be free from shortcomings of the instabilities [19] but suffer from order reduction in velocity and Lagrange multipliers [17]. Towards this end we employ here the primal-dual methodology developed in [20], which overcomes many computational challenging issues, namely, constraint preservation, preserving order of accuracy of the employed time integration operators, and obtaining faster convergence rates of nonlinear iterations for the solution of multi-body dynamical index-3 DAE. In this paper a parallel formulation of the primal-dual technique is presented for index-3 constrained flexible multi-body dynamic systems.

The paper is organized as follows. Following the brief introduction and literature review, the flexible component modeling aspects is described. This is followed by the description of rigid-body modeling, wherein, the selected finite elements are collapsed to a rigid super-element. The constrained system dynamics with attention to both holonomic and non-holonomic systems are described next. The primal-dual technique for the robust simulation of MBD system is described followed the by parallel formulations of this technique. The results illustrate the capabilities and the scalability of the proposed approach followed by pertinent concluding remarks.

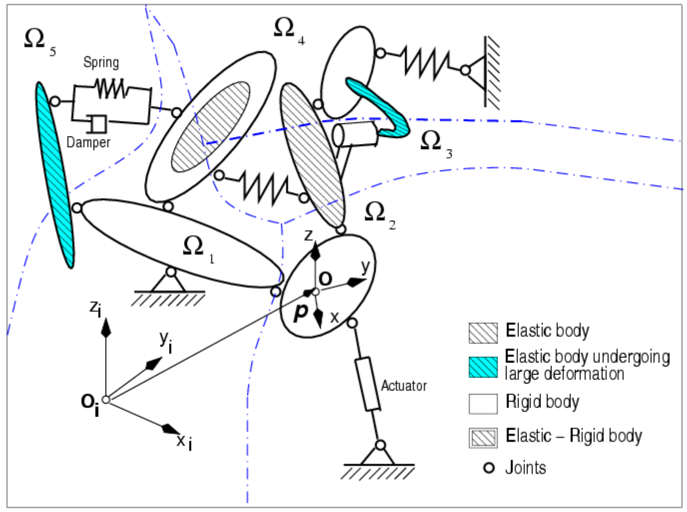

Figure 1: Illustration of complex flexible-rigid multi-body system and their partitioning for parallel processing.

FLEXIBLE COMPONENT MODELING

The differential equations governing the motion of a flexible body Ω are given by

$$\begin{aligned} \rho\ddot{\mathbf{u}} - \nabla.\boldsymbol{\sigma} - \mathbf{b} = 0 \text{ in } \Omega \\ \boldsymbol{\sigma}.\mathbf{n} = \mathbf{t} \text{ on } \Gamma_\sigma \\ \mathbf{u} = \mathbf{u}^* \text{ on } \Gamma_u \end{aligned} \tag{1}$$

where, the boundary Γ of the body is decomposed into two parts Γ_σ and Γ_u, $\mathbf{u}$ are the displacements, ρ is the mass density of the body Ω, and $\boldsymbol{\sigma}$ is the true stress, $\nabla.$ represents the divergence operator with respect to the current coordinates, $\mathbf{b}$ is the body force, $\mathbf{t}$ are the prescribed tractions and $\mathbf{u}^*$ are the prescribed displacements.

Considering a body as rigid can lead to significant computational savings; however it models the motion of a body accurately only in the case when the deformations are small. Thus, in view of the current objectives of simulating lightweight systems under high speed operational conditions, which may undergo large deformation, a model consisting solely of rigid bodies would not be accurate. One therefore resorts to modeling the flexible body by discretizing the equations of motion in space using numerical or modal techniques.

Considerable research has been done during the past few decades for the modeling the equations of motion of deformable bodies. Based on the choice of the reference frame chosen to represent the deformation of points on flexible bodies, these approaches can be classified as: 1) floating frame approach [21], in which an intermediate frame, which decouples the rigid and flexible motion, is defined for each flexible component and the deformations are referred to this frame. Although it is a natural way to extending the rigid body dynamics, this approach is limited to modeling flexible bodies undergoing small deformations, 2) co-rotational approach [22], in which an intermediate frame, which decouples the rigid and flexible motion, is defined for each finite element. Although this approach can be applied for large deformation problems it is more appealing in the case of small deformations for which linear elasticity can be used, and 3) inertial approach [23], in which no intermediate frame is used and all the displacements are referred to the inertial frame. In most current software the inertial approach is used due to its ability to accurately model large deformation and rotations. The equations of motion of a flexible body after carrying out the space discretization using finite elements, in conjunction with the inertial approach, can be written as:

$$
\begin{aligned}
&\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{p}(\mathbf{u}) = \mathbf{f} \\
&\mathbf{M} = \bigcup_{e=1}^{n_e} \mathbf{L}^{(e)^{\mathrm{T}}} \int_{\Omega_e} \mathbf{N}^{\mathrm{T}}\mathbf{N}\, d\Omega \\
&\mathbf{p} = \bigcup_{e=1}^{n_e} \mathbf{L}^{(e)^{\mathrm{T}}} \int_{\Omega_\varepsilon} \mathbf{B}^{\mathrm{T}}(\mathbf{u})\mathbf{S}\, d\Omega \\
&\mathbf{f} = \bigcup_{e=1}^{n_e} \mathbf{L}^{(e)^{\mathrm{T}}} \left[\int_{\Omega_\varepsilon} \mathbf{N}^{\mathrm{T}}\mathbf{b}\, d\Omega + \int_{\Gamma_\varepsilon \cap \Gamma_\sigma} \mathbf{N}^{\mathrm{T}}\mathbf{t}\, dA \right]
\end{aligned}
\tag{2}
$$

where, $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix**,** $\mathbf{p}$ is the vector of the non-linear internal forces, $\mathbf{f}$ is the vector of externally applied forces, $\mathbf{L}^{(e)}$ is the element connectivity matrix, $\mathbf{S}$ is the vector form of the second Piola-Kirchhoff stress, $\mathbf{N}$ is the shape function matrix, $\mathbf{B}$ is the compatibility matrix which relates strains to nodal displacement for linear elasticity and relates the virtual strains to the virtual nodal displacements for non-linear elasticity.

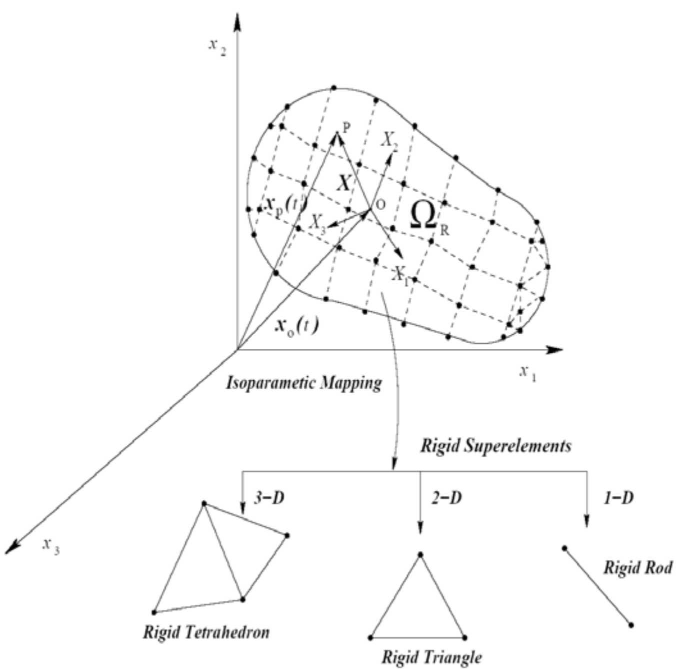

Figure 2: Rigid body Ω_R mapped into a rigid super element, thereby consistently reducing the finite element model and providing the framework for high/low fidelity modeling.

RIGID COMPONENT MODELING

For accurate modeling, ideally, one would like to consider all the components in a complex system as flexible. Although accurate, this approach is not ideally suited for initial and intermediate design stages due to the high computational cost associated with detailed modeling of complex systems. To reduce the computational cost, one of the popular approaches involves the modal reduction, wherein static deformation modes are used to represent the small deformations of a component. For a more general case of large deformation, the modal reduction cannot be used due to the internal forces being a non-linear function of displacements. Another approach to the cost reduction involves the modeling of components using the rigid body hypothesis, which can easily be used in the case when other flexible components undergo large deformation. For designing a new multi-body system in the shortest possible time, one can therefore use the capability of adaptive high/low fidelity modeling, whereby the design can first be evaluated by assuming most of its components to be rigid. Subsequently, after knowing the contribution of each component to the flexibility of the model the number of rigid components can be reduced in a consistent manner towards the final design stages. Thereby adaptively, achieving the objective of accurate modeling of new system together with a short development cycle. In order to provide the designer with the capability to collapse the selected finite elements to a rigid body that reduces the computational cost of the model significantly, we have adopted the approach presented in [24]. This approach, shown in Fig. 2, involves mapping the finite element mesh associated with the component, considered rigid, into a single rigid body super-element using the standard isoparametric mapping. This super-element could be a rod, triangle or a tetrahedron depending on the case of modeling edge, surface or volumetric rigidity. Thus, the equations of motion of a rigid body by collapsing its associated finite elements can be written as

$$\mathbf{M}_r\ddot{\mathbf{u}}_r + \mathbf{G}(\mathbf{u}_r)\boldsymbol{\mu}_r = \mathbf{F}_r \tag{3 a}$$

$$\boldsymbol{\varphi}_r = \mathbf{0} \tag{3 b}$$

where the inertia matrix $\mathbf{M}_r$ of the rigid bodies is given by

$$\mathbf{M}_r = \sum_{i=1}^{n_e} \mathbf{N}_i^{\mathbf{T}} \mathbf{M}_i^{(e)} \mathbf{N}_i \tag{4}$$

and, $\mathbf{u}_r$ are the displacements of the rigid super-element, $\boldsymbol{\varphi}_r$ are the constraint equations restricting the edge length of the rigid super-element to remain constant, $\mathbf{G}(\mathbf{u}_r)$ is the constraint jacobian, $\boldsymbol{\mu}_r$ are the Lagrange multipliers imposing the constraints, $\mathbf{M}_r$ is the equivalent mass matrix of the rigid super-element, $\mathbf{F}_r$ is the equivalent force vector, $\mathbf{N}_i$ is the shape function matrix of the i^{th} element, and $\mathbf{M}_i^{(e)}$ is the mass matrix of the i^{th} element.

CONSTRAINTS IN RIGID-FLEXIBLE SYSTEMS

In a multiple body system such as a vehicle, etc., the interactions between flexible and/or rigid bodies are defined by constraints. A joint or kinematic pair imposes constraints on the relative motion of the two bodies defining the pair. These constraints enable the motion of a multiple body system to be useful for a particular task by reducing the number of degrees of freedom of the system.

Holonomic Constraints

Holonomic constraints arise if the constraint equations are an implicit function of the nodal displacements and time, which have the following representations

$$\boldsymbol{\varphi}(\mathbf{u}, t) = \mathbf{0} \tag{5}$$

If the above equation does not involve the time t, then there is no work-done done by these constraints on the system. A simple example of a holonomic constraint is a revolute or hinge joint which is commonly employed in systems such as vehicles or robotic manipulators. A revolute joint is formed when two bodies are pinned together, thus restricting the relative motion between the two bodies to one rotation about a specific axis.

Non-Holonomic Constraints

Constraints, which are functions of velocities or inequality constraints that cannot be integrated back to the form of holonomic constraints, are called non-holonomic constraints. In vehicle dynamics the constraint for pure rolling of the wheel is a case of non-holonomic constraint, which have the following representations

$$\boldsymbol{\varphi}(\mathbf{u}, \dot{\mathbf{u}}, t) = \mathbf{0} \tag{6}$$

In this study we consider only those non-holonomic constraints that do not do any work on the system.

Equation of motion for Constrained Systems

The equation of motion of system, which satisfies the constraints imposed on it, can be written by imposing the constraints using Lagrange multipliers as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{p}(\mathbf{u}) + \mathbf{G}^{\mathbf{T}}(\mathbf{u}, t)\boldsymbol{\mu} = \mathbf{F} \tag{7 a}$$

$$\boldsymbol{\varphi}(\mathbf{u}, t) = 0 \tag{7 b}$$

The above equations of motion Eq. 7a-b represents a differential equation for $\mathbf{u}$, which depends on the Lagrange multipliers or algebraic variables $\boldsymbol{\mu}$, and the solution is forced to satisfy the algebraic constraints $\boldsymbol{\varphi}$, where $\mathbf{G}$ is the jacobian of the constraints. Thus the equations of motion of a constraint system are called differential algebraic equations (DAEs). It should be noted that unlike the Eqs. 7a-b, which represent the equations of motion of a rigid-flexible body system, Eqs. 3a-b are the equations of motion of a rigid body system and therefore do not contain the internal force vector $\mathbf{p}$.

ROBUST SIMULATION TECHNIQUES FOR CONSTRAINED SYSTEMS

The weak form of the semi-discretized equation of motion of a multiple body system subjected to constraints using any standard time integrator can be represented as

$$\mathbf{M}\ddot{\hat{\mathbf{u}}}_{n+1} + \mathbf{C}\dot{\hat{\mathbf{u}}}_{n+1} + \mathbf{p}(\hat{\mathbf{u}}_{n+1}) + \mathbf{G}^{\mathrm{T}}(\hat{\mathbf{u}}_{n+1})\hat{\boldsymbol{\mu}}_{n+1} = \hat{\mathbf{F}}_{n+1} \tag{8 a}$$

$$\boldsymbol{\varphi}(\mathbf{u}_{n+1}, t) = 0 \tag{8 b}$$

$$\boldsymbol{\varphi}(\mathbf{u}_{n+\frac{1}{2}}, \dot{\mathbf{u}}_{n+\frac{1}{2}}, t) = 0 \tag{8 c}$$

where, $\hat{\mathbf{u}}_{n+1}$, $\dot{\hat{\mathbf{u}}}_{n+1}$, $\ddot{\hat{\mathbf{u}}}_{n+1}$ and, $\hat{\boldsymbol{\mu}}_{n+1}$ are differential and algebraic state variables at time $\hat{t}_{n+1} \in [t_n, t_{n+1}]$. Since we assume there is no work done by constraints in the Eq. 5 and 6, the discrete form of Eq. 5 should also imply the same in order to integrate the equations of motion stably. Therefore, the holonomic constraint equations, Eq. 8a, are satisfied using displacements computed at the end of the time step, and the non-holonomic constraint equations, Eq. 8b, are satisfied using displacements and velocities computed at the mid-point which result in workless constraints. This was first proposed by [25] and later by [26] by means of mean value theorem to discretize the constraint equations. Staggered solution procedure to solve the Eqs. 8a-c is described next. Here we refer it to as primal-dual technique.

Linearizing the equation of motion, Eq. 8a, with respect to the position $\mathbf{u}$ and setting the resulting residual equal to zero, we get

$$\overline{\mathbf{M}}_{\mathbf{u}} \Delta \mathbf{u}_{n+1}^{j+1} = \overline{\mathbf{R}}_u - \hat{\mathbf{G}}_{n+1}^{j\,\mathbf{T}} \hat{\boldsymbol{\mu}}_{n+1}^{j} \tag{9}$$

where,

$$\overline{\mathbf{M}}_u = \left[\ddot{\mathbf{u}}_{,\mathbf{u}}\mathbf{M} + \dot{\mathbf{u}}_{,\mathbf{u}}\mathbf{C} + \mathbf{u}_{,\mathbf{u}}\mathbf{K}_t\right]$$

$$\overline{\mathbf{R}}_u = \hat{\mathbf{F}}_{n+1} - \mathbf{M}\ddot{\hat{\mathbf{u}}}_{n+1} - \mathbf{C}\dot{\hat{\mathbf{u}}}_{n+1} - \mathbf{p}(\hat{\mathbf{u}}_{n+1}) - \mathbf{G}^{\mathrm{T}}(\hat{\mathbf{u}}_{n+1})\hat{\boldsymbol{\mu}}_{n+1}$$

where, $\mathbf{K}_t = \partial\mathbf{P}/\partial\mathbf{u}$ is the tangent stiffness matrix. Linearizing the algebraic equation, Eq. 8b, with respect to the position $\mathbf{u}$, yields

$$\boldsymbol{\varphi}_{n+1}^{j+1} \approx \boldsymbol{\varphi}_{n+1}^{j} + \mathbf{G}_{n+1}^{j} \Delta \mathbf{u}_{n+1}^{j+1} \tag{10}$$

In order to satisfy the constraint equation, the value of $\Delta\mathbf{u}$ is substituted from Eq. 9 into the linearized constraint equation, Eq. 10. Setting the resulting residual equal to zero, yields

$$\mathbf{G}_{n+1}^{j} \overline{\mathbf{M}}_u^{-1} \hat{\mathbf{G}}_{n+1}^{j\,\mathbf{T}} \hat{\boldsymbol{\mu}}_{n+1}^{j} = \boldsymbol{\varphi}_{n+1}^{j} + \mathbf{G}_{n+1}^{j} \overline{\mathbf{M}}_u^{-1} \overline{\mathbf{R}}_u \tag{11}$$

From the above equation the dual variable, $\boldsymbol{\mu}$ can be solved first and subsequently, the primary variables such as $\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}$ can be recovered from Eq. 9 by an iterative procedure. This results in the preservation of constraints and all the underlying properties of the original ODE time integrators, and the accurate solution for the Lagrange multipliers. Thereby, it provides an accurate, efficient and robust formulation for index 3 DAE systems encountered in flexible multiple body systems. The theoretical proofs for this new and novel primal-dual technique for index-3 systems can be found in [20]. This is one of the key aspect of scalable flexible multi-body dynamics technology, which is described next.

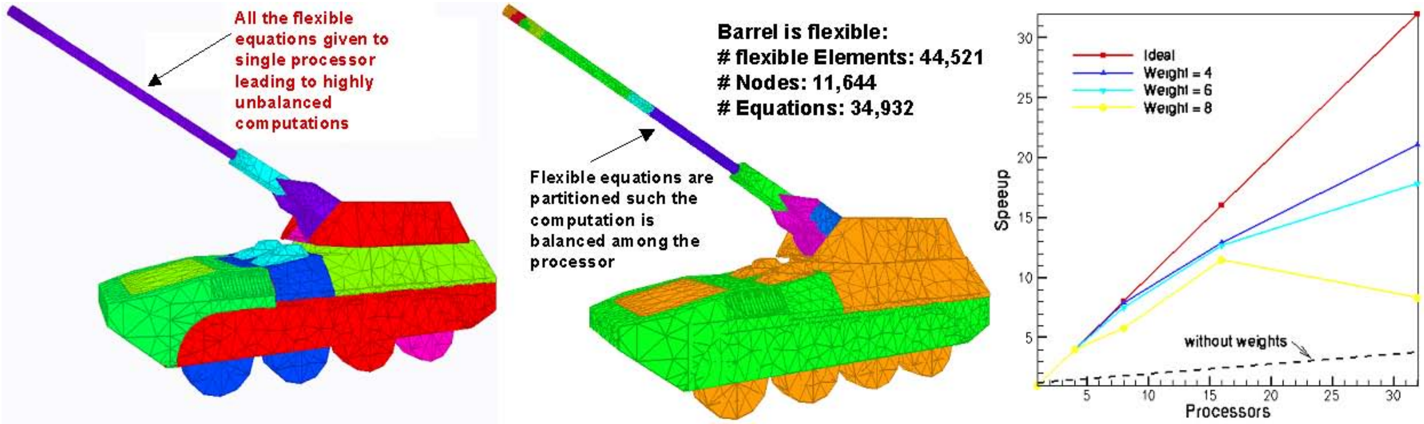

Figure 3: (a) Uniformly weighted mesh partitioning, (b) rigid-flexible weighted mesh partitioning, and (c) parallel performance comparison for different graph node weighting and without weighting. Color coding in (a) and (b) represents a partition of the finite element mesh.

SCALABLE HPC DEVELOPMENTS

The idea of a single multiple body dynamics simulation code which can handle different levels of modeling resolution, from predominantly rigid to predominantly flexible, is critical for many real time and/or large scale systems in which the accuracy of results together with short development cycle are of utmost importance. To the best of our knowledge there is no single scalable HPC code available which can meet the above objectives.

To develop a scalable HPC simulation environment, which is platform independent and can be ported to different HPC architectures, we use message-passing interface (MPI) to develop the proposed program. The parallelization of the simulation code involves parallel mesh and joint partitioning, an implicit scalable solver technology that is robust, and parallel visualization

Mesh Partitioning

The first stage of parallization involves partitioning of the finite element mesh into sub-domains as shown in Fig. 1. Each sub-domain is then attributed to a processor with the objective of balancing the computational loads and minimizing the communications between all the processors such that one can, ideally, achieve perfect scalability (linear reduction/speed-up in execution time when the problem size is kept constant).

In the present approach, the rigid body super-element is formed by collapsing many finite elements, thereby reducing the contribution of these elements to the total computational load. If the finite element mesh is now, naively partitioned by assigning an equal weight to each finite element, the computational loads would be highly unbalanced between processors. We thus apply a mesh partitioning on the finite element mesh where the group of finite elements that are finally mapped to a rigid super-element are assigned less weight than that for the flexible elements, which can deform.

Consider a model of a multi-body system with n_b^r rigid bodies and n_n^r rigid nodes, i.e. nodes that are finally mapped to a rigid super-element. Herein we employ a simple weighting scheme, which attributes the weight w_i^e to an element i in the following manner.

$$w_i = \begin{cases} 1 & \text{, node } i \text{ is rigid} \\ \dfrac{n_n^r}{n_b^r \cdot \omega} + 1 & \text{, node } i \text{ is flexible} \end{cases} \qquad (12)$$

$$w_i^b = \sum_{j=1}^{n_{npe}} L_{ij}^{(e)} w_j^n$$

where, w_i^n is the weight attributed to the node i, ω is the weighting factor which controls the importance given to the flexible nodes in the mesh, n_{npe} are the number of nodes per element and $L_{ij}^{(e)}$ is the element connectivity matrix. It is evident from the above Eq. 12 that the flexible elements, formed by flexible nodes that contribute to the degrees of freedom of the multi-body system, are assigned a higher weight than the rigid elements formed by rigid nodes.

To illustrate the effect of the above weighting scheme on the mesh partition and parallel performance consider a terrain vehicle shown in Fig. 3. While conducting dynamic analysis of the vehicle, the gun barrel and barrel support are considered to be flexible and rest of the vehicle is assumed to be rigid to accurately predict the deformations and stresses in these critical components for various operating loading conditions. Figure 3a and 3b shows mesh partition obtained by using uniform weights and the present scheme of attaching high weights to flexible elements. Figure 3c, shows the parallel performance on the Cray T3E while using different weighing schemes where the factor ω is represented by legend "weight". It is apparent from Fig. 3c that assigning uniform weights to elements results in highly unbalanced computational loads between processors, thereby deteriorating the parallel performance severely. The parallel speedup results are much better for the cases when flexible elements are highly weighted. However, it should be noted that if the system comprises of only rigid-bodies then, a different partitioning approach, which assigns each rigid body to a processor should be used provided the rigid multi-body system has enough partitioned constraint equations to gain any speed-up. The issues related to parallelism for rigid-flexible multi-body system are described next.

Parallel Solution of the Equations of Motion

Joints between rigid bodies are distributed among processors such that each joint is attributed to a unique processor. The constraints and the Jacobian of constraints are thus partitioned as

$$\mathbf{B} = \begin{bmatrix} \cdot & \cdot & \mathbf{P}_1 & \cdot & \cdot \\ \cdot & \cdot & \mathbf{P}_2 & \cdot & \cdot \\ \cdot & \cdot & \dots & \cdot & \cdot \\ \cdot & \cdot & \mathbf{P}_{n-1} & \cdot & \cdot \\ \cdot & \cdot & \mathbf{P}_n & \cdot & \cdot \end{bmatrix} \quad ; \quad \varphi = \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \dots \\ \mathbf{P}_{n-1} \\ \mathbf{P}_n \end{bmatrix} \tag{13}$$

The above partitioning scheme for joints naturally leads to row partitioning of the constraint residual vector and constraint Jacobian matrix. This easily lends to the parallelization of the dual solve given by Eq. 11 without modifying the present parallel finite element technology which is highly scalable.

The primal and dual solve which involve the solution of $\mathbf{u}_{n+1}, \dot{\mathbf{u}}_{n+1}, \ddot{\mathbf{u}}_{n+1}$ and, $\boldsymbol{\mu}_{n+1}$ for serial computing is given by Eq. 9 and Eq. 11, respectively, for any popular implicit time integration scheme. The parallel implementation for the solution of these variables involves the use of highly scalable FETI-DP solver. We first solve for the vector $\hat{\mathbf{S}}_{n+1}$ given by Eq. 14, which is the incremental solution vector for the non-linear dynamic problem without constraints

$$\overline{\mathbf{M}}_{\mathbf{u}} \hat{\mathbf{S}}_{n+1} = \overline{\mathbf{R}}_u \tag{14}$$

To compute the solution vector which satisfies the constraints, the matrix $\hat{\mathbf{H}}_{n+1}$ is first solved for by using Eq. 15

$$\hat{\mathbf{H}}_{n+1} = \overline{\mathbf{M}}_u^{-1} \hat{\mathbf{G}}_{n+1}^{j\,\mathrm{T}} \tag{15}$$

The solution of the dual variable, $\boldsymbol{\mu}$, now involves only a local solve given as

$$\mathbf{G}_{n+1}^{j} \hat{\mathbf{H}}_{n+1} \hat{\boldsymbol{\mu}}_{n+1}^{j} = \boldsymbol{\varphi}_{n+1}^{j} + \mathbf{G}_{n+1}^{j} \hat{\mathbf{S}}_{n+1} \tag{16}$$

and, the primary solution vector can be recovered using a local update given as

$$\Delta \mathbf{u}_{n+1}^{j+1} = \hat{\mathbf{S}}_{n+1} - \hat{\mathbf{H}}_{n+1} \hat{\boldsymbol{\mu}}_{n+1}^{j} \tag{17}$$

RESULTS

First we briefly discuss the validation of the present formulation with experimental results of a slider-crank mechanism studied in [27]. The geometrical and material properties of this slider-crank mechanism are summarized in Table 1 where the subscript 1, 2, 3 correspond to the crank, connecting rod and slider respectively (see Fig 6). Eight-noded quadrilateral elements are used to spatially discretize the system. The crank and connecting rod are modeled with three elements and ten elements respectively, while the slider is modeled with only one element. The connecting rod is modeled as a flexible member as opposed to the crank and slider being modeled as rigid by using two separate triangular rigid super-elements.

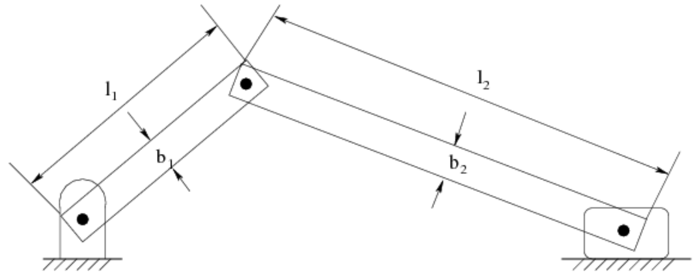

Figure 4: The slider-crank mechanism problem [27] for validation of the present formulation.

Table 1: Geometric and material properties of the slider–crank mechanism where *l* is the length, b is the width, t is the thickness, ν is the poison ratio, ε is the Young's modulus, ρ is the density and M is the mass of the links in the mechanism.

$l_2 = 305\ mm$	$b_2 = 2.06\ mm$	$t_2 = 25.4\ mm$
$\mathcal{E}_2 = 69000\ MPa$	$\rho_2 = 2770\ Kg/m^3$	$\nu_2 = 0.3$
$M_2 = 0.044206\ Kg$	$M_3 = 0.22103\ Kg$	$l_1 = 75.625\ mm$

The bending stress at the mid-point of the connecting rod obtained through numerical simulations is compared with the experimental results. Figure 5 shows the comparison of the experimental and numerical results at a constant crank speed of 500 rpm for one full rotation of the crank. It is evident that the numerical results obtained compares well with the experimental results

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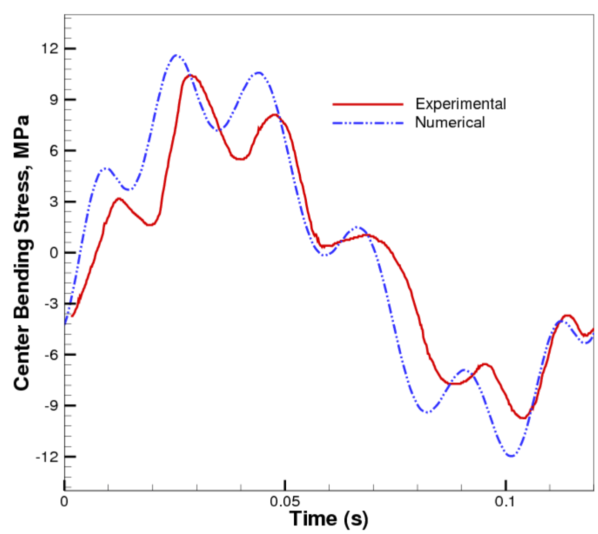

Figure 5: Bending stresses at midpoint of the connecting rod at crank speed of 500 rpm.

Next results are presented to illustrate the parallel scalability of the proposed approach. We have considered a terrain vehicle firing analysis simulation that is critical for the prediction of the dynamic behavior (both gross motion together with deformation) of the vehicle. Here we consider applying the firing load on the recoil housing. The load time history is shown in the Fig 5.

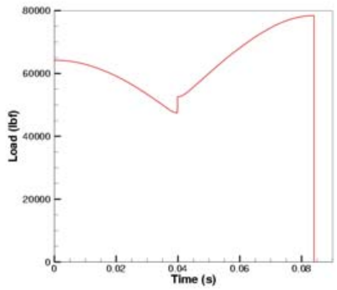

Figure 6: The load time history for the firing analysis.

The results have been obtained employing the optimal dissipative time integration method that has zero-order in displacement and first-order in velocity overshoot characteristics [7]. In addition, to demonstrate the large deformation capability of the code, a barrel of reduced stiffness was considered. In this simulation, the barrel together with barrel support are considered flexible, while the rest of the model is assumed to be rigid. Fig. 6 shows the simulation frame of the firing analysis. It is evident from Fig. 6a – 6c that the barrel undergoes large deformation due to its low stiffness and high loads associated with firing. The present software also helps the designer by predicting the stresses in these flexible components. Fig. 6d – 6f, which shows the acceleration contours, can be used to predict the gravity ("G") force in the crew capsule and other sensitive areas.

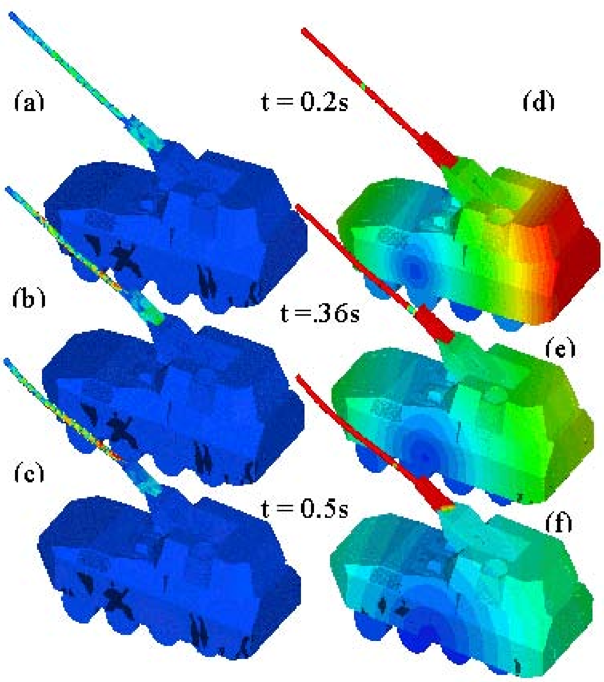

Figure 7: (a) – (c) Von-Mises stress contour plot, (d) – (f) acceleration contour plots during firing analysis of a terrain vehicle with a flexible gun and barrel support, at different time instants.

To study the parallel performance of the HPC simulation code we employed a large finite element model containing one million flexible elements in the barrel and recoil-housing component. The parallel speed-ups are shown in the Fig. 8. It is evident from Fig. 8 that the code is highly scalable and achieves close to linear speedups (up to 128 number of processors were employed for this large-scale problem).

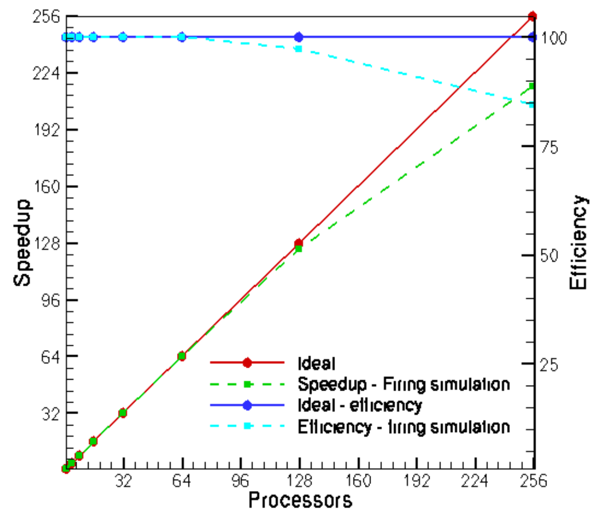

Figure 8: Scalable parallel performance of the proposed approach for flexible-rigid multi-body dynamics implicit computations. The firing analysis simulation of a vehicle is carried out on the Cray T3E HPC consisting of a maximum of 1024 processors. The number of elements is equal to 1,456,686, the number of nodes is equal to 354,434, the number of flexible elements is equal 1,087,100, the nodes is equal to 258,298, and the equations is equal to 774,894.

CONCLUSIONS

The present exposition focuses on the design and development of a single scalable HPC simulation environment to accurately simulate new system designs at varying levels of fidelity. The varying level of fidelity modeling which helps achieve the objective of accurate modeling of new systems together with a short development cycle is implemented by using a non-linear finite element approach to model the flexible components undergoing large deformation, and a rigid body hypothesis for other components. Incorporating a new primal-dual technique for the solution of index-3 DAEs, and a highly scalable FETI-DP solver accomplish robust scalable simulation of the constrained equations of motion.

ACKNOWLEDGMENTS

The authors are very pleased to acknowledge support by the Army High Performance Computing Research Center (AHPCRC) under the auspices of the Department of the Army, Army Research Laboratory (ARL) under contract number DAAD19-01-2-0014. The content does not necessarily reflect

the position or the policy of the government or the policy of the government, and no official endorsement should be inferred.

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