MULTI-OBJECTIVE OPTIMIZATION IN MATERIAL DESIGN AND SELECTION☆

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Abstract—The development or selection of a material to meet given design requirements generally requires that a compromise be struck between several, usually conflicting, objectives. The ways in which multi-objective optimization methods can be adapted to address this problem are explored. It is found that *trade-off surfaces* give a way of visualizing the alternative compromises, and that *value functions* (or *"utility" functions*) identify the part of the surface on which optimal solutions lie. The method is illustrated with examples. © 2000 Acta Metallurgica Inc. Published by Elsevier Science Ltd. All rights reserved.

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1. INTRODUCTION

Real-life decision-making frequently requires that a compromise be reached between conflicting objectives. The compromises required to strike a balance between wealth and quality of life, between the performance and the cost of a car, or between health and the pleasure of eating rich foods, are familiar ones. Similar conflicts arise in the choice of materials. The objective in choosing a material is to optimize a number of *metrics of performance* in the product in which it is used. Common among these metrics are *cost*, *mass*, *volume*, *power-to-weight ratio*, and *energy density*, but there are many more. Conflict arises because the choice that optimizes one metric will not, in general, do the same for the others; then the best choice is a compromise, optimizing none but pushing all as close to their optima as their interdependence allows. This paper is concerned with multi-objective optimization of material choice. It draws on established methods for multi-objective optimization [1–5] and for material selection [6] illustrating how the first can be applied to the second. The methods are equally applicable to material selection and to material design.

2. OPTIMIZED MATERIALS SELECTION

2.1. Performance metrics, control variables and objective functions

Any engineering component has one or more *functions*: to support a load, to contain a pressure, to transmit heat, and so forth. In designing the component, the designer has an *objective*: to make it as cheap as possible, perhaps, or as light, or as safe, or some combination of these. This must be achieved subject to *constraints*: that certain dimensions are fixed, that the component must carry the given load or pressure without failure, that it can function in a certain range of temperature, and in a given environment, and many more. Function, objectives and constraints (Table 1) define the boundary conditions for selecting a material and—in the case of load-bearing components—a shape for its cross-section [6].

The performance of the component, measured by *performance metrics*, P_j, depends on *control variables*, x_i. The control variables include the dimensions of the component, the mechanical, thermal and electrical loads it must carry, and the properties of the material from which it is made. Performance is described in terms of the control variables by one or more *objective functions*. An objective function is an equation describing a performance metric, P, expressed such that performance is inversely related to its value, requiring that minimum be sought for P. Thus

Function	"What does the component do?"
Objective	"What is to be maximized or minimized?"
Constraints[a]	"What non-negotiable conditions must be met?"
	"What negotiable but desirable conditions ... ?"

[a] It is sometimes useful to distinguish between "hard" and "soft" constraints. Stiffness and strength might be absolute requirements (hard constraints); cost might be negotiable (a soft constraint).

$$P_j = f_j[(\text{Loads}, F), (\text{Geometry}, G), (\text{Material}, M)] \tag{1}$$

or

$$P_j = f_j(F, G, M)$$

where "f" means "a function of". *Optimum design* is the selection of the material and geometry which minimizes a given performance metric, *P*. *Multi-objective optimization* is a procedure for simultaneously optimizing several interdependent performance metrics $P_1, P_2, \ldots, P_j$.

2.2. Single objective optimization and material choice

The mode of loading that most commonly dominates in engineering is not tension, but bending. Consider, as an example, the performance metric for a stiff panel of length ℓ, width b and thickness t (Fig. 1), with the objective of minimizing its mass. The objective is to minimize the mass m of the panel, described by

$$m = \ell b t \rho \tag{2}$$

where ρ is the density of the material of which it is made. The length ℓ, width b and force F per unit width are specified; the thickness t is free. We can reduce the mass by reducing t, but only so far, because the panel must meet the constraint on its stiffness S, meaning that it must not deflect more than δ under a load Fb. To achieve this we require that

$$S = \frac{Fb}{\delta} = \frac{C_1 EI}{\ell^3} \geq S^* \tag{3}$$

where S^* is the desired stiffness, E is Young's modulus, C_1 is a constant which depends on the distribution of load and I is the second moment of the area of the section, which, for a panel of section
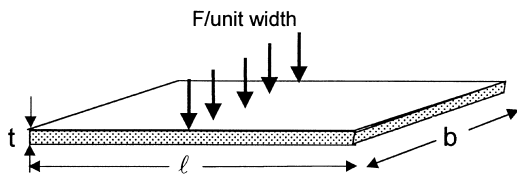

Fig. 1. A panel of length ℓ, width b and thickness t, loaded in bending by a force F per unit width.

$b \times t$ is

$$I = \frac{bt^3}{12}. \tag{4}$$

Using equations (3) and (4) to eliminate t in equation (2) gives the objective function for the performance metric m:

$$m \geq \left(\frac{12 S^* b^2}{C_1}\right)^{1/3} \ell^2 \left(\frac{\rho}{E^{1/3}}\right). \tag{5}$$

All the quantities in equation (5) are specified by the design except for the group of material properties in the last parentheses, $\rho/E^{1/3}$. The values of the performance metric for competing materials therefore scale with this term, which is called a *materials index*. Taking material M_0 as the reference (the incumbent in an established design, or a convenient standard in a new one), the performance metric of a competing material M_1 differs from that of M_0 by the factor

$$\frac{m_1}{m_0} = \frac{(\rho_1/E_1^{1/3})}{(\rho_0/E_0^{1/3})} \tag{6}$$

where the subscript "0" refers to M_0 and the "1" to M_1. If the constraint were that of strength rather than stiffness, the constraining equation becomes that for failure load, F_f per unit width, meaning the onset of yielding:

$$F_f = 2 C_2 \frac{I \sigma_y}{bt\ell} \geq F_f^*$$

where C_2, like C_1, is a constant that depends only on the distribution of the load. The objective function becomes

$$m \geq \left(\frac{6 F_f^* b^2}{C_2}\right)^{1/2} \ell^{3/2} \left(\frac{\rho}{\sigma_y^{1/2}}\right) \tag{7}$$

where σ_y is the yield strength of the material of which the panel is made and $F_f^* b$ is the desired minimum failure load. Here the materials index is $\rho/\sigma_y^{1/2}$. Taking material M_0 as the reference again, the performance metric of a competing material M_1 differs from that of M_0 by the factor

$$\frac{m_1}{m_0} = \frac{(\rho_1/\sigma_{y,1}^{1/2})}{(\rho_0/\sigma_{y,0}^{1/2})}. \tag{8}$$

More generally, if the performance metrics for a reference material M_0 are known, those for competing materials are found by scaling those of M_0 by the ratio of their material indices. There are many such indices. A few of those that appear most commonly are listed in Table 2.

Selection of a material to optimize a single objective is simply a case of identifying the index charac-

Table 2. Material indices[a] (the "materials" part of a performance equation)

Function, objective and constraint (and example)	Index[b]
Tie, minimum weight, stiffness prescribed (*cable support of a light-weight tensile structure*)	ρ/E
Beam, minimum weight, stiffness prescribed (*aircraft wing spar, golf club shaft*)	$\rho/E^{1/2}$
Beam, minimum weight, strength prescribed (*suspension arm of automobile*)	$\rho/\sigma_y^{2/3}$
Panel, minimum weight, stiffness prescribed (*automobile door panel*)	$\rho/E^{1/3}$
Panel, minimum weight, strength prescribed (*table top*)	$\rho/\sigma_y^{1/2}$
Column, minimum weight, buckling load prescribed (*push-rod of aircraft hydraulic system*)	$\rho/E^{1/2}$
Spring, minimum weight for given energy storage (*return springs in space applications*)	$E\rho/\sigma_y^2$
Precision device, minimum distortion, temperature gradients prescribed (*gyroscopes; hard-disk drives; precision measurement systems*)	α/λ
Heat sinks, maximum thermal flux, thermal expansion prescribed (*heat sinks for electronic systems*)	α/λ
Electromagnet, maximum field, temperature rise and strength prescribed (*ultra high field magnets; very high speed electric motors*)	$1/\kappa C_p\rho$

[a] The derivation of these and many other indices can be found in Ref. [6].
[b] ρ = density; E = Young's modulus; σ_y = elastic limit; λ = thermal conductivity; α = thermal expansion coefficient; κ = electrical conductivity; C_p = specific heat.

terizing the performance metric, and choosing materials with the smallest value of this index.

2.3. Multi-objective optimization and trade-off surfaces

When there are two or more objectives, solutions rarely exist that optimize all at once. The objectives are normally non-commensurate, meaning that they are measured in different units, and in conflict, meaning that any improvement in one is at the loss of another. The situation is illustrated for two objectives by Fig. 2 in which one performance metric, P_2, is plotted against another, P_1. Each bubble describes a solution. The solutions that minimize P_1 do not minimize P_2, and vice versa. Some solutions, such as that at A, are far from optimal: other solutions exist which have lower values of both P_1 and P_2. Solutions like A are said to be *dominated* by others. Solutions like that at B have the characteristic that no other solution exists with lower values of both P_1 and P_2. These are said to be *non-dominated* solutions. The line or surface on which they lie is called the *non-dominated* or *optimum* trade-off surface [2]. The values of P_1 and P_2 corresponding to the non-dominated set of solutions are called the Pareto set [2, 7].

The trade-off surface identifies the subset of solutions that offer the best compromise between the objectives, but it does not distinguish between them. Three strategies are available to deal with this.

1. The trade-off surface like that of Fig. 2 is established and studied, using intuition to select between non-dominated solutions.
2. All but one of the objectives are re-formulated as constraints by setting lower and upper limits for them, thereby allowing the solution which mini-
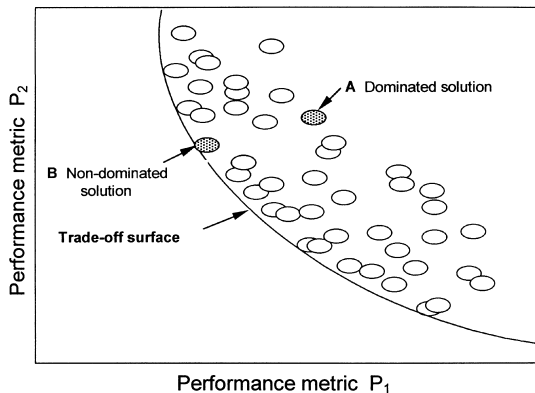

Fig. 2. Dominated and non-dominated solutions, and the optimum trade-off surface.
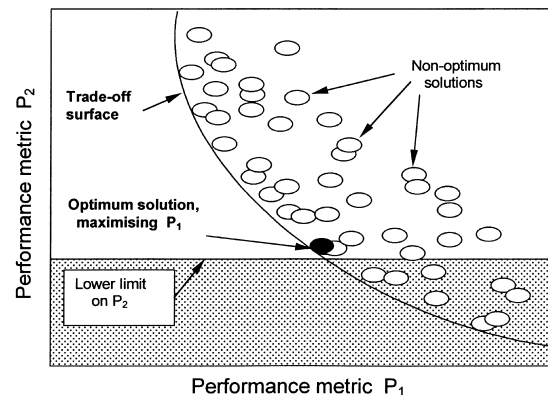

Fig. 3. Imposing limits on all but one of the performance metrics allows the optimization of the remaining one, but this defeats the purpose of multi-objective optimization.
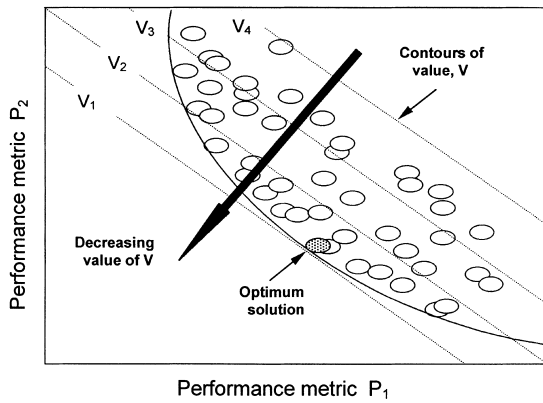
Fig. 4. A value function V plotted on the trade-off diagram. The solution with the lowest V is indicated. It lies at the point at which the value function is tangent to the optimum trade-off surface.
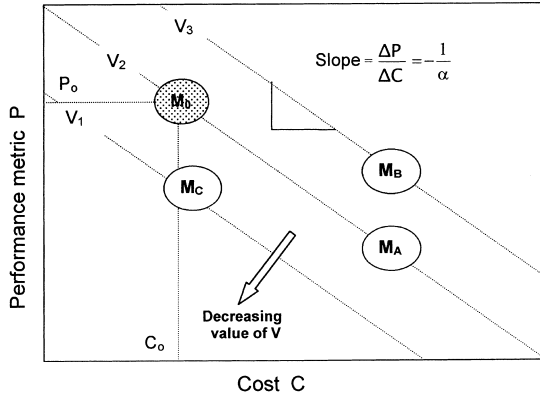

Fig. 5. The trade-off between performance and cost. Neither material M_A nor M_B are viable substitutes for M_0. Material M_C is a viable substitute because it has a lower value of ΔV.

mizes the remaining objective read off, as illustrated in Fig. 3.
3. A composite objective function or *value function*, V, is formulated; the solution with the minimum value of V is the overall optimum, as in Fig. 4. This is explored next.

2.4. Value functions

Define the value function†

$$V = \alpha_1 P_1 + \alpha_2 P_2 + \ldots + \alpha_i P_i \ldots \qquad (9)$$

where the values of α are *exchange constants*: they relate the performance metrics P_1, P_2, …to value, V, which is measured in units of currency ($, £, DM, FF, etc.). The exchange constants are defined by

$$\alpha_1 = \left(\frac{\partial V}{\partial P_1} \right)_{P_2, \ldots, P_i, \ldots} \qquad (10a)$$

$$\alpha_2 = \left(\frac{\partial V}{\partial P_2} \right)_{P_1, \ldots, P_i, \ldots} \qquad (10b)$$

that is, they measure the change in value for a unit change in a given performance metric, all others held constant. If the performance metric P_1 is mass m (to be minimized), α_1 is the change in value V associated with unit increase in m. If the performance metric P_2 is heat transfer Q per unit area, α_2 is the change in value V associated with unit increase in Q. The best solution is the one with the smallest value of V, which, with properly chosen values of

α_1 and α_2, now correctly balances the conflicting objectives.

With given values of V and exchange constants α_i, equation (9) defines a relationship between the performance metrics, P_i. In two dimensions, this plots as a family of parallel lines, as shown in Fig. 4. The slope of the lines is fixed by the ratio of the exchange constants, α_1/α_2. The best solution is that at the point at which a value-line is tangent to the trade-off surface because it is the one with the smallest value of V.

2.5. Minimizing cost as an objective

Frequently one of the objectives is that of minimizing cost‡, C, so that $P_1 = C$. Since we have chosen to measure value in units of currency, unit change in C gives unit change in V, with the result that

$$\alpha_1 = \left(\frac{\partial V}{\partial C} \right)_{P_2, \ldots, P_i, \ldots} = 1 \qquad (10c)$$

and equation (9) becomes

$$V = C + \alpha_2 P_2 + \ldots + \alpha_i P_i \ldots . \qquad (11)$$

As a simple example, consider the substitution of a new material, M, for an incumbent, M_0, based on cost C and one other performance metric, P. Substitution is potentially possible if the value V of M is less than that, V_0, of the incumbent M_0. Thus substitution becomes a possibility when

$$V - V_0 = (C - C_0) + \alpha(P - P_0) \leq 0 \qquad (12)$$

or

$$\Delta V = \Delta C + \alpha \Delta P \leq 0$$

from which

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† For the use of value functions for material selection, see Refs [6, 8, 9].

‡ For background in cost modelling see Refs [10–15].

Table 3. Exchange constants α for transport systems

Sector: transport systems	Basis of estimate	Exchange constant £/kg ($/lb)
Family car, structural	fuel saving	0.5–1.5 (0.4–1.1)
Truck, structural components	payload	5–10 (4–8)
Civil aircraft, structural	payload	100–500 (75–300)
Military vehicle, structural	payload, performance	500–1000 (350–750)
Space vehicle, structural	payload	3000–10,000 (2000–75,000)
Bicycle, structural components	perceived value (price–mass plots)	80–1000 (50–700)

$$\frac{\Delta P}{\Delta C} \leq -\frac{1}{\alpha} \qquad (13)$$

defining a set of potential applications for which M is a better choice than M_0.

To visualize this, think of a plot of the performance metric, P_1, against cost, C, as shown in Fig. 5. The incumbent M_0 is centred at P_0, C_0; the potential substitute at P, C. The line through M_0 has the slope defined by equation (13), using the equality sign. Any material which lies on this line, such as M_A, has the same value V as M_0; for it, ΔV is zero. Materials above, such as M_B, despite having a lower (and thus better) value of P than M_0, have a higher value of V. Materials below the line, such as M_C, have a lower value of V, a necessary condition for substitution. Remember, however, that while negative ΔV is a necessary condition for substitution, it may not be a sufficient one; sufficiency requires that the difference in value ΔV be large enough to justify the investment in new technology.

The population of materials on Fig. 5 is small. When the population is large, the optimum choice is found by seeking the point at which a line with the slope given by equation (13) is tangent to the optimum trade-off surface, as in Fig. 4.

2.6. Values for the exchange constants, α_i

An exchange constant is a measure of the value, real or perceived, of a performance metric. Its magnitude and sign depend on the application. Thus the value of weight saving in a family car is small, though significant; in aerospace it is much larger. The value of heat transfer in house insulation is directly related to the cost of the energy used to heat the house; that in a heat exchanger for power electronics can be much higher. The value of performance can be real, meaning that it measures a true saving of cost, energy, materials, time or information. But value can, sometimes, be perceived, meaning that the consumer, influenced by scarcity, advertising or fashion, will pay more or less than the true value of these metrics.

In many engineering applications the exchange constants can be derived approximately from technical models. Thus the value of weight saving in transport systems is derived from the value of the
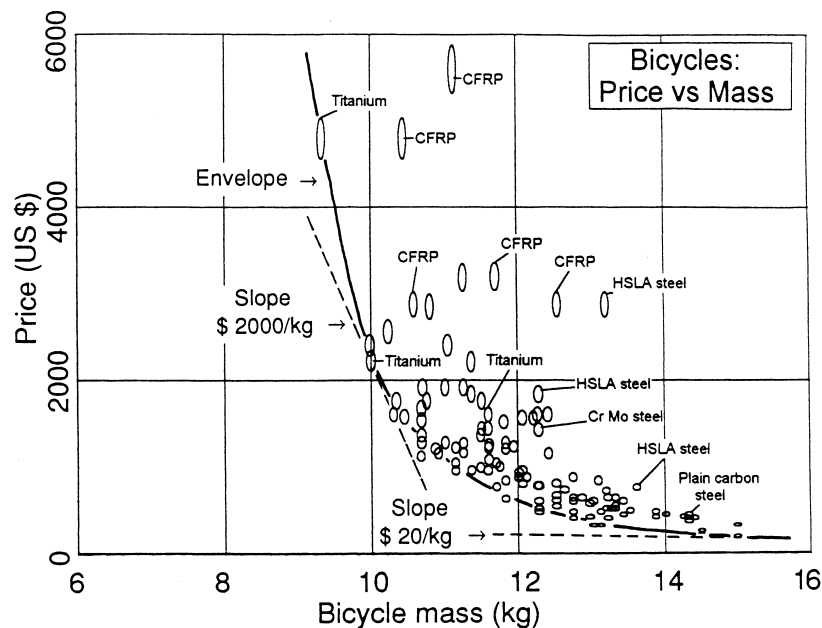

Fig. 6. A plot of price against mass for bicycles. The lower envelope of the data defines a non-dominated or optimal trade-off line. The exchange constant is approximated by the slope of the line. It is low for cheap, heavy bikes, but becomes very large for light, expensive ones.
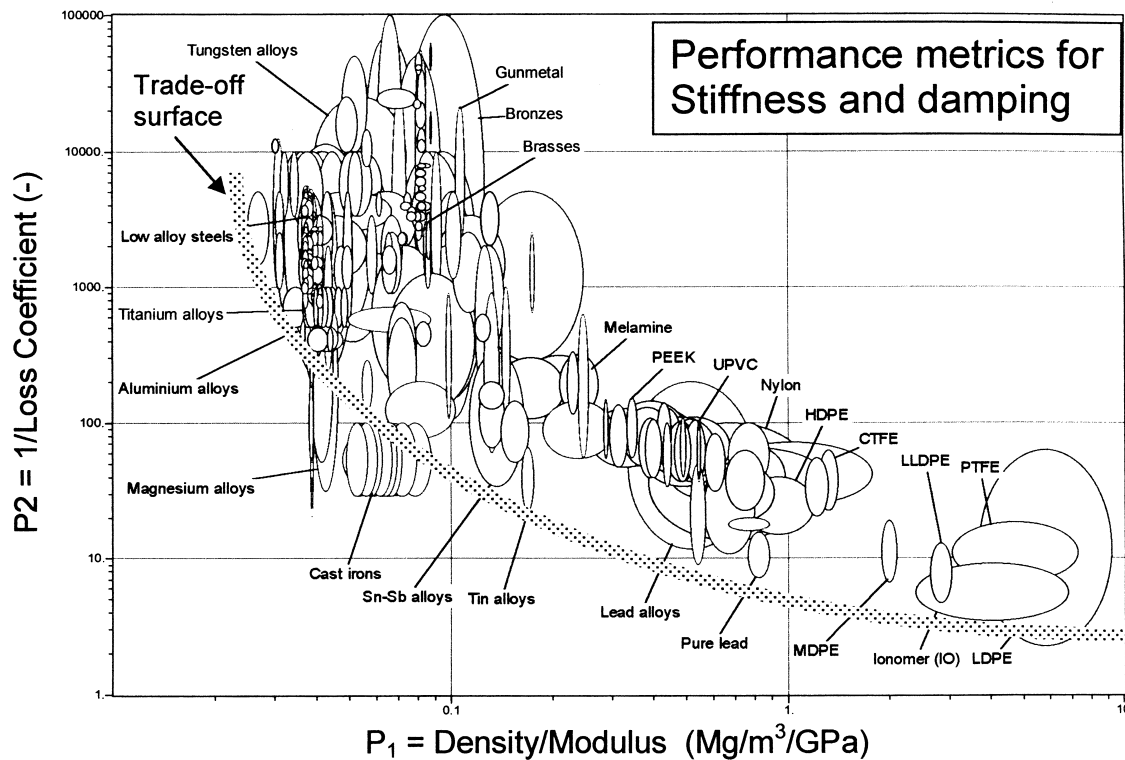
Fig. 7. A trade-off plot for performance metrics $P_1 = \rho/E$ and $P_2 = 1/\eta$. The shaded band shows the optimum trade-off surface.

fuel saved or that of the increased payload which this allows (Table 3). The value of heat transfer can be derived from the value of the energy transmitted or saved by unit change in the heat flux per unit area. Approximate exchange constants can sometimes be derived from historical pricing data; thus the value of weight saving in bicycles can be found by plotting the price† P of bicycles against their mass m, using the slope $(-dP/dm)$ as an approximate measure of α (Fig. 6). Finally, exchange constants can be found by interviewing techniques [16, 17] that elicit the value to the consumer of a change in one performance metric, all others held constant.

The values of α in Table 3 describe simple trade-offs between cost and performance. Circumstances can change these, sometimes dramatically. The auto-maker whose vehicles fail to meet legislated requirements for fleet fuel consumption will assign a higher value to weight saving than that shown in Table 3; so, too, will the aero-engine maker who has contracted to sell an engine with a given power-to-weight ratio if the engine is overweight. These special cases are not uncommon, and can provide the first market opportunity for a new material.

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† For any successful product the cost C, the price P and the value V are related by $C < P < V$, since if $C > P$ the product is unprofitable, and if $P > V$ no one will buy it. Thus P can be viewed as a lower limit for V.

3. APPLICATIONS

3.1. Simple trade-off between properties

Consider selection of a material for a design in which it is desired, for reasons of vibration control, to maximize the specific modulus E/ρ (E is Young's modulus and ρ is the density) and the damping, measured by the loss coefficient η. We identify two performance metrics, P_1 and P_2, defined such that minima are sought for both:

$$P_1 = \frac{\rho}{E} \qquad (14a)$$

and

$$P_2 = \frac{1}{\eta}. \qquad (14b)$$

Figure 7 shows the trade-off plot. Each bubble on the figure represents a material; the dimensions of the bubble show the ranges spanned by these property groups. Materials with high values of P_1 have low values of P_2, and vice versa, so a compromise must be sought. The optimum trade-off surface, marked, identifies a subset of materials with good values of both performance metrics. If high E/ρ (low P_1) is of predominant importance, then aluminium alloys are a good choice; if greater damping (lower P_2) is required, magnesium alloys or cast irons are a better choice; and if high damping is the
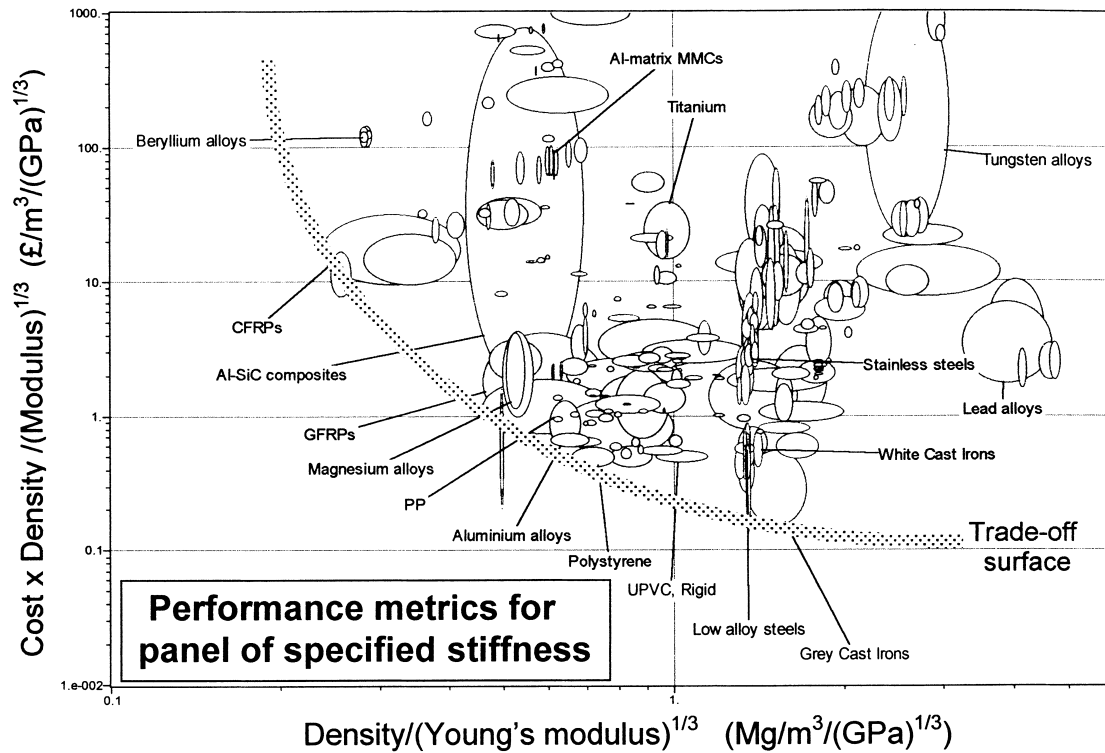
Fig. 8. The performance metrics (measured by the material indices) for cost and mass of a panel of specified stiffness, plotted against each other. The trade-off front (shaded) separates the populated section of the figure from that which is unpopulated.

over-riding concern, tin or lead alloys become attractive candidates. It is sometimes possible to use judgement to identify the best position on the trade-off surface (strategy 1, above). Alternatively (strategy 2) a limit can be set for one metric, allowing an optimum for the other to be read off. Setting a limit of $\eta > 0.1$, meaning $P_2 \leq 10$, immediately identifies commercial lead alloys as the best choice in Fig. 7. Finally (strategy 3) it is possible to define the value function

$$V = \alpha_1 P_1 + \alpha_2 P_2 = \alpha_1 \frac{\rho}{E} + \alpha_2 \frac{1}{\eta} \qquad (15)$$

and to seek materials which minimize V. Contours of constant V, like those of Fig. 4, have slope

$$\left(\frac{\partial P_2}{\partial P_1}\right)_V = -\frac{\alpha_1}{\alpha_2}. \qquad (16)$$

The point at which one contour is tangent to the trade-off surface identifies the best choice of material. Implementation of this strategy requires values for the ratio α_1/α_2 which measures the relative importance of stiffness and damping in suppressing vibration. Technical modelling can permit this to be evaluated: one example is given in Ref. [18].

3.2. Co-minimizing mass and cost

One of the commonest trade-offs is that between mass and cost. Consider, as an example, co-minimizing the mass and cost of the panel of specified stiffness analysed in Section 2.2. The mass of the panel is given by equation (5) that we rearrange to define the performance metric P_1

$$P_1 = \frac{m}{\beta} = \left(\frac{\rho}{E^{1/3}}\right) \qquad (17)$$

with β, a constant for a given design, given by

$$\beta = \left(\frac{12S^*b^2}{C_1}\right)^{1/3} \ell^2. \qquad (18)$$

The cost C of the beam is simply the material cost per kg, C_m, times the mass m, giving the second performance metric P_2:

$$P_2 = \frac{C}{\beta} = \left(\frac{C_m \rho}{E^{1/3}}\right). \qquad (19)$$

Figure 8 shows the trade-off plot. The horizontal axis P_1 is simply the material index $\rho/E^{1/3}$. The vertical axis, similarly, is the index $C_m\rho/E^{1/3}$. Conventional alloys (cast irons, steels, aluminium alloys) lie in the lower part of the diagram. Beryllium alloys, CFRPs and Al-based MMCs lie
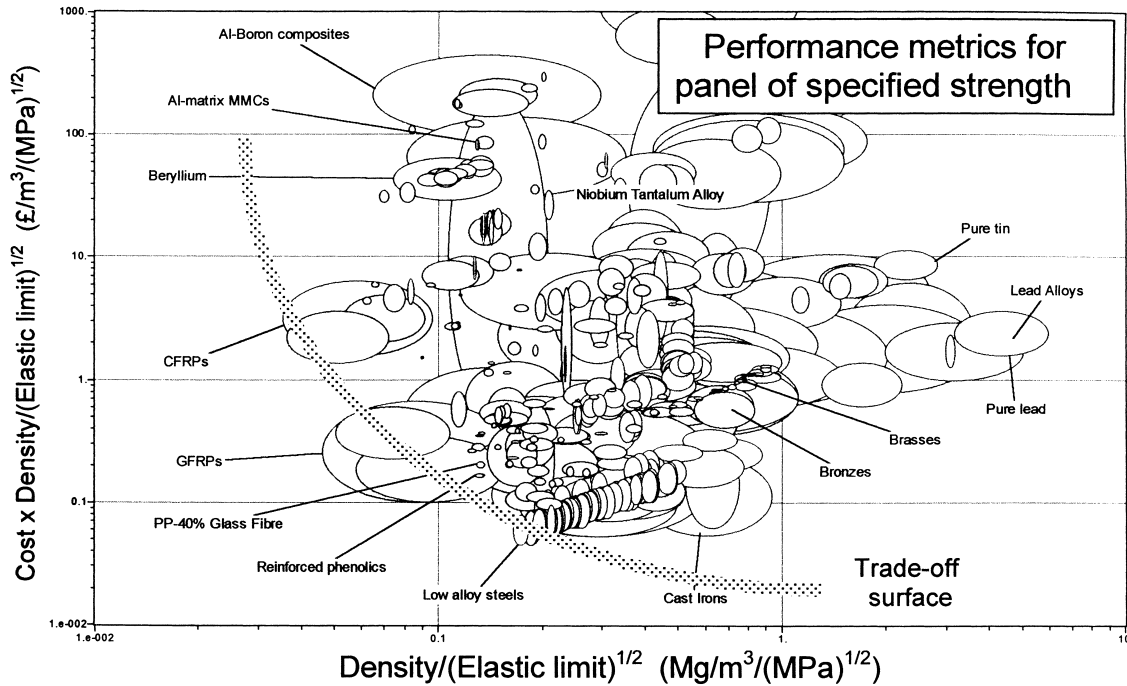
Fig. 9. The performance metrics (measured by the material indices) for cost and mass of a panel of specified strength, plotted against each other. The trade-off front separates the populated section of the figure from that which is unpopulated.

in the central and upper parts. Figure 9 shows the corresponding plot when the constraint on stiffness is replaced by that on strength. Proceeding as before, but using equation (7) instead of equation (5), we define the performance metrics:

$$P_1 = \left(\frac{\rho}{\sigma_y^{1/2}}\right) \quad (20a)$$

and

$$P_2 = \left(\frac{C_m \rho}{\sigma_y^{1/2}}\right). \quad (20b)$$

As before, both are simple material indices. Cast irons and steels lie on the optimum trade-off surface at low values of P_2; GFRPs and CFRPs also lie on the surface, but at low values of P_1.

In Figs 8 and 9, the materials that perform well by both criteria lie on or near the optimal trade-off front, indicated by the shaded band. The front characterizes the best achievable compromise for a

panel of specified stiffness (Fig. 8) or strength (Fig. 9) with minimum weight and cost. But at which part of the front should the choice be made?

To answer this question for the panel of specified stiffness we define the value function

$$V = \alpha_1 P_1 + \alpha_2 P_2 = \alpha_1 \left(\frac{\rho}{E^{1/3}}\right) + \left(C_m \frac{\rho}{E^{1/3}}\right) \quad (21)$$

(since α_2, relating value to cost, is unity). Values of α_2 relating value to mass, are listed in Table 3. The equation is evaluated in Table 4 for two extreme values of α_1. When α_1 has the low value of £0.5/kg, nodular cast irons are the best choice. But if α_1 is as high as £500/kg, SR-200 grade beryllium is a better choice than any of the other materials.

For the panel of specified strength of Fig. 9 the value function becomes

$$V = \alpha_1 \left(\frac{\rho}{\sigma_y^{1/2}}\right) + \left(C_m \frac{\rho}{\sigma_y^{1/2}}\right), \quad (22)$$

Table 4. The selection of panel materials: stiffness constraint

Material	ρ (Mg/m^3)	E (GPa)	C_m (£/kg)	P_1	P_2	V ($\alpha_1 =$ £0.5/kg)	V ($\alpha_1 =$ £500/kg)
Cast iron, nodular	7.30	175	0.25	1.31	0.33	**0.99**	655.0
Low-alloy steel (4340)	7.85	210	0.45	1.32	0.59	1.25	660.6
Al 6061–T6	2.85	70	0.95	0.69	0.66	1.01	345.6
Al-6061–20% SiC, PM	2.77	102	25	0.59	14.8	15.1	309.8
Ti-6-4 B265 grade 5	4.43	115	20	0.91	18.2	18.7	473.2
Beryllium grade SR-200	1.84	305	250	0.27	67.5	67.6	**202.5**

Table 5. The selection of panel materials: strength constraint

Material	ρ (Mg/m^3)	σ_y (MPa)	C_m (£/kg)	P_2	P_1	V ($\alpha_1 = $£0.5/kg)	V ($\alpha_1 = $£500/kg)
Cast iron, nodular	7.30	240	0.25	0.47	0.12	0.36	235.1
Low-alloy steel (4340)	7.85	1400	0.45	0.21	0.09	**0.20**	105.1
Al 6061–T6	2.85	270	0.95	0.17	0.16	0.25	85.2
Al-6061–20% SiC, PM	2.77	410	25	0.14	3.50	3.57	73.5
Ti-6-4 B265 grade 5	4.43	1020	20	0.14	2.80	2.87	**72.8**
Beryllium grade SR-200	1.84	350	250	0.10	25.0	25.1	75.0

evaluated in Table 5. Here low-alloy steels are the best choice when α_1 is low; B265 grade 5 titanium alloys, when α_1 is high. The same information can be displayed graphically by plotting contours of V on Figs 8 and 9; the point at which a contour is tangent to the optimum trade-off surface identifies the best choice. But when the value function combines more than two metrics of performance as in equation (9), graphical methods cease to be useful and ranking by V, as in Tables 4 and 5, becomes the best approach.

3.3. Cost effective materials to minimize thermal distortion in precision devices

Precision devices, by which we mean precision machine tools, hard-disk drives, guidance gyroscopes and the like, present special problems of materials selection. The accuracy of such devices is frequently limited by the dimensional changes caused by temperature gradients. Compensation for thermal expansion is of course possible provided the device is at a uniform temperature. Thermal gradients are the real problem: they cause change of shape—that is *distortion*—for which compensation is not possible [18, 19]. What then are good materials for cost-effective precision devices?

Figure 10 shows, schematically, the simplest of such devices: a hand-held micrometer. It consists of a *force loop* (the frame), an *actuator* (the threaded drive) and a *sensor* (the verneer)—all precision instruments have these features. We aim to choose a material for the force loop, balancing cost against performance. For a force loop of fixed dimensions the volume is constant and the material cost is
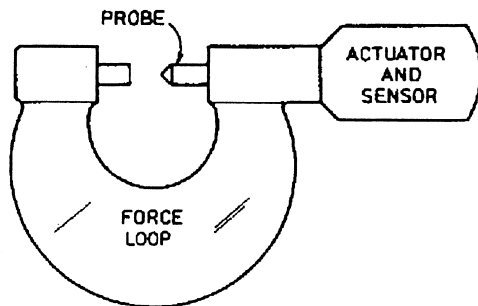
equal to

$$P_1 = C_m \rho v \tag{23}$$

where C_m is the cost per kg of the material in the shape of the force loop, ρ is its density and v is the volume of the material in the force loop. The force loop will, in general, support heat sources: the fingers of an operator of the device in Fig. 10, or more usually, electrical components which generate heat. Susceptibility to thermal distortion is assessed by considering the simple case of one-dimensional heat flow through a panel with one side at temperature T and the other, connected to the heat source, at $T + \Delta T$. In the steady state, Fourier's law relates the heat flux q to the temperature gradient dT/dx:

$$q = -\lambda \frac{dT}{dx} \tag{24}$$

where λ is the thermal conductivity. The strain is related to temperature by

$$\varepsilon = \alpha_T \Delta T \tag{25}$$

where α_T is the thermal expansion coefficient and T_0 is ambient temperature. The distortion is proportional to the *gradient* of the strain, and we use this as the performance metric:

$$P_2 = \frac{d\varepsilon}{dx} = \alpha_T \frac{dT}{dx} = \left(\frac{\alpha_T}{\lambda}\right)q. \tag{26}$$

Thus for a given geometry and heat flow q, the distortion $d\varepsilon/dx$ is minimized by selecting materials with the smallest values of the property group α_T/λ.

Assume, reasonably, that the material substitution for the frame involves no change of shape, thus minimizing other design changes. Then the cost is that of the material and its processing to shape. Define the value function

$$V = \alpha_1 P_1 + \alpha_2 P_2 = C_m \rho v + \alpha_2 q \left(\frac{\alpha_T}{\lambda}\right)$$

setting $\alpha_1 = 1$ as before. Dividing by v gives

$$\frac{V}{v} = (C_m \rho) + \alpha_2 q \left(\frac{\alpha_T}{\lambda}\right). \tag{27}$$

Figure 11 shows the trade-off between the two parenthetical groups of material properties. Rearranging this equation gives a linear relationship
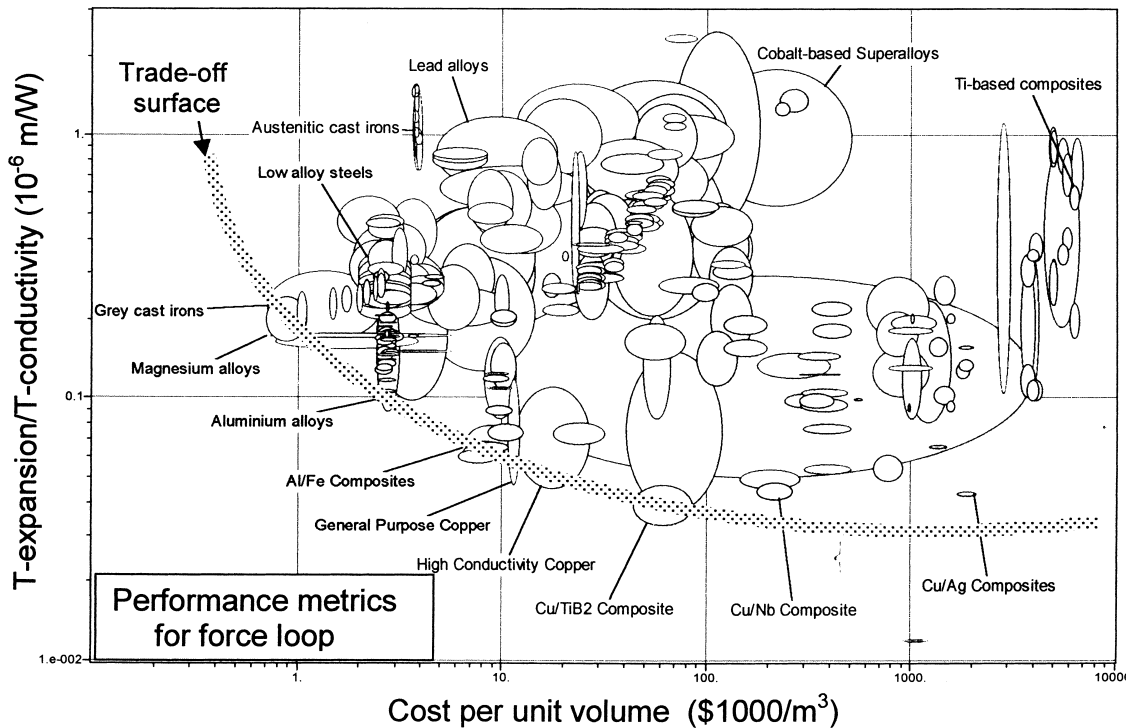

Fig. 10. A hand-held micrometer—the simplest example of a precision measuring device.

Fig. 11. A trade-off plot for the material groups determining the performance metrics P_2 and cost C showing the trade-off front. Low values of the exchange constant α_2 give high tangent slopes, and vice versa.

between the property groups (α_T/λ) and $(C_m\rho)$

$$\left(\frac{\alpha_T}{\lambda}\right) = -\frac{v}{\alpha_2 q}(C_m\rho) + \frac{1}{\alpha_2 q}V. \qquad (28)$$

This equation describes a family of lines of slope $-v/\alpha_2 q$ on Fig. 11, each line corresponding to a value of V (as in Fig. 4). Thus large devices such as machine tools (v large) in which some distortion can be tolerated (α_2 small) lead to lines with a steep, negative slope; the tangent to the trade-off surface occurs at cast irons, magnesium or aluminium alloys. Small, distortion-sensitive, devices are characterized by a small negative slope; then the best choice is copper or one of the copper-based composites identified on the figure.

4. CONCLUSIONS

The property profiles of engineering materials are very diverse. Optimum selection requires that the best match be found between the available profiles and the requirements of the design. Methods exist for achieving this when the design has a single objective. But it is rare that a design has a single objective; almost always there are several, and optimized selection requires that a balance be struck between them.

This paper reviews methods of dealing with opti-mal selection of discrete entities to meet multiple objectives, and adapts these methods to the specific case of material selection. Methods of developing performance metrics characterizing each objective are illustrated. Often, the performance metrics can be reduced to a simple combination of material properties like $\rho/E^{1/3}$ or α_T/λ. Trade-off plots allow the identification of an optimal trade-off surface on which the best choices lie. Value functions combining the performance metrics in a properly balanced way contain exchange constants that relate the performance metrics. If values for the exchange constants are known, materials can be ranked by value, identifying those that offer the best compromise. Estimates for the exchange constants can sometimes be made by modelling, and when this is difficult it may still be possible to devise limits between which they must lie, allowing the selection to proceed. This method is illustrated by a number of examples.

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