Multiple Model MIMO Predictive Control for Variable Speed Variable Pitch Wind Turbines

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Abstract— **A multivariable control strategy based on model predictive control techniques for the control of variable speed variable pitch wind turbines is proposed. The proposed control strategy is described for the whole operating region of the wind turbine (both partial and full load regimes). Pitch angle and generator torque are controlled simultaneously to maximize energy capture, mitigate drive train transient loads and smooth the power generated while reducing the pitch actuator activity. This has the effect of improving the efficiency and the power quality of the electrical power generated, and of increasing the life time of the system's mechanical parts. Furthermore, safe and acceptable operation of the system is guaranteed by incorporating most of the constraints on the physical variables of the WECS in the controller design. In order to cope with nonlinearities in the WECS and continuous variations in the operating point, a multiple model predictive controller is suggested which provides near optimal performance throughout the whole operating region.**

I. INTRODUCTION

CONTROL systems play a very important role in modern Wind Energy Conversion Systems (WECSs). A well designed WECS control system enables more efficient energy generation, better power quality and the alleviation of aerodynamic and mechanical loads resulting in increased life of the installation. Consequently, such a control system will have a direct impact on the cost of energy produced [1]-[3].

In general, a variable speed variable pitch WECS has two operating regions with different control objectives, as shown in Fig. 1 [1]. The partial load regime includes all wind speeds between the cut in wind speed, v_{ci}, and the rated wind speed, v_r (wind speed at which the system rated power is achieved). In this region, the control system is required to adjust the turbine rotor speed, ω_t, such that maximum energy is extracted. When the wind speed is above v_r and below the cut out wind speed, v_{co}, the wind turbine is operating in the full load regime. In this region the control system is required to regulate both the output power and the

generator (turbine) speed to their rated values.

Design of WECS control systems is not a straight forward task. One of the main reasons for this is that the controlled system is a multiple-input multiple-output (MIMO) system with strongly coupled variables. Furthermore, the system non-linearity, the stochastic variations of the input power and the presence of physical constraints on the system variables render the control design task more difficult.
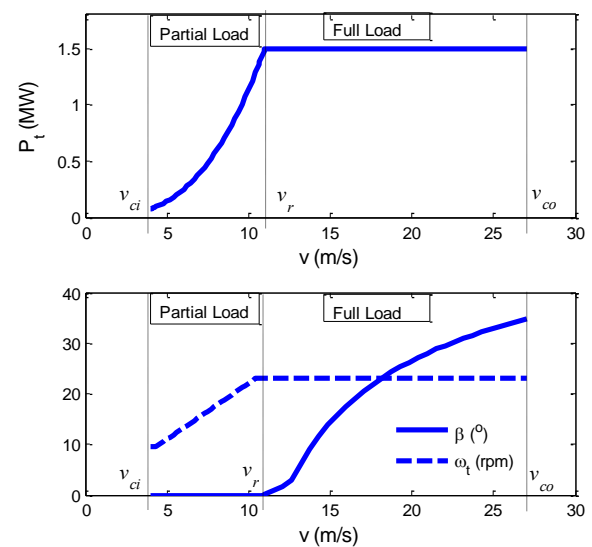

Fig. 1. Ideal Power curve for a WECS.

The literature of WECS control system design is vast. Most of the papers focus on designing the control system in either the partial load regime or the full load regime.

Many control techniques were proposed to control the WECS in the partial load regime [1], [4]-[8].The design of the classical Proportional Integral (PI) controller is described in [4] and [8]. To cope with the system non-linearity, it is proposed in [5] and [6] to use a gain-scheduling Linear Quadratic Gaussian (LQG) controller. In [1] and [7], a gain scheduled H_∞ controller is suggested.

Recently, many papers focusing in the control of variable speed variable pitch WECSs operating in the full load regime have appeared [8]-[10]. Most of the work reported ignores the multivariable nature of the problem [8]-[9]. Classical PI controllers are used in [8]. A PI controller in the power loop and an adaptive self-tuning regulator in the speed loop are proposed in [9]. A multivariable gain scheduled H_∞ controller is proposed in [10].

The presence of two control regions with different control goals and control structures requires the ability to switch between these controllers. This topic has been considered in

several recent studies indicating that when wind speed fluctuates around its rated value, undesirable drive train transient loads and electric power overshoots can occur [1].

This paper is motivated by the lack of papers considering the design of an overall control strategy that can work for both partial and full load regimes. Furthermore, as discussed in [1], [10]-[11], recognizing the multivariable nature of the problem and designing a MIMO controller will lead to much superior performance as compared to the decentralized approach commonly used in the literature. To the knowledge of the authors, the only work that describes the design of a multivariable controller that can work for both partial and full load regimes is found in [1],[12], where a multivariable gain scheduled H_∞ controller is proposed.

In this paper, a new control strategy based on Model Predictive Control (MPC) techniques is proposed for controlling variable speed variable pitch WECSs in both partial and full load regimes. The main advantage of the proposed strategy is that it is a multivariable control method that effectively uses the full capability of the controlled system to obtain the desired performance in the whole operating region of the WECS, while keeping the system variables within safe operating limits.

II. MODELING OF WECS

A model of the entire WECS can be structured as several interconnected subsystems as shown in Fig. 2 [1]. Details of individual blocks are given in Appendix A [1]-[3] and [9].
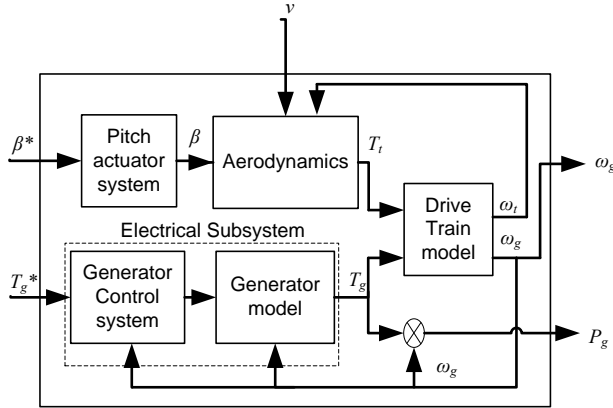

Fig. 2. WECS model.

A. Linearized WECS model

The overall WECS model described in (A.1)-(A.10) is nonlinear. Linearizing the turbine torque equation in (A.4) yields (1)-(2).

$$\delta T_t = L_\omega(\bar\omega_t, \bar v, \bar\beta)\,\delta\omega_t + L_v(\bar\omega_t, \bar v, \bar\beta)\,\delta v + L_\beta(\bar\omega_t, \bar v, \bar\beta)\,\delta\beta \tag{1}$$

$$\left.\begin{aligned}
L_\omega(\bar\omega_t, \bar v, \bar\beta) &= \frac{\partial T_t}{\partial \omega_t}\Big|_{(\bar\omega_t, \bar v, \bar\beta)}\\
L_v(\bar\omega_t, \bar v, \bar\beta) &= \frac{\partial T_t}{\partial v}\Big|_{(\bar\omega_t, \bar v, \bar\beta)}\\
L_\beta(\bar\omega_t, \bar v, \bar\beta) &= \frac{\partial T_t}{\partial \beta}\Big|_{(\bar\omega_t, \bar v, \bar\beta)}
\end{aligned}\right\} \tag{2}$$

The symbol δ is used to represent the deviation of the variable from its operating point value while the over bar in $\bar{\cdot}$ denotes the value of the variable at the operating point. The WECS operating point is completely defined by $\bar v$ [1].

Now the linearized state space representation of the WECS, (A.1)-(A.10), can be written as (3)-(5) where $\boldsymbol{x} \overset{\text{def}}{=} [\delta\omega_t \quad \delta\omega_g \quad \delta T_{tw} \quad \delta T_g \quad \delta\beta]^T \in \mathbb{R}^5$ is the state vector, $\boldsymbol{u} \overset{\text{def}}{=} [\delta T_g^* \quad \delta\beta^*]^T \in \mathbb{R}^2$ is the control input and $\boldsymbol{y} \overset{\text{def}}{=} [\delta\omega_g \quad \delta P_g]^T \in \mathbb{R}^2$ is the measured output. The symbols T_g, T_{tw}, $\omega_t, \omega_g, \beta$ and P_g denote the generator torque, the drive train torsional torque, the turbine speed, the generator speed, the pitch angle and the generator power, respectively.

$$\dot{x}(t) = \widetilde{\boldsymbol{A}}x(t) + \widetilde{\boldsymbol{B}}_u\boldsymbol{u}(t) + \widetilde{\boldsymbol{B}}_v\delta v(t)$$
$$y(t) = \widetilde{\boldsymbol{C}}x(t) \tag{3}$$

$$\widetilde{\boldsymbol{A}} = \begin{bmatrix} \frac{L_\omega}{J_t} & 0 & -\frac{i}{J_t} & 0 & \frac{L_\beta}{J_t} \\ 0 & 0 & \frac{1}{J_g} & -\frac{1}{J_g} & 0 \\ k_s i + \frac{iB_s}{J_t}L_\omega & -k_s & -\left(\frac{i^2 B_s}{J_t} + \frac{B_s}{J_g}\right) & 0 & \frac{iB_s}{J_t}L_\beta \\ 0 & 0 & 0 & -\frac{1}{\tau_g} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \tag{4}$$

$$\widetilde{\boldsymbol{B}}_u = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\tau_g} & 0 \\ 0 & \frac{1}{\tau} \end{bmatrix}, \widetilde{\boldsymbol{B}}_v = \begin{bmatrix} \frac{L_v}{J_t} \\ 0 \\ \frac{iB_s}{J_t}L_v \\ 0 \\ 0 \end{bmatrix}, \widetilde{\boldsymbol{C}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & \bar{T}_g & 0 & \bar\omega_g & 0 \end{bmatrix} \tag{5}$$

It can be seen that the system is MIMO and that the system dynamics vary when the average wind speed varies.

III. CONTROL PROBLEM DESCRIPTION

There are two operating zones with different control objectives - the partial and the full load regimes.

In the partial load regime, the primary objective is to control the turbine rotor speed to maximize the wind turbine's aerodynamic efficiency. This is achieved by manipulating the generator torque set point and fixing the pitch angle at its optimal value (usually very close to zero). The main control challenge is to design a controller that can maximize the energy conversion efficiency while minimizing transient loads on the drive train [5].

In the full load regime, the main control objective is to regulate both the generator power, P_g, and the generator speed, ω_g, at their rated values $P_{g,rat}$ and $\omega_{g,rat}$, respectively, by manipulating the generator torque set point T_g^* and the pitch angle set point β^*.

One of the main challenges for designing WECS control systems in the full load regime is the presence of severe fluctuations in the input turbine power, P_t, caused by erratic variations in the wind speed. Fluctuations in P_t can lead to large variations in the drive train torsional torque, T_{tw}, and in the electric power supplied to the grid. These, in turn, can cause reduction in the life time of the WECS components and voltage flicker problems [8]-[9].

A common challenge in both partial and full load regimes is the presence of nonlinearity in the system dynamics and the continuous variation of the operating point.

It is clear that there are many design aspects that must be considered in the design of an effective control system for WECSs. The most important ones are summarized below.

- Maximizing the energy capture.
- Smoothing the electrical power supplied to the system in the full load regime.
- Minimizing transient loads on the drive train and the control activity of the pitch actuator system.
- Ensuring good performance of the closed loop system over the whole operating range of the WECS.
- Keeping system variables within acceptable limits.

The last aspect is explained as follows. For WECSs, there are some physical actuator limits such as the ones the amplitude and speed of the pitch servos. Ignoring such constraints during the controller design can lead to performance degradation [13]. Furthermore, due to safety and operational issues, there are maximum limits on the generated power and the turbine speed that must be maintained during operation. A violation of these limits during system operation can cause WECS disconnection.

IV. CLASSICAL WECS CONTROL STRATEGY

This section describes a control strategy that is commonly used in commercial WECSs [12]. The objective is to explain the shortcomings of this classical strategy and the motivation for the proposed control strategy described in Section V.

The classical controller is shown in Fig. 3. The main part of the controller is a set of two PI controllers for tracking of the generator speed. In partial load operation, the pitch angle set point β^* is fixed at zero and the generator torque set point T_g^* is manipulated by a PI controller so that the generator speed ω_g tracks the desired generator speed set point ω_g^*. In the full load regime, the generator torque set point is fixed at its rated value $T_{g,rat}$, while the pitch angle set point is used as control signal to regulate the generator speed at its rated value. Using this approach, the power is indirectly regulated at its rated value. Generally, these PI controllers are gain scheduled, in order to take into account the variations in the aerodynamics. Furthermore, a bumpless switching between the partial and the full load control configurations is implemented.

The main shortcomings of this classical strategy are explained as follows. In the partial load regime, the main criticism for this approach is that the controller design does not allow a fine tuning of the trade-off between the energy performance and the reliability demands in terms of transient loads in the drive train [2]. Furthermore, when the system is operating near the rated wind speed, the partial load control structure focuses on controlling the generator speed only irrespective of the generator power. Due to wind speed fluctuations, significant power and drive train torsional torque overshoots can occur [1]. Finally, in the full load

regime, using the pitch actuator alone to regulate the generator speed can cause large pitch activity and severe power fluctuations. This in turns reduces the life time of the equipments and deteriorates the power quality produced.
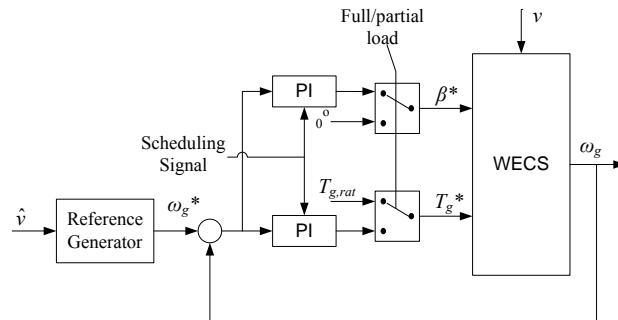

Fig. 3. Classical control strategy using two PI controllers.

V. PROPOSED CONTROL STRATEGY

The proposed control strategy based on Multiple Model Predictive Control (MMPC) is described below.

A. Review of Model Predictive control

Model-based predictive control (MPC) has been successfully used in many industrial applications in recent decades for many reasons such as [13]- [16]:

- MPC is very suitable for MIMO control problems.
- MPC algorithms can directly take into account constraints on the system variables.
- MPC is based on optimal control techniques. In fact, under some conditions, the MPC controller coincides with the famous LQG controller.

The main drawback of using MPC controllers is the requirement to solve a quadratic programming (QP) problem on line. This has restricted the use of MPC to applications with slow dynamics. However, due to the advances in computational power and in optimization algorithms, MPC can be used now for systems with fast dynamics [15].

B. Multiple Model Predictive Control (MMPC) for variable speed variable pitch WECSs

The use of linear model predictive control with a non-linear plant, such as the one considered in Section II, in which the operating point is continuously changing can lead to degradation in the closed loop performance [13]. There has been extensive research effort to extend the applicability of MPC to non-linear systems [17]. One of the most straight forward approaches is to use MMPC [18]. In the MMPC approach, the whole operating region is divided into *M* sub-regions with *M* linearized models that adequately represent the local system dynamics within each sub-region. A linear MPC controller based on each model is designed. Finally, a criterion by which the control system switches one controller to another as operating conditions change is defined.

The proposed WECS control strategy MMPC is shown Fig. 4. The main components of the MMPC controller are the prediction model bank, the optimization problem and the state estimator [13], [16]. Details of these selections for the

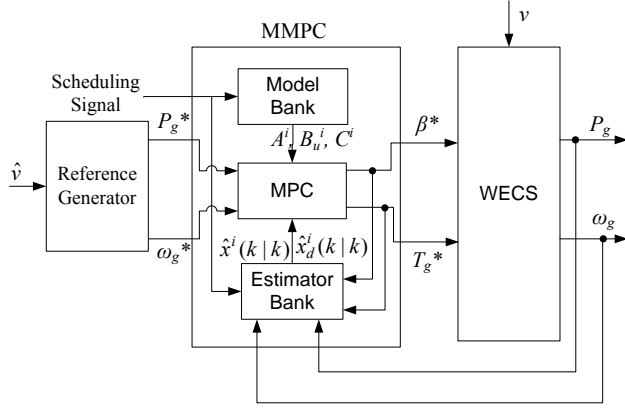case of variable speed variable pitch WECS are given below.

Fig. 4. Proposed control strategy using MMPC.

1) Prediction model bank

A model bank (6), consisting of M linearized models that represent the WECS dynamics in the whole operating region shown Fig. 1 must be available.

$$\boldsymbol{x}^i(k+1) = \boldsymbol{A}^i\boldsymbol{x}^i(k) + \boldsymbol{B}_u^i\boldsymbol{u}(k) + \boldsymbol{B}_d^i\boldsymbol{d}^i(k)$$
$$\boldsymbol{y}^i(k) = \boldsymbol{C}^i\boldsymbol{x}^i(k) + \boldsymbol{D}_d^i\boldsymbol{d}^i(k) \quad , i = 1, \dots M \quad (6)$$

For the case of the WECS, $\boldsymbol{u}(k)$, $\boldsymbol{x}^i(k)$ and $\boldsymbol{y}^i(k)$ of model i in (6) at the sampling instant k are defined in (7).

$$\boldsymbol{x}^i(k) \stackrel{\text{def}}{=} \begin{bmatrix} \delta\omega_t^i(k) & \delta\omega_g^i(k) & \delta T_{tw}^i(k) & \delta T_g^i(k) & \delta\beta^i(k) \end{bmatrix}^T$$
$$\boldsymbol{u}(k) \stackrel{\text{def}}{=} \begin{bmatrix} \delta T_g^*(k) & \delta\beta^*(k) \end{bmatrix}^T$$
$$\boldsymbol{y}^i(k) \stackrel{\text{def}}{=} \begin{bmatrix} \delta\omega_g^i(k) & \delta P_g^i(k) \end{bmatrix}^T$$
$$(7)$$

The fictitious unmeasured disturbance $\boldsymbol{d}^i(k) \in \mathbb{R}^{n_d}$ is used to represent the effect of actual unmeasured disturbances on the plant and it is modeled using (8).

$$\boldsymbol{x}_d^i(k+1) = \overline{\boldsymbol{A}}^i\boldsymbol{x}_d^i(k) + \overline{\boldsymbol{B}}^i n_d(k)$$
$$\boldsymbol{d}^i(k) = \overline{\boldsymbol{C}}^i\boldsymbol{x}_d^i(k) + \overline{\boldsymbol{D}}^i n_d(k) \quad (8)$$

It is assumed that $n_d(k)$ is zero mean white noise.

The matrices $\boldsymbol{A}^i$, $\boldsymbol{B}_u^i$ and $\boldsymbol{C}^i$ in (6) with sampling period, T_s, can be obtained by discretizing the linearized continuous plant model in (3) at different operating wind speeds, $\overline{v}^i$.

Combining (6) and (8), the augmented prediction model bank used in the MPC formulation is given by:

$$\begin{bmatrix} \boldsymbol{x}^i(k+1) \\ \boldsymbol{x}_d^i(k+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}^i & \boldsymbol{B}_d^i\overline{\boldsymbol{C}}^i \\ \boldsymbol{0} & \overline{\boldsymbol{A}}^i \end{bmatrix}\begin{bmatrix} \boldsymbol{x}^i(k) \\ \boldsymbol{x}_d^i(k) \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_u^i \\ \boldsymbol{0} \end{bmatrix}\boldsymbol{u}(k)$$
$$\boldsymbol{y}^i(k) = \begin{bmatrix} \boldsymbol{C}^i & \boldsymbol{D}_d^i\overline{\boldsymbol{C}}^i \end{bmatrix}\begin{bmatrix} \boldsymbol{x}^i(k) \\ \boldsymbol{x}_d^i(k) \end{bmatrix}$$
$$(9)$$

2) Optimization problem

Assuming knowledge of the estimates of the plant states $\widehat{\boldsymbol{x}}^i(k|k)$ and disturbance states $\widehat{\boldsymbol{x}}_d^i(k|k)$ given the data up to time k, the MPC solves the quadratic program in (10)-(13).

$$\min_{\substack{\forall \Delta T_g^*(k+j), \Delta\beta^*(k+j) \\ j=0,1,\dots,N_c-1}} \left\{ \begin{matrix} \sum_{j=1}^{j=N_p} q_1^i(e_1^i)^2 + q_2^i(e_2^i)^2 \\ \sum_{j=0}^{j=N_c-1} r_1^i\Delta T_g^{*2}(k+j) + r_2^i\Delta\beta^{*2}(k+j) \\ +r_3^i\beta^{*2}(k+j) \end{matrix} \right\}$$
$$(10)$$

Subject to:

Prediction model equations in (9)   (11)

$$\boldsymbol{x}^i(k) = \widehat{\boldsymbol{x}}^i(k|k), \; \boldsymbol{x}_d^i(k) = \widehat{\boldsymbol{x}}_d^i(k|k) \quad\quad (12)$$

$$\Delta\beta_{min} \leq \Delta\beta^*(k+j) \leq \Delta\beta_{max}, j = 1,2,\dots,N_c \quad \text{(13-a)}$$

$$\beta_{min} \leq \beta^*(k+j) \leq \beta_{max}, j = 1,2,\dots,N_c \quad \text{(13-b)}$$

$$0 \leq T_g^* \leq T_{g,max}, j = 1,2,\dots,N_c \quad\quad \text{(13-c)}$$

$$\omega_g^i(k+j) \leq \omega_{g,max}, j = 1,2,\dots,N_p \quad\quad \text{(13-d)}$$

$$P_g^i(k+j) \leq P_{g,max}, j = 1,2,\dots,N_p \quad\quad \text{(13-e)}$$

Here, $e_1^i \stackrel{\text{def}}{=} P_g^*(k+j) - P_g^i(k+j)$, $e_2^i \stackrel{\text{def}}{=} \omega_g^*(k+j) - \omega_g^i(k+j)$, $\bullet_{max} (\bullet_{min})$ is used to denote the maximum (minimum) dynamical limit of $\bullet$. The limits $T_{g,max}$, $\omega_{g,max}$ and $P_{g,max}$ are ususally higher than the rated generator torque, speed and power, respectively. The pitch angle set point control move, $\Delta\beta^*(k)$, is defined as $\beta^*(k) - \beta^*(k-1)$. Knowing T_s and the actual pitch rate limits, $\dot{\beta}_{max}$ and $\dot{\beta}_{min}$, the limits $\Delta\beta_{max}$ and $\Delta\beta_{min}$ can be calculated.

3) State estimation

In this paper, the state estimates are computed using an observer bank consisting of M state observers in (14)-(15).

$$\begin{bmatrix} \widehat{\boldsymbol{x}}^i(k|k) \\ \widehat{\boldsymbol{x}}_d^i(k|k) \end{bmatrix} = \begin{bmatrix} \widehat{\boldsymbol{x}}^i(k|k-1) \\ \widehat{\boldsymbol{x}}_d^i(k|k-1) \end{bmatrix} +$$
$$\boldsymbol{K}^i\left(\boldsymbol{y}(k) - (\boldsymbol{C}^i + \boldsymbol{D}_d^i\overline{\boldsymbol{C}}^i)\widehat{\boldsymbol{x}}^i(k|k-1)\right) \quad (14)$$

$$\begin{bmatrix} \widehat{\boldsymbol{x}}^i(k+1|k) \\ \widehat{\boldsymbol{x}}_d^i(k+1|k) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}^i & \boldsymbol{B}_d^i\overline{\boldsymbol{C}}^i \\ \boldsymbol{0} & \overline{\boldsymbol{A}}^i \end{bmatrix}\begin{bmatrix} \widehat{\boldsymbol{x}}^i(k|k) \\ \widehat{\boldsymbol{x}}_d^i(k|k) \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_u^i \\ \boldsymbol{0} \end{bmatrix}\boldsymbol{u}(k)$$
$$(15)$$

The gain, $\boldsymbol{K}^i$, is designed using Kalman filtering techniques [16] based on model i in (9).

4) Bumpless switching between different MPC controllers

At any sampling instant, only one QP in (10)-(13) based on one of the M models of the model bank (6) is solved. However, all M estimators in the estimator bank continuously receive the current control signal and measured signals and update their internal state estimates. Consequently, the internal states of all estimators are kept up to date, thus reducing the transients in the state estimation when a new MPC controller becomes active.

Switching between different MPCs is based on the value of a scheduling signal as shown in Fig. 4. In the case of the WECS, the scheduling signal can be the generator speed and pitch angle or an average wind speed estimate [10], [12].

Due to wind speed fluctuations, it is important to ensure bumpless switching between different MPC controllers. This is ensured since the MMPC algorithm calculates the control increments, $\Delta\beta^*(k)$ and $\Delta T_g^*(k)$, and by continuously updating all estimators in the estimator bank.

C. MMPC tuning for variable speed variable pitch WECSs

1) MMPC Weight selection

In this paper the whole operating region ($v_{ci} \leq \overline{v} \leq v_{co}$), with v_{ci} and v_{co} given in Appendix B, will be partitioned into six operating sub-regions defined in Table I. For each of these sub-regions, a linearized model (3) at $\overline{v}^i$ given in Table

I is assumed to be available. Using these models, six MPC controllers are designed.

Sub-region, i	Speed range (m/s)	$\overline{v}^i$ (m/s)	Weights
1	4-9	6.5	$q_1 = 0$, $q_2 = 1$, $r_1 = 1$, $r_2 = 1000$, $r_3 = 1000$
2	9-11	10	$q_1 = 0$, $q_2 = 2$, $r_1 = 1$, $r_2 = 0.01$, $r_3 = 0.15$
3	11-14	12.5	$q_1 = 50$, $q_2 = 20$, $r_1 = 100$, $r_2 = 3$, $r_3 = 0$
4	14-18	16	*Same as sub-region 3*
5	18-22	20	*Same as sub-region 3*
6	22-26	24	*Same as sub-region 3*

The first sub-region ($i = 1$) in Table I corresponds to the partial load regime. To achieve the control objectives for this region stated in Section III, the weights r_2^1 and r_3^1 in (10) should be set to large values to force the pitch angle set point to be fixed at zero. Since the objective is to force the generator speed to track its set point, the weight q_1^1 should be set to zero while the weights q_2^1 and r_1^1 in (10) should be selected to achieve the desired trade-off between energy maximization and drive train transient load minimization.

The second sub-region ($i = 2$) in Table I corresponds to partial load operation near the rated wind speed. The weights q_1^2, q_2^2 and r_1^2 should be selected similar to those in sub-region 1. However, the weights r_2^1 and r_3^1 should be reduced in comparison with those in sub-region 1 and the constraint (13-e) should be replaced with (16). This selection allows the pitch system to be activated when required to prevent the power from exceeding its rated value when the wind speed fluctuates near the rated wind speed. Consequently, power and drive train twist torque overshoots are eliminated.

$$P_g^i(k + j) \leq P_{g,rat} , j = 1, 2, \ldots, N_p \qquad (16)$$

The four sub-regions corresponding to $i = \{3,4,5,6\}$, represent the WECS operation in the full load regime. The weight r_3^i should be set to zero to allow the pitch angle to take any required value. The weights q_1^i, q_2^i, r_1^i and r_2^i are tuned to achieve the desired trade-off between output power smoothing, generator speed regulation, drive train transient loads reduction and pitch angle activity, respectively.

The other MMPC parameters such as the sampling time, the prediction horizon, the control horizon are chosen as:

$$T_s = 50 \, ms , N_p = 20 , N_c = 10 \qquad (17)$$

2) Offset free tracking MMPC

It is important to design the MMPC to guarantee offset free tracking for the WECS. This property ensures that when the system reaches steady state, the controlled variables are equal to their desired set points in the presence of plant/model mismatch, constant exogenous disturbances and step variations in the set points.

Following the guidelines in [19], it will be assumed that there are two integrated white noise unmeasured disturbances, $d(k) \in \mathbb{R}^2$, that enter at the input of the system. This can be achieved by using (18).

$$B_d = B_u, D_d = 0, \overline{A} = \overline{B} = \overline{C} = I_2 \text{ and } \overline{D} = 0 \qquad (18)$$

This selection ensures offset free tracking and regulation in both partial and full load regimes, respectively [19].

VI. SIMULATION RESULTS

In this section, the performance of the proposed control strategy is compared with the classical control strategy described in Section IV. In all simulations, the designed controllers are tested on the non-linear model in Appendix A, with data given in Appendix B. The speed, torque and power signals have been normalized based on the per unit system in [3]. For the sake of comparison, it will be assumed that an estimate of the average wind speed is available and is used as the scheduling signal in both strategies.

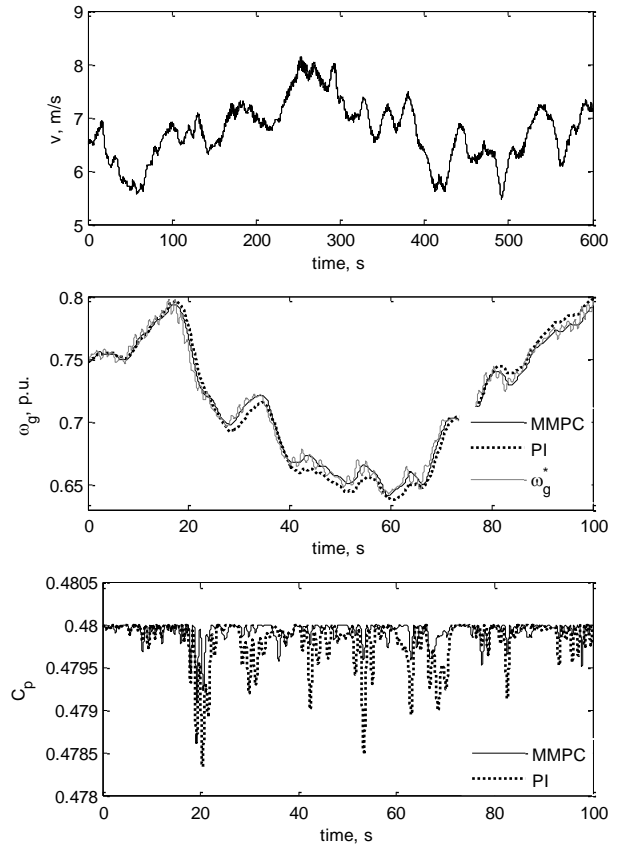### A. Partial load with variable-speed operation (low wind speed)

Fig. 5. Simulation results for low wind speeds.

A simulation result for partial load operation is shown in Fig. 5. It can be seen that the MMPC controller allows better tracking of the generator speed and higher aerodynamic efficiency compared to the classical strategy. This is achieved while the drive train torsional torque is almost identical for both cases (the figure is not shown here). Table II indicates a slight increase in the average power produced and around 35% decrease in the standard deviation of λ around its optimum value when using the MMPC controller.

TABLE II
LOW WIND SPEEDS STATISTICS

Quantity	MPC	PI	MPC/PI
AVG(P_g)	0.2136	0.211	1.011
STD(λ)	0.0605	0.0925	0.654

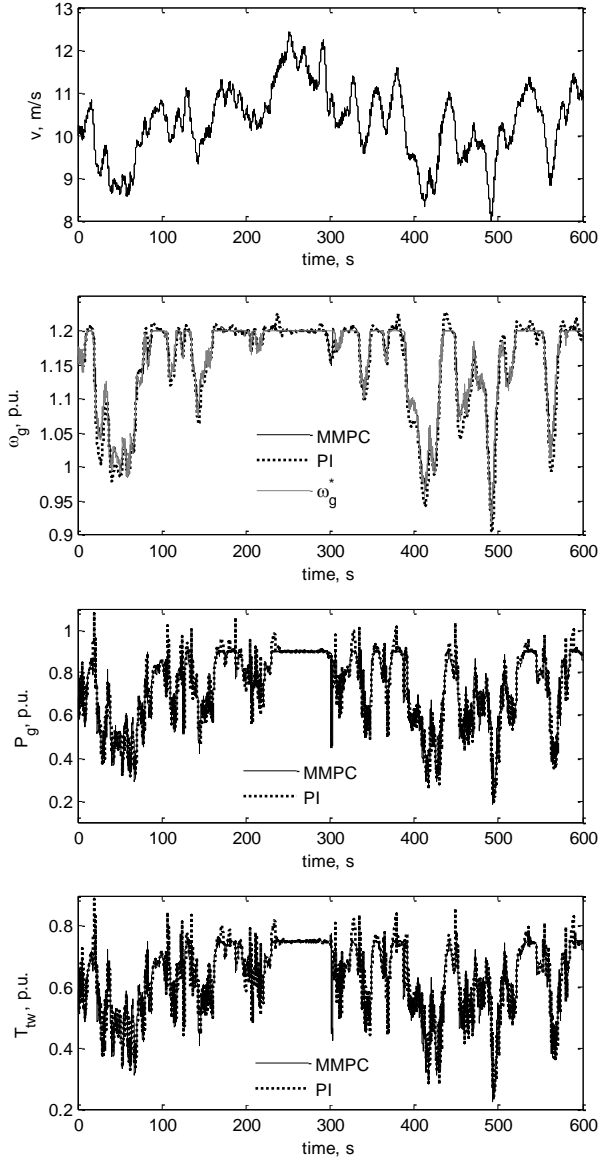*B. Partial load operation and near rated speed operation (medium wind speed)*

Fig. 6. Simulation results for medium wind speeds.

TABLE III
MEDIUM WIND SPEEDS STATISTICS

Quantity	MPC	PI	MPC/PI
STD($\dot{\beta}$)	0.2813	0.1439	1.95
Max(T_{tw})	0.7740	0.8200	0.944
Max(P_g)	0.9014	1.0766	0.837

A simulation result for partial load operation near the rated wind speed (11m/s) is shown in Fig. 6. It can be seen that power and drive train torque overshoots occur when using the classical strategy. The MMPC eliminated these overshoots and P_g and ω_g stay within the rated values.

However, this is achieved by increasing the pitch activity as shown in Table III. Despite this increase, the value of the pitch activity is still very small when compared to the values obtained at full load in table IV.
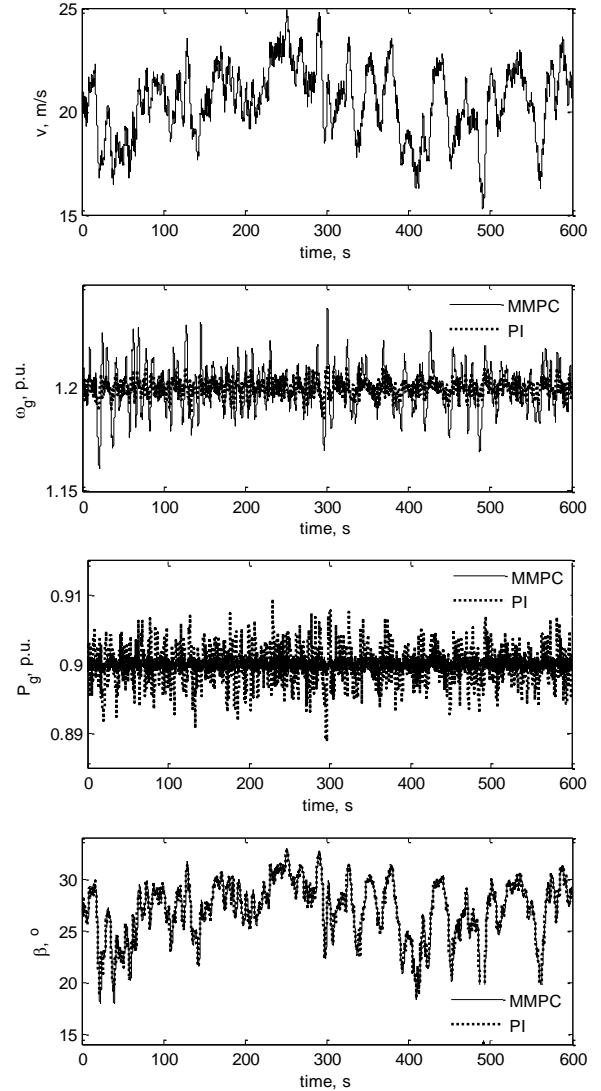
C. Full load operation (High wind speed)

Fig. 7. Simulation results for high wind speeds.

TABLE IV
HIGH WIND SPEEDS STATISTICS

Quantity	MPC	PI	MPC/PI
STD($\dot{\beta}$)	1.1673	1.4295	0.8166
STD(P_g)	0.00037	0.0028	0.1321
STD(ω_g)	0.0108	0.0037	2.9189

A simulation result for full load operation of the WECS is shown in Fig. 7. It can be observed that when using the MMPC controller, the power fluctuations are much reduced. The price is an increase in the generator speed fluctuations in comparison with the classical strategy. This does not represent a problem as long as it is guaranteed to keep the speed below its maximum limit. Furthermore, Table IV shows about 18% reduction in STD($\dot{\beta}$) and more than 85% reduction in STD(P_g) when using the MMPC controller.

VII. Conclusion

A multivariable control strategy based on model predictive control techniques is proposed to control variable speed variable pitch WECSs over their full operating ranges. In the partial load regime, the MPC controller can be designed to provide the desired tradeoff between energy maximization and reduction of the drive train torsional torque. Near the rated wind speed, power and drive train torsional torque overshoots are eliminated. In the full load regime, the MPC controller can be designed to provide the desired tradeoff between power smoothing and speed regulation while reducing drive train torsional torque fluctuations and pitch actuator activity. Furthermore, the MPC controller provides the desired WECS performance while keeping the system variables within safe operating limits. Performance of the multivariable MPC controller is compared with the classical PI control strategy. Simulation results showed the superiority of the proposed control over the classical gain scheduled PI control strategy over the whole operating region of the WECS.

Appendices

A. Appendix A

1) Pitch actuator system

$$\dot{\beta} = -\frac{1}{\tau}\beta + \frac{1}{\tau}\beta^* \tag{A.1}$$

$$\beta_{min} \leq \beta \leq \beta_{max} \tag{A.2}$$

$$\dot{\beta}_{min} \leq \dot{\beta} \leq \dot{\beta}_{max} \tag{A.3}$$

Here, τ is the time constant of the pitch system and $\bullet_{max}$ ($\bullet_{min}$) is the maximum (minimum) limit of $\bullet$.

2) Aerodynamic system

$$T_t = \frac{C_P(\lambda,\beta)}{\lambda}\frac{1}{2}\rho\pi R^3 v^2 \tag{A.4}$$

$$C_P(\lambda,\beta) = 0.5176\left(\frac{116}{\lambda_i} - 0.4\beta - 5\right)e^{-\frac{21}{\lambda_i}} + 0.0068\lambda \tag{A.5}$$

$$\frac{1}{\lambda_i} = \frac{1}{\lambda+0.08\beta} - \frac{0.035}{\beta^3+1} \tag{A.6}$$

Here, T_t is the turbine torque, R is the blade length; $C_P(\lambda,\beta)$ is the power coefficient, $\lambda \overset{\text{def}}{=} \frac{\omega_t R}{v}$ is the tip speed ratio and ω_t is the speed of the low speed shaft.

3) Drive train model

$$\frac{d\omega_t}{dt} = -\frac{i}{J_t}T_{tw} + \frac{1}{J_t}T_t \tag{A.7}$$

$$\frac{d\omega_g}{dt} = \frac{1}{J_g}T_{tw} - \frac{1}{J_g}T_g \tag{A.8}$$

$$\frac{dT_{tw}}{dt} = k_s i\omega_t - k_s\omega_g - \left(\frac{i^2 B_s}{J_t} + \frac{B_s}{J_g}\right)T_{tw} + \frac{iB_s}{J_t}T_t + \frac{B_s}{J_g}T_g \tag{A.9}$$

Here, J_t and J_g are the inertia of the turbine and the generator, respectively; T_{tw} is the drive train torsional torque; i is the gear ratio; k_s, B_s are the shaft stiffness and damping coefficients, respectively.

4) Generator model

$$\dot{T_g} = -\frac{1}{\tau_g}T_g + \frac{1}{\tau_g}T_g^* \tag{A.10}$$

Here, T_g, T_g^* and τ_g are the generator torque, torque set point and time constant, respectively.

5) Wind speed model

Wind speed, $v(t)$, is modeled as (A.11). The model used here is based on [1]. The model includes tower shadow, wind shear and rotational sampling effects.

$$v(t) = v_m(t) + v_t(t) \tag{A.11}$$

B. Appendix B

System rated power = 1.5 MW(0.9 p.u.); R = 35 m; v_{ci}/ v_r/ v_{co} = 4/11/26 m/s; i = 62.6; Turbine inertia constant = 3 s; Generator inertia constant = 0.5 s; Shaft stiffness = 0.5 pu; Shaft damping = 0.01 pu; β_{min}/β_{max} = 0/45 º; $\dot{\beta}_{min}$/$\dot{\beta}_{max}$ = -10/10 º/s; , τ_g = 5 ms; Rated/Max generator speed = 1.2 p.u./1.3 p.u

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