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Synchronization and control of directed transport in chaotic ratchets via active control

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Abstract

Using a technique derived from nonlinear control theory, we demonstrate that two identical inertial ratchets transporting particles in two directions can be synchronized such that both ratchets transport particles in a desired direction. This novel approach to control of directed transport is applicable when there are multiple co-existing attractors in phase space transporting particles in different directions. Numerical simulations are employed to illustrate the approach.

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1. Introduction

Recently, there has been an increased interest in the study of transport phenomena of nonlinear systems that can extract usable work from unbiased nonequilibrium fluctuations. These so-called ratchets (or Brownian ratchets) [1,2] can be modeled by a Brownian particle undergoing random walk on a periodic asymmetric potential and being acted upon by an external time-dependent force of zero average. Research activities in this area is partly motivated by the challenge to model and control some biological processes at both micro and macro scales as found in transport of ion channels and muscle operations respectively [3]. Another source of motivation is the potential for technological applications aimed at devising mechanisms for sorting, separating, pumping and controlling tiny particles at nanoscales and micro scales (see Refs. [2,4] and references therein). Outstanding experimental realization of some of these devices have been carried out. Specifically, the control of motion of vortices in superconductors [5], particles in asymmetric silicon pores [6], charged particles through artificial pores [7], among others, have been reported recently.

Several attempts have been made to understand the generation of unidirectional motion from nonequilibrium fluctuations. The vast majority of the models described in the literature considers the overdamped cases in which the effect of the inertial term is neglected [1,8]. Recently, ratchet models wherein the inertial term is considered have been extensively investigated since it was first studied by Jung et al. [9]. These ratchets possess, in general, a classical chaotic dynamics that modifies significantly the transport properties [9,10]. For instance, current reversal and multiple current reversals have been attributed to changes in the bifurcation structure. In addition, the implication of chaotic dynamics in deterministic ratchets has been recently addressed in the quantum domain, together with the possible connection with quantum chaos [11].

In a different context, Savel'ev et al. [12] examined the transport properties of binary mixture of interacting particles and showed that attracting or repelling interaction among identical particles can result in the amplification (inversion) of their net current. This is potentially useful for enhancing and regulating transport (e.g. through synthetic ion channels) and separation of repelling particles. Interaction among identical and

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nonidentical particles can lead to synchronized dynamics when a threshold is reached. Synchronization of two coupled chaotic ratchets has been recently investigated [13] and it is believed that the synchronization of coupled ratchets could provide some information regarding the transport properties of inertia ratchets [13]. Synchronization phenomena in coupled or driven nonlinear oscillator are in general of fundamental importance in nonlinear dynamics and have been extensively investigated both theoretically and experimentally since the seminal work of Pecora and Carroll in 1990 [14]. Chaos synchronization is closely related to the observer problem in control theory [15]. The problem may be treated as the design of control law for full chaotic observer (the slave system) using the known information of the plant (the master system) so as to ensure that the controlled receiver synchronizes with the plant. Hence, the slave chaotic system traces the dynamics of the master in the course of time.

Various techniques have been proposed for achieving stable synchronization between identical and non-identical systems. Notable among these methods, the active control scheme [16] has received a considerable attention in the last few years due to its simplicity and robustness. Applications to various systems abound, some of which includes the Lorenz, Chen and Lü system [17], geophysical system [18], spatiotemporal dynamical system [19], the so-called unified chaotic attractor [20], electronic circuits, which model a third-order "jerk" equation [21], the Bloch equation [22] and most recently in RCL-shunted Josephson junction [23].

In the present Letter, we employ the active control technique to first, examine the synchronization behavior of two inertial ratchets with identical system parameters; both evolving from different initial conditions. Secondly, we explore the synchronization property to show that the direction of particle transport in inertial ratchets can be reversed or controlled by using active control technique.

2. The chaotic ratchet model

Let us consider the one-dimensional problem of a particle driven by a periodic time-dependent external force under the influence of an asymmetric potential of the ratchet type [9,10]. The time average of the external force is zero. In the absence of stochastic noise, the dynamics is exclusively deterministic. The dimensionless equation of motion for a particle of unit mass moving in the ratchet potential $V(x)$ is given by (see Ref. [10] for instance):

$$\ddot{x} + b\dot{x} + \frac{dV(x)}{dx} = a\cos(\omega_D t), \tag{1}$$

where time t has been normalized in the unit of ω_0^{-1}, ω_0 being the frequency of the linear motion around the minima of $V(x)$. b is the damping parameter, while a and ω_D are the amplitude and frequency of the driving force respectively; $V(x)$ is the dimensionless potential given by

$$V(x) = C - \frac{1}{4\pi^2\delta}\left[\sin 2\pi(x - x_0) + 0.25\sin 4\pi(x - x_0)\right]. \tag{2}$$

The constant $C \simeq 0.0173$ and $\delta \simeq 1.600$. The potential is shifted by a value x_0 in order that the minimum of $V(x)$ is located at the origin (see Fig. 1(a)). We note that apart from its periodicity, the ratchet potential (2) has an infinite number of potential wells; so that the orbits transport particles from one well to another. Thus, in the Poincaré section representation, one can utilize this periodicity to collapse the dynamics to a unit cell within a phase space region for which $-0.5 \leqslant x \leqslant 0.5$ (Fig. 1(b)).

The extended phase space in which the dynamics is taking place is three-dimensional, since we are dealing with an inhomogeneous differential equation with an explicit time dependence. Eq. (1) can be expressed in autonomous form and then solved numerically using the fourth-order Runge–Kutta algorithm. Since the equation is nonlinear, its solution therefore allows the possibility of periodic and chaotic orbits. Fig. 1(b)–(d) shows the chaotic behavior of system (1).

3. Synchronization of inertial ratchets using active control

Let us consider two identical ratchets in a master–slave configuration, such that the master ratchet with the subscript 1 is to control the slave ratchet with subscript 2. Without losts of generality, we can express the master ratchet system as

$$\begin{aligned} \dot{x}_1 &= y_1, \\ \dot{y}_1 &= a\cos(w_D t) - by_1 - \frac{dV(x_1)}{dx_1}, \end{aligned} \tag{3}$$

and the slave ratchet as

$$\begin{aligned} \dot{x}_2 &= y_2 + u_1(t), \\ \dot{y}_2 &= a\cos(w_D t) - by_2 - \frac{dV(x_2)}{dx_2} + u_2(t), \end{aligned} \tag{4}$$

where $u_1(t)$ and $u_2(t)$ are control functions to be determined. In order to estimate the control functions, we subtract Eq. (3) from Eq. (4) and defining the error system as the difference between the master and the slave ratchets:

$$x_3 = x_2 - x_1; \qquad y_3 = y_2 - y_1, \tag{5}$$

we obtain

$$\begin{aligned} \dot{x}_3 &= y_3 + u_1(t), \\ \dot{y}_3 &= -by_3 - \frac{dV(x_2)}{dx_2} + \frac{dV(x_1)}{dx_1} + u_2(t). \end{aligned} \tag{6}$$

We redefine the control functions to eliminate all items that cannot be shown in the form x_3 and y_3:

$$\begin{aligned} u_1(t) &= V_1(t), \\ u_2(t) &= \frac{dV(x_2)}{dx_2} - \frac{dV(x_1)}{dx_1} + V_2(t). \end{aligned} \tag{7}$$

Thus,

$$\begin{aligned} \dot{x}_3 &= y_3 + V_1(t), \\ \dot{y}_3 &= -by_3 + V_2(t), \end{aligned} \tag{8}$$

which can be written as $[\dot{x}_3, \dot{y}_3]^T = \mathbf{M} + V(t)$, where $\mathbf{M} = \left(\begin{smallmatrix} 0 & 1 \\ 0 & -b \end{smallmatrix}\right)$ is a feedback matrix and $V(t) = [V_1(t), V_2(t)]^T$. The error dynamics (8) is a full state controllable entity so that

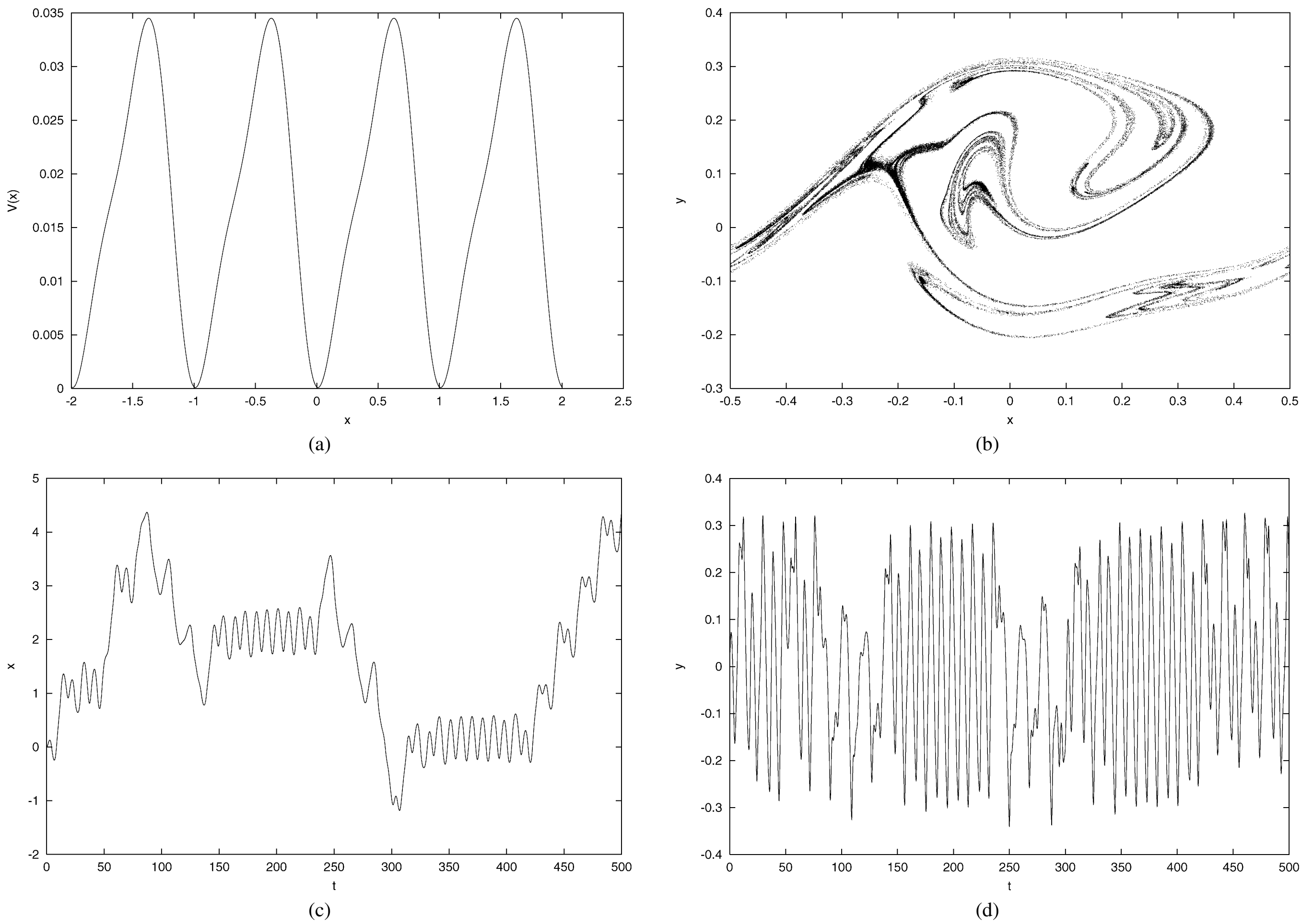

Fig. 1. Ratchet dynamics (a) the ratchet potential, (b) chaotic attractor (in the Poincaré section), (c) intermittent chaotic transport (x-variable), (d) y variable. $a = 0.08092844$, $b = 0.1$, $\omega_D = 0.67$.

feedback gains can be designed to stabilize the error system $[x_3, y_3]^T$, such that error signals asymptotically converge to zero as $t \to \infty$. We choose a constant matrix $\mathbf{K}$ which will control the error dynamics (8) such that:

$$\begin{pmatrix} V_1(t) \\ V_2(t) \end{pmatrix} = \mathbf{K} \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}. \tag{9}$$

There are several choices of $\mathbf{K}$ that could lead to the stability of the closed-loop system (8). In principle, the elements of matrix $\mathbf{K}$ could be chosen such that the feedback matrix $\mathbf{M}$ has all the eigenvalues with negative real parts. Here, we choose

$$\mathbf{K} = \begin{pmatrix} -1 & -1 \\ 0 & b-1 \end{pmatrix}. \tag{10}$$

With matrix (10), the error system (8) is stable and the close-loop system has eigenvalues $(-1, -1)$. This choice will lead to the synchronization of the master–slave ratchets.

4. Numerical results

We first illustrate the synchronization behavior of two identical chaotic ratchets. In our numerical simulations, we used the 4th-order Runge–Kutta scheme with a time-step of $0.02\pi/\omega_D$; where ω_D, the frequency of the external driving force is fixed at 0.67 throughout the Letter. Other parameters of the master–slave ratchet system were chosen such that the dynamics is chaotic as shown in Fig. 1. The initial conditions for the master are $x_1(0) = y_1(0) = 0$ while the initial conditions for the slave are: $x_2(0) = 0.5$, $y_2(0) = 0.1$. When the control is switched off, the chaotic transport of the master–slave ratchet is as shown in Fig. 2(a) and when the control is switched on, the two ratchets are synchronized as shown in Fig. 2(b).

In order to examine the control of directed transport, we make the following choice of initial conditions: $x_1(0) = -0.10$, $y_1(0) = 0.25$, $x_2(0) = 0.43$, $y_2(0) = -0.12$ and simulate the system for $a = 0.156$ as employed in Ref. [24]. These initial conditions and parameter setting are of particular interest as they corresponds to the case where two attractors co-exist in phase space as already presented in [24]; and the situation is analogous to a mixture of identical or nonidentical particles [12]. Here, the master system corresponds to the chaotic attractor transporting particles in the positive direction while the slave is the periodic attractor, which transports particles in the negative direction as shown in Fig. 3(a). It should be noted that our numerical results shown in Fig. 3(a), is obviously the same as Fig. 4 in Ref. [24]. It is very easy to show that the direction of particle transport can be reversed or controlled at any given time to follow the direc-

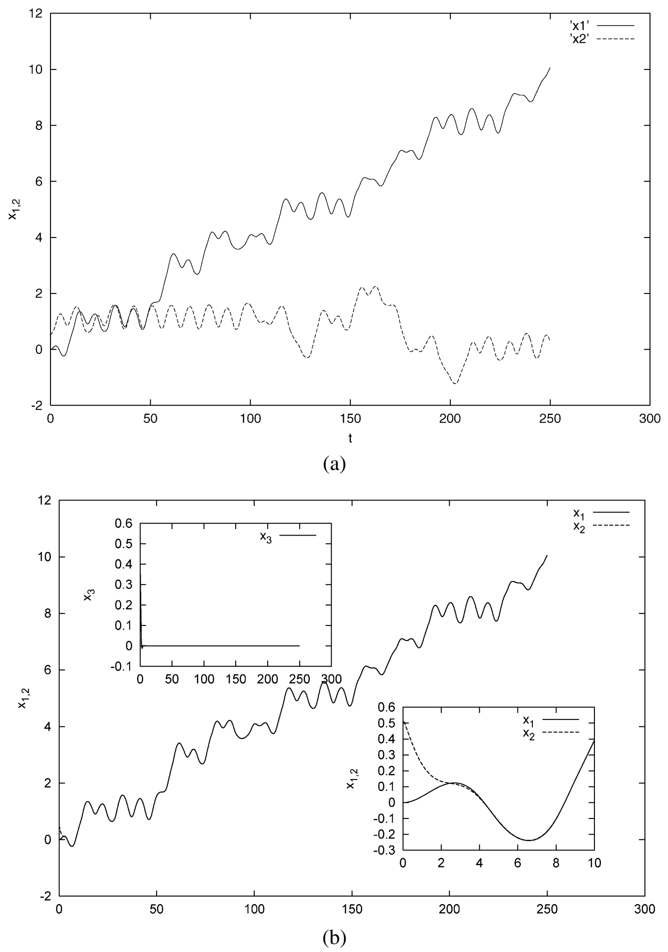

Fig. 2. Synchronization dynamics of the master–slave ratchets when the control is (a) switched off (b) switched on at $t = 0$. $x_1(0) = y_1(0) = 0$ and $x_2(0) = 0.5$, $y_2(0) = 0.1$ and $\omega_D = 0.67$. The inset (bottom-right) in (b) is the zoom showing the initial transient ($t = 4$ s) before complete synchronization is achieved. The inset (top-left) in (b) is the error dynamics (x_3 vs. t) showing asymptotic convergence to zero as $t \to \infty$.

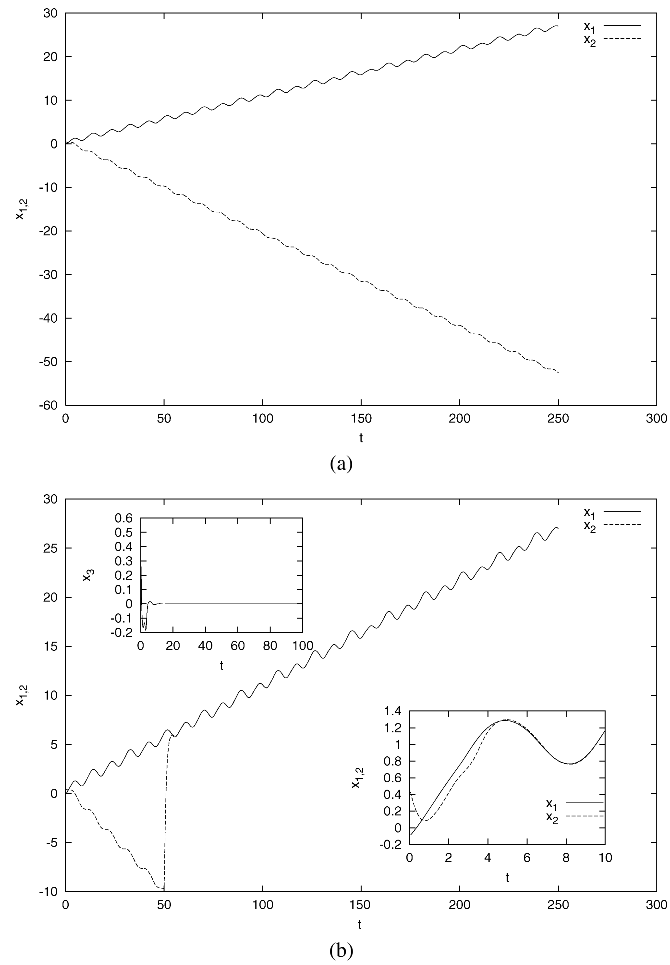

Fig. 3. Synchronization dynamics of the master–slave ratchets for initial conditions: $x_1(0) = -0.10$, $y_1(0) = 0.25$, $x_2(0) = 0.43$, $y_2(0) = -0.12$ and $a = 0.156$, $b = 0.1$, $\omega_D = 0.67$. In (a) control is switched off while in (b) control of particle transport to the positive direction in a chaotic manner is achieved when control is switched on at $t = 50$. Inset (top-left) in (b) is the error dynamics (x_3 vs. t) while the inset (bottom-right) is the zoom of the initial transient when control is activated $t = 0$.

tion of another attractor by using the mechanism of active control. We illustrate this in Fig. 3(b); where the controllers have been activated at a target time of 50 s. Obviously, the periodic attractor now follows the direction of the chaotic attractor, both transporting particles in the positive direction.

5. Concluding remarks

In summary, we have demonstrated a specific application of the active control in nonequilibrium statistical physics wherein the direction of particle transport can be reversed by designing appropriate active controllers that ensures stable synchronization between the master ratchet and the slave ratchet. Thus, by identifying the directions of particle transport for co-existing attractors, one can decide a desired direction of particle transport and the time for which a reverse to this direction is required. With appropriate controllers, any desired direction of transport can be achieved. Finally, we note that the active control method has been found effective in synchronizing nonidentical chaotic systems as well as systems with parameter mismatch (see for example Refs. [17,21,25]). The implication is that, in mixtures of interacting particles, one can employ the active control technique to control the motion of identical or nonidentical particles moving in a ratchet potential as well as the motion of identical particles moving in nonidentical ratchet potentials. Detailed study of these interesting cases have already been carried out together with its connection to particle separation and will be reported elsewhere. As illustrated in [16], implementation of the active control scheme is visible and simple; and could be extended to ratchet devices.

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