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VARIATION PROPAGATION ANALYSIS ON COMPLIANT ASSEMBLIES CONSIDERING CONTACT INTERACTION

Kang Xie
Department of Mechanical Engineering –
Engineering Mechanics
Michigan Technological University
Houghton, Michigan 49931

Lee Wells
Department of Mechanical Engineering –
Engineering Mechanics
Michigan Technological University
Houghton, Michigan 49931

Jaime A. Camelio
Department of Mechanical Engineering –
Engineering Mechanics
Michigan Technological University
Houghton, Michigan 49931

Byeng D. Youn
Department of Mechanical Engineering –
Engineering Mechanics
Michigan Technological University
Houghton, Michigan 49931

ABSTRACT

Dimensional variation is inherent to any manufacturing process. In order to minimize its impact on assembly products is important to understand how it propagates through the assembly process. Unfortunately, manufacturing processes are complex and in many cases highly non-linear. Traditional assembly models have represented assembly as a linear process. However, assemblies that include the contact between their components and tools show a highly non-linear response. This paper presents a new assembly methodology considering the contact effect. In addition, an efficient to predict output response is presented. The enhance dimension reduction method (eDR) is used to accurately and efficiently predict the statistical response of the assembly to variation on the input parameters.

INTRODUCTION

Dimensional variation is inherent to any manufacturing process. Therefore, it is important to understand how it propagates through the process. Fast and accurate evaluation models of process variation are critical in determining the final dimensional variation of a product and in selecting robust product/process design. Unfortunately, manufacturing processes are complex and in many cases highly non-linear restraining the potential of the analysis. In general, the lack of efficient modeling tools limited the analysis capabilities to simplified linear models.

One commonly used non-linear manufacturing process is the compliant assembly process. Compliant assembly is defined as the process of joining flexible or non-rigid parts. Many products, including automobiles, aircraft, furniture, and home appliances, are constructed primarily from compliant parts. In many of these products, the number of parts can be very large, such as the several hundred compliant parts that form a typical auto body assembly. Since parts and fixtures inherently have geometrical variation, understanding how these variations propagate through the system is of significant interest to the design and control of such systems. Two approaches have been widely adopted to model assembly processes: rigid body analysis [1-2] and compliant analysis [3-6]. However, all these methodologies are based on linearized models. Finally, Cai et al. [7] introduced the contact non-linear effect on the assembly of compliant parts. They used a second order Taylor expansion (TSE) method to estimate the non-linear effects. However, TSE methods are efficient and accurate only on close-to-linear methods.

Variation propagation analysis is defined as mechanism by which input uncertainty is propagated to output uncertainty through a system or process. The system or process may consist of subsystems or sub-processes. Input uncertainty includes any type of parameters or variables that are uncertain, as shown in Table 1. Although uncertainty propagation has been extensively investigated in many engineering fields, uncertainty propagation is still a state of art, mainly due to its expensiveness and inaccuracy for nonlinear behavior of systems.

Table 1 Sources of uncertainty

Source	Uncertainty Type	Examples
Product	Shape	*Circularity*
	Size	*Length; Thickness*
	Configuration	*Angles*
	Material	*Young's Modulus*
Process	Geometrical	*Fixtures position; Welding gun location*
	Process Parameters	*Pressure; Sequence Welding temperatures; Current Welding speed; Welding direction*

Accordingly, many different methods have been developed for uncertainty propagation analysis. These methods can be categorized into three approaches: sampling techniques, expansion techniques, and Advanced First-Order Second Moment (AFOSM).

The most common sampling techniques are Monte Carlo Simulation (MCS) and Design of Experiments (DOE). In general, these methods are quite comprehensive and easy to use but prohibitively expensive to achieve good accuracy. Simulation methods [3] can be expensive for predicting high reliability, whereas DOE [8] can be costly for high dimensional problems, so-called a curse of dimensionality. Therefore, sampling techniques are often used for verification or benchmarking studies.

There exist three types of expansion methods: Taylor series expansion, perturbation method, and Neumann expansion. Taylor series expansion method is sometimes called Root Sum Squares (RSS) method. It yields highly inaccurate estimates for nonlinear system. Hence, its application has been restricted to linear or mildly nonlinear systems. In addition to such difficulty, it requires a second-order sensitivity analysis for uncertainty control and management, which is expensive and complicated [9]. In the perturbation method, the solution is approximately represented in a perturbed form. Thus, it can be applied to diverse systems represented by differential, integral, and algebraic equations. Its primary disadvantages are the lack of applicability to experiments and computational expensiveness when the dimension of the system is large [10]. Similarly, the main limitation of Neumann expansion method is the requirement that the perturbation terms must be small. Further, this method is in general difficult to apply in conjunction with modeling complex nonlinear systems, as the model equations are often mathematically intractable [11]. It is quite interesting that the common drawback of expansion methods is inaccuracy of uncertainty characterization for nonlinear systems.

Depending on the order of system approximation, uncertainty propagation can be analyzed using First-Order Reliability Method (FORM) and using Second-Order Reliability Method (SORM). These methods accurately predict a tail approximation of the probability distribution for a system but, respectively, require first-order and second-order derivatives for system performances with respect to input uncertainties [12]. Thus, the application of AFOSM is limited to relatively simple mathematical models.

Current assembly models for geometrical variation propagation prediction are limited to linear sensitivity analysis. However, real assembly processes are more complex and heavily subject to uncertainties of system parameters. In order to extend the capabilities of current models, it is necessary to create new methods that predict geometrical variation propagation by taking into account the non-linear effects due to the contact between the components and tools in the physical assembly process.

This paper presents a new methodology to predict the effect on assembly dimensions due to variation on geometrical dimensions on the assembly components. The methodology considers the components interaction due to the physical contact between the components and tools (clamps and welding guns). These interactions produce additional deformations in the components during the assembly process. In addition, due to the limitation of traditional uncertainty propagation methods a new methodology for uncertainty propagation in non-linear assembly systems is presented.

The paper is organized as follows. Section 2 reviews the traditional rigid and compliant assembly methodologies. Section 3 presents the new methodology to predict assembly variation propagation in non-linear contact assemblies. The enhanced dimension reduction (eDR) method is presented in Section 4. The eDR method allows predicting the output distribution for the assembly based on given distribution of the input parameters. In Section 5, a case study of a hood bracket assembly is discussed. The case study shows the application of the new methodology. Finally, Section 6 draws the conclusions.

2. TRADITIONAL SHEET METAL ASSEMBLY MODELING

Several models have been proposed to predict how variation propagates during assembly. Initial approaches were focused on rigid part assembly using either the Root Sum Squares (RSS) method or Monte Carlo Simulation. Detailed review and discussion can be found in Chase and Parkinson [13]. Recently, multi-level variation propagation models have also been developed. Mantripragada and Whitney [2] proposed a state transition model to predict the variation propagation in multi-stage assembly systems. Ding et al. [1] presented a state space model for dimensional control in sheet metal assembly assuming rigid parts. For compliant assembly, Liu and Hu [3] proposed a compliant assembly model to analyze the effect of deformation and springback on assembly variation by applying linear mechanics and statistics. Using finite element methods (FEM), they constructed a sensitivity matrix to establish a linear relationship between the incoming part deviation and the output assembly deviation. Camelio et al. [4] extended this approach to multi-station systems using a state space representation.

Assembly variation is estimated as a function of the components' geometry, process layout and the contribution of various sources of variation. Three main sources of variation have been identified in compliant sheet metal assembly: component variation, fixture variation and joining method induced variation. Part variation includes the mean deviation, μ, and the variance of the deviation, σ^2, on parameters that describe the geometry of the component. A deviation is the difference between the actual part dimension and the nominal dimension. Part deviation can be denoted as a vector $V \in R^n x1$, in which the elements correspond to deviations at each parameter. Traditional assembly modeling approaches define part deviation as point based considering only key control characteristics.

Liu and Hu [3] presented the method of influence coefficients (MIC) to predict the impact of the part deviation, V_u, on the assembly deviation, V_a. Finite element methods and MIC are used to obtain the sensitivity matrix, S, for a sheet metal assembly. The elements of the sensitivity matrix, s_{ij}, measure the sensitivity of the assembly at node i to the incoming part deviation at node j. This approach considers a linear relationship between the incoming parts deviation and the final assembly deviation. Therefore, the assembly deviation, V_a, can be calculated using Eq. (1). By definition V_a is the assembly deviation vector, where the column elements represent the assembly deviation at the key measurement points. V_u is the component deviation vector, where the elements represent the component deviation at the welding nodes.

$$\mathbf{V_a} = \begin{bmatrix} v_{a_1} \\ v_{a_2} \\ \vdots \\ v_{a_m} \end{bmatrix} = \sum_{j=1}^{n} \begin{bmatrix} s_{1j} \\ s_{2j} \\ \vdots \\ s_{mj} \end{bmatrix} \cdot v_{u_j} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ s_{m1} & s_{m2} & \cdots & s_{mn} \end{bmatrix} \cdot \begin{bmatrix} v_{u_1} \\ v_{u_2} \\ \vdots \\ v_{u_n} \end{bmatrix} \quad (1)$$

$$\text{or} \quad \mathbf{V_a} = \mathbf{S} \cdot \mathbf{V_u}$$

The station level model presented in Eq. (1) describes the assembly variation behavior at a single station. However, sheet metal assembly processes are typically multilevel hierarchical manufacturing processes, where parts are joined together at different sequential or parallel stations. Dimensional variation will propagate from station to station based on incoming parts variation, fixture variation and the joining process variation. The propagation effect of the dimensional variation can be modeled as a linear time discrete system, where the variable time, k, represents the station location (Eq. 2), **A** is the state matrix, **X** is the state vector, **B** is the input matrix, **U** is the input vector, **W** is a perturbation vector, **Y** is the measurement/observation vector, **C** is the observation matrix, and **V** is a measurement system noise vector.

$$\begin{aligned} \mathbf{X}(k) &= \mathbf{A}(k) \cdot \mathbf{X}(k-1) + \mathbf{B}(k) \cdot \mathbf{U}(k) + \mathbf{V}(k) \\ \mathbf{Y}(k) &= \mathbf{C}(k) \cdot \mathbf{X}(k) + \mathbf{V}(k) \end{aligned} \quad (2)$$

Camelio et al. [4] developed a methodology to analyze the propagation of variation in compliant multi-station assembly systems using a state space representation. Based on their model, the state space equation can be rewritten as:

$$\begin{aligned} \mathbf{X}_k = (\mathbf{S}_k - \mathbf{P}_k + \mathbf{I})(\mathbf{X}_{k-1} + \mathbf{M_i}(\mathbf{X}_{k-1} - \mathbf{U}_k^{3-2-1})) \\ - (\mathbf{S}_k - \mathbf{P}_k)(\mathbf{U}_k^{N-3} + \mathbf{U}_k^{g}) + \mathbf{W}_k \end{aligned} \quad (3)$$

where $\mathbf{S_k}$ is the sensitivity matrix (similar to MIC), $\mathbf{P_k}$ is the part deformation matrix, and $\mathbf{M_k}$ is the relocation matrix associated with station k. $\mathbf{U^{3\text{-}2\text{-}1}}$ is the variation vector of a *3-2-1* locating fixture, $\mathbf{U^{N\text{-}3}}$ is the variation vector for a *N-2-1* fixture with N>3, and $\mathbf{U^g}$ is the variation vector for welding guns. **W** represents the noise which is the propagated variation not accounted by this model.

As presented in this section, traditional assembly models are linear and point based. In order to include non-linear contact effects, a new methodology to represent the assembly process is needed.

3. MODELING ASSEMBLY VARIATION INCLUDING CONTACT CONSIDERATIONS

Although the Method of Influence Coefficients (MIC), presented by Liu and Hu [3] and widely used on assembly variation simulation, can precisely and efficiently predict the assembly distribution based on the linear mechanics, it cannot be directly used for problems that behave in the nonlinear domain. One of the limitations of the MIC is that the assembly deformation is considered linear and no consideration to part interference is included. Therefore, the parts are allowed to penetrate each other when get in contact during the assembly process. In addition, the MIC approach constructs the sensitivity matrix evaluation using the response of a nominal assembly under external displacements for each individual component and the assembly. An equivalent force for each source of variation is generated by exerting the corresponding deviation of the component departing from a nominal position. The forces and displacements are estimated using a finite element model. Then, the clamping effect is simulated by applying the equivalent force in the opposite direction to make the component to recover it's nominal position. This approach differs significantly from the real assembly process, limiting its capacity to represent the process under non-linear conditions. As it was mentioned earlier, non-linear behavior on assembly systems is common under the contact interaction between parts and due to welding distortion effects.

Based on these limitations, a new methodology to represent the assembly process is needed. Considering the actual capabilities of commercial available finite element packages, a good method will be a model that can represent the assembly process as close to reality as possible. The proposed new approach is able to better simulate the real assembly and allows system parameterization. The system parameterization is a powerful tool to characterize the response of the assembly for different sources of variation. Combining the FEA tool with the enhanced Dimensional Reduction Method (eDRM), the new methodology can efficiently and precisely handle nonlinear contact problems.

Traditionally, an assembly process of sheet metal parts considers five steps: 1) the parts are located in the assembly station, a 3-2-1 locating fixture is used; 2) additional locators or clamps are closed to nominal, deforming the sheet metal part if the part is non-nominal; 3) the welding gun(s) is closed to nominal, producing additional deformations; 4) the parts are joined together using, in general, resistance spot welding; 5) the welding gun and clamps are released; and 6) the assembly springback. In order to precisely represent the assembly process, a similar process is simulated in finite elements using Abaqus.

Considering the six steps in the assembly process presented, a new methodology based on finite elements was developed. The two objectives of the new methodology are: 1) to represent the assembly process as real as possible using finite elements; and 2) to incorporate the effect of the physical contact between the components and tools in the assembly. Two types of contact are considered the tool/part interactions and the part/part interactions. The new methodology consists of four steps that represent the assembly process of compliant sheet metal parts:

Step 1 The parts are located in the station. This is equivalent to construct the finite element model. The compliant parts are represented as shell elements. The locators are simulated as single point displacement constraints. In addition, to enhance the capability of the model to handle the interaction between parts and tools, contact pairs elements are defined. The contact areas include both the contact between components or parts and the contact between the tools and the components. The parts are modeled including any deviation from its nominal shape. In order to model uncertainty in some dimensions, the model is parameterized. Therefore the components geometry and mesh can be modified by a small set of parameters.

Step 2 The clamps are closed deforming the individual components to their nominal position. Each clamp is modeled as a rigid body. In general, the stiffness of the clamps is much larger than the stiffness of the individual components. Therefore, this is a reasonable assumption. At this step, the clamp is moved towards the part, the part is deformed due to the contact between the clamp and the part.

Step 3 The welding gun is closed and the parts are joined together. The welding process is simulated in three sub-steps. First, the contact elements in each component corresponding to the welding area are assigned as bonded. Bonding is one property available for the contact pairs to determine the behavior of elements that become in contact. Second, an additional type of elements is introduced. The welding nugget is represented by connector elements. These elements constrain the nodes in the different parts to share the same DOF. Finally, an additional finite element tool, micro-gap adjusting, is used to ensure the welding area between parts is completely in contact.

Step 4 The parts are released. At this state, the welding gun tools and additional locators and clamps are removed, then, the assembly springbacks.

One of the main limitations of modeling contact elements is convergence of the finite element model. The proposed methodology has shown reliable results avoiding convergence issues. Several measures were taken to improve the convergence of the contact model. Some of the measures that improve the performance of the simulation and that are include in the methodology are: 1) a fine mesh and fillets near the contact areas are used to avoid a single node penetration; 2) connector elements representing the weld nugget are used to ensure sufficient constraint between welded parts; 3) micro-gap adjusting is used to overcome the micro gap, separation errors between the parts caused by the calculation errors in the finite element.

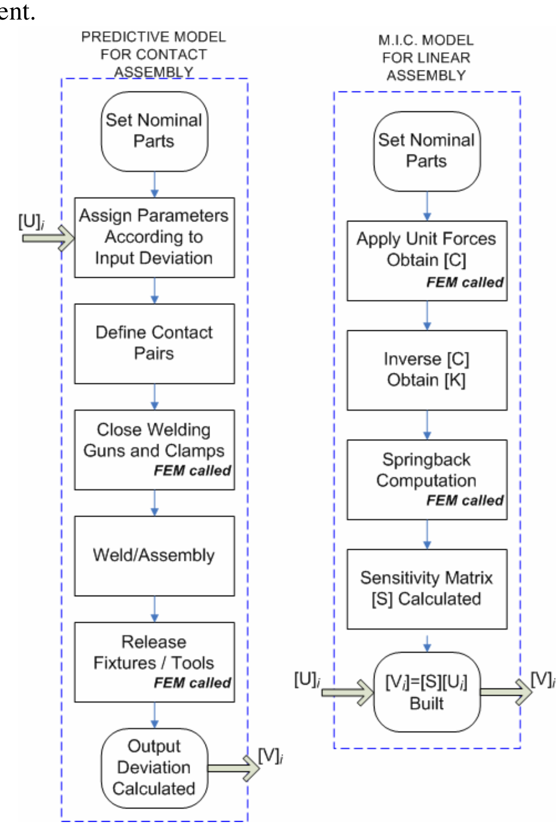


Fig. 1 Predictive contact assembly and MIC

This simulation process differs from the MIC presented by Liu and Hu [3] because it simulates the assembly process as a complete set of sequential operations without incorporating the use of equivalent forces or displacements to determine the final springback. Figure 1 shows the steps of each methodology. The main limitation of the new approach is that requires a complete simulation to evaluate the final springback of the assembly for each set of input deviation. Due to the non-linear response of the contact behavior, an expensive Monte Carlo simulation is required to completely describe final distribution of the output dimensions considering some variability in the input parameters. To overcome this limitation, an efficient method to predict the variation propagation in non-linear contact assembly processes is presented in the next section.

4. ENHANCED DIMENSIONAL REDUCTION METHOD

4.1 DIMENSION-REDUCTION (DR) METHOD

In general, statistical moments (or PDF) of a certain system response can be calculated as

$$\mathscr{E}\left\{Y^m(\mathbf{X})\right\} = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} Y^m(\mathbf{x}) \cdot f_{\mathbf{X}}(\mathbf{x}) \cdot d\mathbf{x} \tag{4}$$

In Eq. (4), a major challenge is found in multi-dimensional integration of joint probability density function of system inputs. To resolve this difficulty, the DR method uses an additive decomposition [14, 15], which converts a multi-dimensional integration in Eq. (4) into multiple one-dimensional integrations. The additive decomposition is defined as

$$Y(X_1,...,X_N) \cong \sum_{j=1}^{N} Y(\mu_1,...,\mu_{j-1}, X_j, \mu_{j+1},...,\mu_N) - (N-1)Y(\mu_1,...,\mu_N) \tag{5}$$

In aid of the additive decomposition, uncertainty quantification of system responses becomes much simpler. For reliability and quality assessment, the math statistical moments for the responses are considered in Eq. (6) as

$$\mathscr{E}\left[Y^m(\mathbf{X})\right] \simeq \mathscr{E}\left\{Y_a{}^m\right\} = \int_{-\infty}^{\infty} Y_a{}^m \cdot f_{\mathbf{X}}(\mathbf{x}) \cdot d\mathbf{x} \tag{6}$$

$$\text{where } Y_a = \sum_{j=1}^{N} Y(\mu_1,...,\mu_{j-1}, X_j, \mu_{j+1},...,\mu_N) - (N-1) \cdot Y(\mu_1,...,\mu_N)$$

Uncertainty of system responses can therefore be evaluated through multiple one-dimensional numerical integrations. The remaining challenge of the problem is how to carry out one-dimensional integration effectively. Using numerical integration, the one-dimensional integrations will be performed with integration weights $w_{j,i}$ and points $x_{j,i}$ using Eq.(7).

$$\mathscr{E}\left[\sum_{j=1}^{N} Y^m(\mu_1,...,\mu_{j-1}, X_j, \mu_{j+1},...,\mu_N)\right] \cong \sum_{j=1}^{N}\sum_{i=1}^{k-1} w_{j,i} Y^m(\mu_1,...,\mu_{j-1}, x_{j,i}, \mu_{j+1},...,\mu_N) \tag{7}$$

The number of integration points determines computational efficiency of the DR method. In general, the univariate DR method uses kN+1 integration points where N is the number of input random parameters and k is the integration points along each axis excluding the sample at the mean. It is suggested that k must be maintained at 2, or at most, 4, for large-scale engineering problems.

Using the DR method, three major disadvantages are inaccuracy, inefficiency, and/or singularity for nonlinear applications. For highly nonlinear problems, the use of 2N+1 or 4N+1 integration points is not sufficient enough to capture the true nature of the problem. Inaccuracy can be resolved via increasing the number of integration points. However, this increases computational cost substantially. The DR method suggests the use of a moment based quadrature rule. It requires only statistical information of the random input parameters and generates the integration points and weights for numerical integration. Unfortunately, the large amount of integration points to characterize a nonlinear problem requires high order statistical moments of the input parameters to be known. It has been shown in Ref. 3 that the use of high order statistical moments creates a singularity problem in determining the weights and integration points.

4.2 ENHANCED DIMENSION REDUCTION (EDR) METHOD

The DR method is enhanced by incorporating a more robust one-dimensional numerical integration scheme. It is referred to as the enhanced Dimension-Reduction (eDR) method [16]. Compared to the DR method, the eDR method increases the accuracy by using a stepwise moving least squares approximation. It generates approximate response values, $\hat{Y}(\mu_1,...,\mu_{j-1}, x_{j,i}, \mu_{j+1},...,\mu_N)$, at all integration points along each random input in Eq. (8). This approximate response allows the incorporation of any numerical integration method.

$$\begin{aligned}&\mathscr{E}\left[\sum_{j=1}^{N} Y^m(\mu_1,...,\mu_{j-1}, X_j, \mu_{j+1},...,\mu_N)\right] \\ &\cong \sum_{j=1}^{N}\sum_{i=1}^{n} w_{j,i} Y^m(\mu_1,...,\mu_{j-1}, x_{j,i}, \mu_{j+1},...,\mu_N) \\ &\cong \sum_{j=1}^{N}\sum_{i=1}^{n} w_{j,i} \hat{Y}^m(\mu_1,...,\mu_{j-1}, x_{j,i}, \mu_{j+1},...,\mu_N)\end{aligned} \tag{8}$$

To recover the disadvantages of the DR method due to a moment based quadrature rule, it is suggested in [16] that the eDR method use the adaptive Simpson rule as an alternative integration approach. This allows more flexibility for the eDR method to handle any distribution type encountered in practical engineering problems. This is possible since the stepwise moving least squares approximation produces highly accurate one-dimensional responses. Consequently, the number of integration points can be increased to as many as possible without evaluating actual system responses. Therefore, the eDR method turns out to be very efficient and accurate through the new one-dimensional integration scheme. The highest order of error in applying the eDR method is

$$\mathscr{E}\left\{Y^m(\mathbf{X})\right\} - \mathscr{E}\left[\sum_{j=1}^{N} Y^m(\mu_1,...,\mu_{j-1}, X_j, \mu_{j+1},...,\mu_N)\right] = \frac{1}{2!2!}\sum_{i<j}\frac{\partial^4 y}{\partial x_i^2 \partial x_j^2}(0) I\left[x_i^2 x_j^2\right] \quad (9)$$

The numerical procedure for the eDR method is as follows:

Step 1 Define a reasonable set of sample points to be used for the stepwise moving least squares approximation, usually 2N+1 or 4N+1, depending on available resources and any prior knowledge of system nonlinearity. It is suggested that for a 4N+1 sample size, μ, $\mu \pm 1\sigma$, and $\mu \pm 3\sigma$, should be used. However, if one parameter is known to be extremely non-linear, additional sample points for that variable should be performed. It should be noted that this does not require additional samples along the remaining dimensions. The dimensional reduction method does not require orthogonal arrays due to its one dimensional nature.

Step 2 Perform one-dimensional function approximations for all random input parameters using the stepwise moving least squares approximation.

Step 3 Perform numerical integration using the adaptive Simpson rule to calculate statistical moments for all approximate functions in Eq. (5)

Step 4 Create probability density (or distribution) function based on statistical moments using the modified Gamma distribution [17] or a Pearson system [18].

4.3 VERIFICATION OF THE EDR METHOD

To verify that the eDR method can handle nonlinear, multi-dimensional problems where input parameters are non-normally distributed, a vehicle side impact problem is considered for uncertainty propagation [5]. In fact, when all random inputs are set to normal distribution, the errors of statistical moments are maintained below 1%. So, in the vehicle side impact problem, the velocity of the door is considered with non-normal and skewed distributions for random inputs. An explicit relationship for the velocity of the door at the B-pillar of the car frame, V_d, has been determined [11] as

$$V_d = 0.75 - 0.489X_1X_4 - 0.843X_2X_3 + .0432X_5X_6 - .0556X_5X_7 - .000786X_7^2 \quad (10)$$

where all random variables are defined in Table 2. This problem encompasses three different distribution types; normal distributions, $N(\mu,\sigma)$, beta distributions, $\beta(\mu,\sigma,q,r)$, and uniform distributions, $U(L,U)$, where μ is the mean, σ is the standard deviation, q and r are the beta distribution parameters, L is the lower bound, and U is the upper bound. Both X_3 and X_4 are positively skewed, whereas others are symmetric.

Table 2 Distribution Parameters for Random Inputs

Random Variable		Parameters
X_1	the floor side inner	$N(1,.05)$
X_2	the door beam	$N(1,.05)$
X_3	the door belt line	$\beta(1,.05,5,36.44)$
X_4	the roof rail	$\beta(1,.05,5,36.44)$
X_5	Material property of door beam	$\beta(0.3,.006,5,36.44)$
X_6	material property of inner B-Pillar	$U(0,10)$
X_7	the barrier collision point width	$U(0,10)$

For this problem, the $4N+1$ sampling points are used to generate the stepwise moving least squares approximation and they are chosen using the moment-based quadrature rule. As shown in Fig. 2, it shows a good agreement of probability distribution for the velocity of the door at the B-pillar between the MCS and the eDR method. As well, Table 3 displays the resulting statistical information of the response from the eDR method as well as the Monte Carlo simulation with 100,000 samples. The skewness is quite small and thus the error in the skewness appears to be relatively large. But, it is negligible since the percentile error becomes highly susceptible to a small fluctuation. This is justified by the resulting histogram of the simulation and the PDF of the eDR (generated using the modified Gamma distribution), shown in Fig. 2.

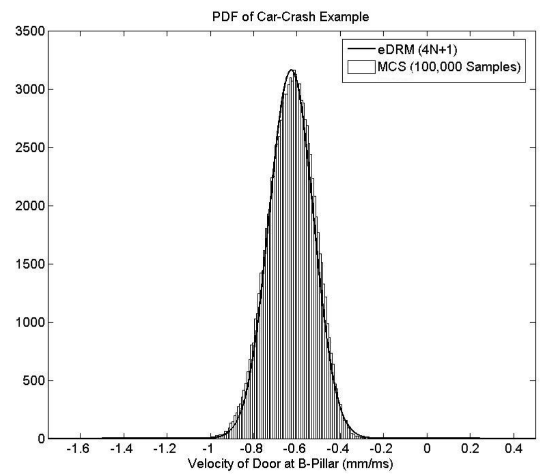


Fig. 2 Histogram (MCS) and PDF (eDR Method)

Table 3 Results Side Crash Example

Method	Mean	Std. Dev.	Skewness	Kurtosis
MCS	-0.6615	0.2331	-0.4364	2.7327
eDR	-0.6606	0.2328	-0.4380	2.7181
Error, %	0.1325	0.1307	0.3605	0.5327

5. CASE STUDY

The proposed methodology is illustrated with an example representing the assembly of a hood-pin bracket. As seen in Fig. 3, the bracket consists of the attachment element and the locating pin for the hood. The location of the pin is a critical dimension that determines the appearance (gap and flushness) and closure effort of the hood. The material of each component is a mild steel with young modulus E = 20,700 N/mm^2 and Poison's ratio υ = 0.3. The approximate length and height of this assembly is 150mm by 150mm.

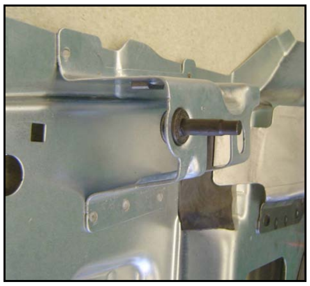


Fig. 3 Hood bracket

The ABAQUS model was developed to replicate the assembly process, as shown in Fig 4. The assembly of the bracket includes two components: the bracket itself including the locating pin for the hood and the fender. Three contact areas have been identified for this assembly. Each contact area defines a set of contact element pairs between the two components. These contact areas 1, 2 and 3 are circled in Fig. 4. Two additional contact areas are identified in the model; these areas correspond to the contact between the welding tool and the components. These contact pairs (4 and 5) are indicated by rectangles in Figure 4. The friction coefficient is assigned with value of 0.1. In order to improve the convergence of the model, a fine mesh is considered around the contact areas. The contact pairs are used to avoid the penetration between components and between a tool (clamp or welding gun) and the assembly components.

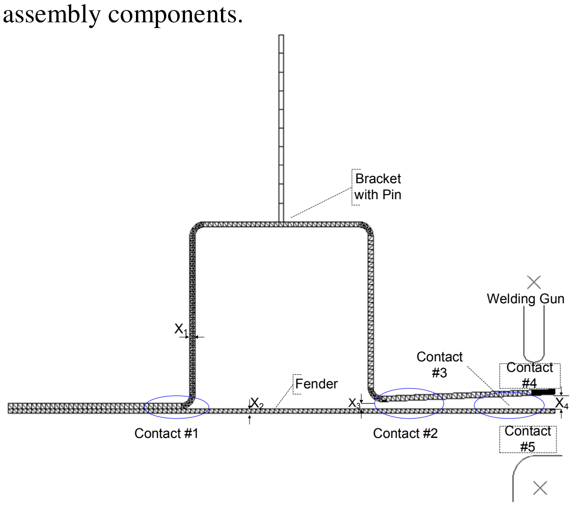


Fig. 4 Hood bracket assembly model

Four possible sources of variation have been simulated in the assembly. The sources of variation are represented as dimensional geometric errors in each component. As shown in Fig. 4, the sources of variation are: the material thickness of the bracket and fender, x1 and x2, respectively; the corner gap, x3, and the flange gap, x4. These dimensional errors are normally caused by the manufacturing process producing these two components. The FE model is parameterized to include these four variables., which are specified in Table 4. A Beta distribution is used to describe the random behavior of the input variables to avoid outliers in the distributions that may cause difficulty on the convergence of the finite element model. The key product characteristic in the bracket assembly is the angle and location of the hood pin. The objective of the analysis is to determine the angle change on the pin after assembly. Simulations are conducted to determine the statistical distribution and parameters that describe the random nature of the angle after assembly due to the input variation on the variables x_1, x_2, x_3 and x_4.

Table 4 Random Input Parameters

Random Variable	Physical Property	Distribu -tion	Mean (mm)	Range (mm)
x_1	Thickness Component 1	Beta	1.1	0.9~1.3
x_2	Thickness Component 2	Beta	1.0	0.7~1.3
x_3	Corner Gap	Beta	1.5	0.0~3.0
x_4	Flange Gap	Beta	3.0	0.0~6.0

The assembly process is simulated following the methodology presented in Fig. 1. First, the assembly components are located in the station using a set of locators. The displacement of the left flange on the bracket is constrained in all three Degree of Freedoms (DOFs) (t_x, t_y, and r_{xy} in 2D plane model) where t_x, t_y, and r_{xy} are translational displacements along horizontal and vertical axes and rotational displacement on the 2D plane, respectively. The fender is located using two locators at each extreme constraining the three DOFs. The assembly simulation begins approaching the lower electrode of the welding gun to the fender and moving the upper electrode downward in order to close the gap between the right ends of the two components. After the right end of components has been deformed, the vertical distance between the welding gun tools is maintained at the combined thickness of the two parts (e.g., x_1+x_2). Then, both components are joined together using the bonded contact property and the connector elements in the welding area not allowing any separation. Finally, the welding gun and the fixture at the right end of fender are removed, which results in the assembly springback. Figure 5 shows the change in the hood-pin angle before and after assembly. As can be seen in the figure, the angle of the pin does not recover its nominal position due to the new assembly constraints.

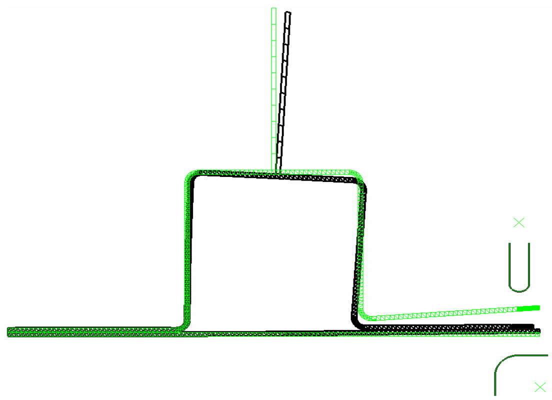
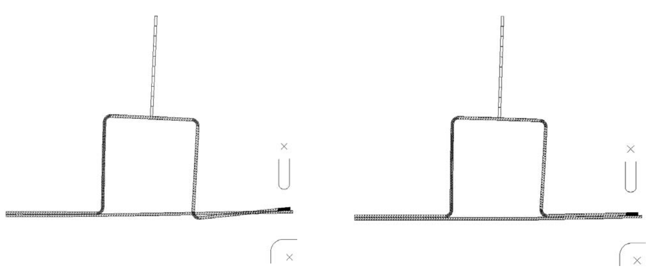


Fig. 5 Finite element results from before and after assembly

5.1 RESULTS FROM THE NON-CONTACT MODEL AND CONTACT MODEL

In order to compare the results from the new contact assembly methodology with respect to the traditional linear non-contact assembly modeling, an MCS without considering contact between the components or tools was conducted. The difference of the effect between the non-contact model and contact model is showed graphically in Fig. 6. As shown in the figure, the main limitation of the linear assembly modeling is the penetration of the bracket into the fender.. Figure 6a shows the expected results using traditional MIC. As can be seen, the components penetration remains after assembly. Even that this condition is physically impossible, these results are common in assembly modeling. The proposed methodology result for contact assemblies is presented in Fig. 6b. Penetration between components is eliminated. This solution provides a closer estimation of the physical phenomenon.

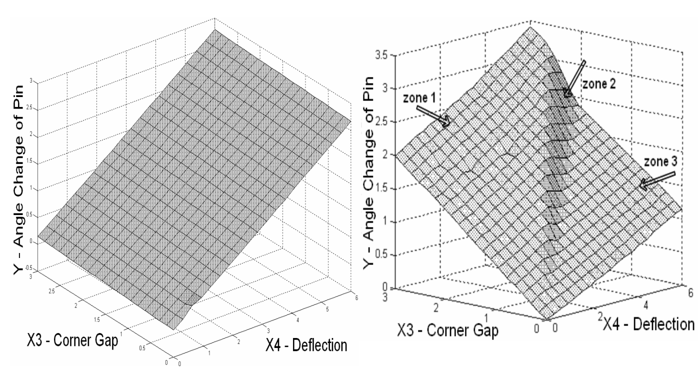


a) Non-Contact Model b) Contact Linear Model

Fig. 6 Finite element results after assembly, non-Contact and contact linear models

5.2 NON-LINEAR CONTACT MODELING RESULTS

Using the eDR method and the predictive non-linear contact assembly model, the assembly of the hood-bracket was studied. The relationship between the input variables x_i, shown in Table 4, and output variable y, the hood-pin angle, was studied. First, the non-linearity of the assembly response was analyzed. After an initial analysis, the variables x_3 and x_4 were identified to be significant to the response non-linearity. In other words, the pin angle is more sensitive to changes on the shape of the bracket (parameters x_3 and x_4) than to changes in the thicknesses of the components (parameters x_1 and x_2). The non-linearity relation between the pin angle and the variables x_3 and x_4, was investigated by building a response surface with multiple simulation runs, as shown in Fig. 7. For this study, two cases were considered: compliant assemblies with and without contact considerations. In the case of the linear model without contact, 15 points equally distributed along each range of x_3 and x_4 were sampled, while keeping x_1 and x_2 at their mean values. The pin angle was determined for the 225 data points. Figure 7a shows the results for the non-contact model results. As can be seen, the angle of the pin is independent of the variable x_3. This can be explained because any change in the variable x_4 only increases the penetration between the components without affecting the springback of the assembly. In contrast, the response surface for the contact model was built with 20 equally spaced samples along x3 and x4 range. So, a total of 400 FE analyses were conducted to study the non-linearity of the pin-angle. Figure 7b shows the nonlinear relation between the input variables x_3, x_4 and the output variable y for the contact model. Three zones can be identified in the figure. First, Zone 1 corresponds to the cases where the two components never become in contact, except for the welding area. Zone 2 corresponds to the cases where the two components experience a weak contact. Weak contact means that the contact is observed as the corner section of upper component pushes the bottom component down, however no significant component deformation occurs. Finally, Zone 3 corresponds to the case when the components significantly deform during the assembly process due to the component interference. During this interaction the corner section of upper component first pushes the bottom component down until reach the most deformation, and then the corner section of upper component is forced to move to left along the top surface of bottom component to compensate the deformation on the flange.



a) Non-Contact Model b) Contact Model

Fig. 7 Angle Response with respect to variables x_3 and x_4

5.3 PIN ANGLE PREDICTION USING EDR METHOD AND MONTE-CARLO SIMULATION

Direct MCS is performed by artificially generating a set of random numbers (5,000 sample size) for variables x_1, x_2 , x_3 and x_4. Those variables are assumed to follow an independent Beta distribution with random properties in Table 4. The sample size is chosen based on the following criteria: accuracy and efficiency of the predictive model. The predicted model will be compared between the MCS and the eDR method in terms of the PDF and statistical moments.

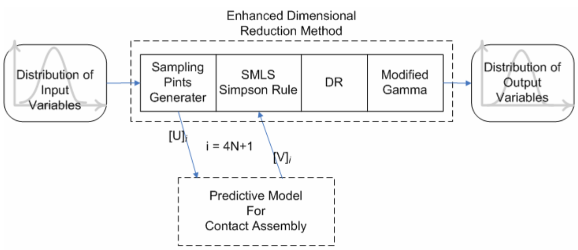


Fig. 8 The eDR Method with predictive contact model

The flow chart for the enhanced Dimensional Reduction Method combined with Predictive Model for Contact Assembly is showed in Figure 8.

Table 5 4*N*+1 Sampling points for the eDR Method

X_1	X_2	X_3	X_4	*Y*
0.9	1	1.5	3	1.8054
1	1	1.5	3	1.7514
1.2	1	1.5	3	1.6357
1.3	1	1.5	3	1.582
1.1	0.7	1.5	3	1.4966
1.1	0.85	1.5	3	1.6002
1.1	1.15	1.5	3	1.7696
1.1	1.3	1.5	3	1.8276
1.1	1	0	3	0.54317
1.1	1	0.75	3	0.99872
1.1	1	2.25	3	2.2272
1.1	1	3	3	2.6692
1.1	1	1.5	3	1.6933
1.1	1	1.5	0	1.0414
1.1	1	1.5	1.5	1.3444
1.1	1	1.5	4.5	1.7431
1.1	1	1.5	6	2.0324

The eDR method used 4*N*+1 sampling points to predict random behavior of the pin angle, where $N = 4$, generating a total of 17 sample points, which are evaluated using the FE analysis. The sample points are chosen by the advised method discussed in Step 1 of the eDR method. These sample points are listed in Table 5. While employing the eDR method for uncertainty propagation of the pin angle, one dimensional response approximation must be performed along each random variable using the SMLS method. To show the accuracy of the stepwise moving least square method, the response with respect to x_4 and 25 FE analyses, where x_1, x_2 and x_3 remain constant are shown in along Fig 9. As can be seen the stepwise moving least squares method accurately approximates the actually relationship. The response with respect to x_4 is the most non-linear of the four input parameters, therefore showing that all four responses can be accurately approximated with this method.

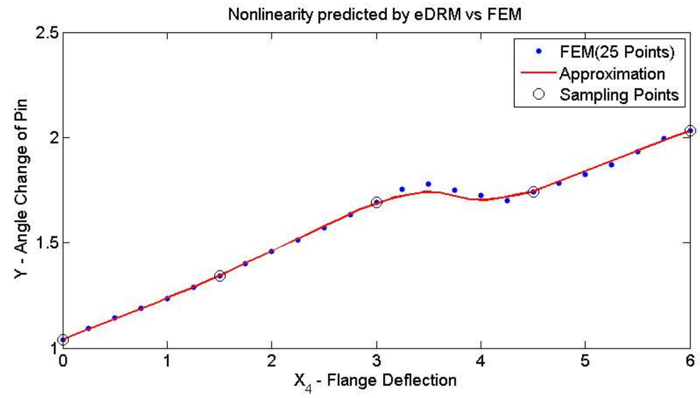


Fig. 9 Nonlinearity prediction using Stepwise Moving Least Squares method

As shown in Table 6, the eDR method estimated random behavior of the pin angle accurately and efficiently, compared to the MCS. It is also verified from the PDF approximation, as shown in Fig. 10.

Table 6 Comparison of the methods

Model	No Contact	With Contact		
Method	MCS	eDRM	MCS	Error (%)
Mean(mm)	1.3653	1.5811	1.6028	1.3539
Std. Dev.	0.4577	0.4402	0.4423	0.4771
No of FEAs	5,000	17	5,000	
CPU Time (minutes)	7,500	34	10,000	

The compared results of non-contact and contact models are showed in Table 6. MCS was used for the non-contact model to predict the output distribution. For the contact model, MCS along with the eDR method were used to approximate the output uncertainty. The error percentage between the solutions of the MCS and the eDR method for the contact model is also shown in the table. It is evident that the eDR makes a good agreement in the mean and standard deviation with the MCS. As shown in Fig. 7, the pin angle response must hold highly nonlinear interactions between x_3 and x_4, since the stepwise response runs along the diagonal direction of x_3 and x_4. According to the error analysis of the eDR method, numerical error can be accumulated from bi-quadratic terms or higher. It is thus obvious that numerical error is mainly due to highly nonlinear interaction between x_3 and x_4. As well, it is expected that the MCS yield a minor degree of the error due to a finite number of samples.

As shown in Fig. 10, there is a good correlation between the histogram from the MCS and the PDF from the eDR method. So, this case study shows that the eDR method produces an excellent estimate of the uncertainty propagation of the highly nonlinear assembly process.

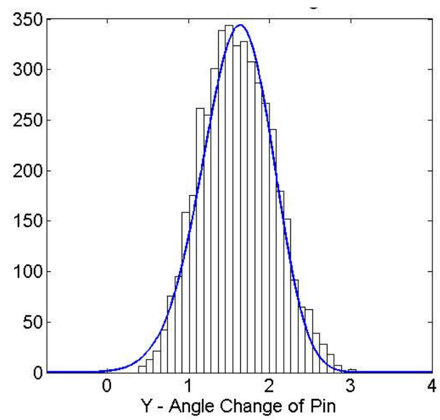


Figure 10 MCS Histogram and eDR Method PDF

6. CONCLUSIONS

This paper presents a new methodology for variation propagation modeling on compliant assemblies that includes the contact effect between components and assembly tools (fixtures and welding tools). The methodology is based on finite element method. A parametric model is used in order to incorporate the input variation from different variables. In addition, several elements to improve finite element convergence were implemented. The new model response from contact assembly is non-linear; therefore, the traditional sensitivity analysis is not adequate to estimate the statistical response of the characteristics of the assembly. In order to improve the efficiency of the methodology compared with MCS methods, the eDR method is used to sample and estimate the statistical response of the system. A case study is presented for the assembly of an automotive hood-bracket. The proposed methodology combined with eDRM produces an excellent estimate of the uncertainty propagation on highly nonlinear assembly processes.

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