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A Continuum Dislocation Theory Model for Predicting the Wall Thickness Changes of Hollow Drawn Tubing[1]

An expression, based on some concepts of continuum dislocation theory, is developed for predicting the wall thickness changes in hollow drawn tubing. This expression predicts the observed wall thickening and thinning behavior and incorporates the parametric influences on the changes of wall thickness.

Introduction

THE application of continuum dislocation theory to plasticity problems has been limited. The only analysis which has been treated in detail is the indentation or punch problem [1].[2]

In this paper a theoretical expression is advanced, based on the concepts of a continuous distribution of dislocations, for predicting the wall thickness changes associated with hollow tube drawing. Classical interpretation of this mode of tube drawing has been only moderately successful for "thin-wall" tubes. Even then, certain discrepancies between theory and experiment are observed. The analysis to follow will be examined in terms of the experimental observations previously reported by the author [2].

Two theoretical analyses, by Swift [3] and Moore and Wallace [4] have endeavored to predict the wall thickness changes of hollow drawn tubes. Particular attention was given to the increase in wall thickness observed during the early stages of the reduction sequence followed by wall thinning at the heavier reductions. Swift's theory was based entirely on "thin-wall" tubing assumptions. Moore and Wallace, however, extended Swift's analysis by including the effect of γ_0 on the wall thickness behavior, where γ_0 is defined as the ratio of the initial wall thickness to the initial outer radius.

There are two facets which the Moore-Wallace theory cannot explain. Their analysis results in a differential slope solution which predicts that the wall thickness must increase during the initial stages of deformation or the reduction sequence. While their analysis was based primarily on "thin-wall" considerations, experimental evidence [2] for "thick-wall" tubes of a number of materials has shown various degrees of the wall thinning at all stages of reduction. A more general theory would have to predict both the wall thickening and thinning behavior. Secondly, the Moore-Wallace theory predicts, for an infinitely thin-wall tube ($\gamma_0 \simeq 0$) that the wall increases and then commences to thin over a reduction range of approximately 0.5. However, experimental observations on thin-wall tubes [2] suggest that thickening continues until reductions near 100 percent are attained.

Both theories take into consideration the influence of die-angle and work hardening on the wall thickness changes. Experimental evidence [2] indicates that a greater wall thickening tendency occurs by decreasing the die-angle. A similar trend is observed for a material with a larger work hardening capacity. Both theories do not agree with experiment in terms of the die-angle, for they predict a wall thickening trend with increasing die-angle. As for the influence of work hardening, the Moore-Wallace theory indicates trends which agree with experiment, whereas Swift's theory is not in agreement.

[1] Work performed under the auspices of the United States Atomic Energy Commission.

[2] Numbers in brackets designate References at end of paper.

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Nomenclature

γ_0, γ = initial and final ratio of wall thickness to outer radius
t_0, t = initial and final wall thickness
a_0, a = initial and final inner radius
b_0, b = initial and final outer radius
l_0, l = initial and final length
$\alpha_{hi}, \alpha_{z\theta}$ = dislocation density tensors
ϵ_{hkl} = permutation tensor
e_{nk} = elastic strain tensor
$e_{ki}^*, e_\theta^*, e_r^*$ = plastic strain tensors
ω_{ki}^* = plastic rotation tensor
k_0 = geometry and parametric constant
B = parametric constant
β = semidie angle
n = work-hardening function

The theoretical analysis to be presented incorporates the parametric influences on the wall thickness changes for the tube over the entire tubing range of $0 \leq \gamma_0 \leq 1$. There are several aspects of the hollow drawing process which makes the application of continuum dislocation theory seem very promising. One of the advantages is that the dislocation density distribution is a state quantity. This property is independent of the path used to reach a final reduction.

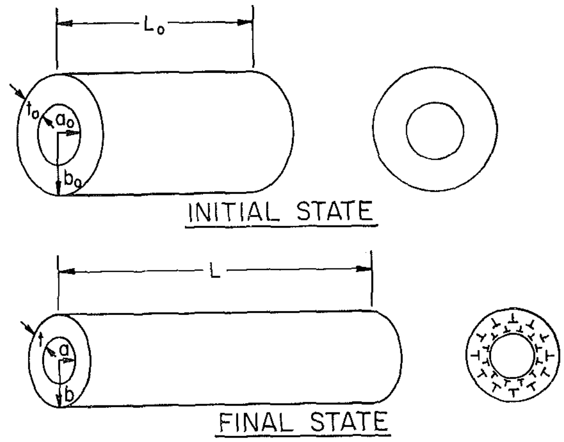

Fig. 1 The initial and final states of a hollow drawn tube

Fundamental Relations

The deformation produced in hollow drawing is indicated in Fig. 1. In this communication an attempt is made to find a theoretical relationship between the change in wall thickness $(t - t_0)/t$ and the reduction, $(b_0 - b)/b_0$ for various values of $\gamma_0 = t_0/b_0$, where t_0, t, b_0, and b are the initial and final wall thickness and outer radius, respectively. The material is assumed to be incompressible, and constancy of volume is expressed by

$$\pi(b_0^2 - a_0^2)l_0 = \pi(b^2 - a^2)l, \tag{1}$$

or

$$\log (l/l_0) + \log \frac{2b - t}{2b_0 - t_0} + \log t/t_0 = 0. \tag{2}$$

In order to obtain a relationship between wall thickness and outer radius it is necessary to examine some of the fundamental concepts of continuum dislocation theory. The dislocation density tensor α_{hi} is given by Kröner [5] and Mura [6] as

$$\alpha_{hi} = -\epsilon_{hlk}e_{ki,l}^* - \epsilon_{hlk}\omega_{ki,l}^*, \tag{3}$$

where ϵ_{hlk} is the permutation tensor and e_{ki}^*, ω_{ki}^* are the plastic strain and plastic rotation, respectively. Mura [7] has shown that the plastic rotation is zero and the rotation is simply a lattice rotation for dislocation distributions in deformed cylindrical geometries. Thus

$$\omega_{ki}^* = 0. \tag{4}$$

Then expression (3) becomes, in terms of cylindrical coordinates,

$$\alpha_{z\theta} = -\left(\frac{de_\theta^*}{dr} + \frac{e_\theta^* - e_r^*}{r}\right), \tag{5}$$

where $\alpha_{z\theta}$ is the density of dislocations whose line direction is parallel to the z-axis of the cylinder and which have the Burger's vector in the θ direction. Kröner [8] and Mura [6] also give the relation

$$(1/2)\epsilon_{ijk}\alpha_{hk,j} + (1/2)\epsilon_{hmn}\alpha_{in,m} = \epsilon_{hmn}\epsilon_{ijk}e_{nk,jm}, \tag{6}$$

where e_{nk} is the elastic strain. It is assumed that a state of large plastic strain exists and that the elastic contributions are negligible. The right-hand side of (6) is then equal to zero and, as a result, the expression in terms of cylindrical coordinates becomes

$$\frac{1}{r}\frac{d}{dr}(r\alpha_{z\theta}) = 0. \tag{7}$$

Solution

The solution of the differential equation (7) has the form

$$\alpha_{z\theta} = K_0/r, \tag{8}$$

where K_0 is a constant of integration. From the boundary condition that there is no net dislocation distribution (random array) for the initial state prior to deformation, the constant may be chosen as

$$K_0 = k_0 \log (l/l_0), \tag{9}$$

which yields for (8)

$$\alpha_{z\theta} = (k_0/r) \log (l/l_0). \tag{10}$$

The hoop, e_θ^*, and radial, e_r^*, strains are assumed to be represented in (5) as mean strains. Therefore, from (2)

$$e_\theta^* = \log \frac{2b - t}{2b_0 - t_0}, \qquad e_r^* = \log t/t_0. \tag{11}$$

A substitution of (10) and (11) into (5) leads to the relation

$$k_0 \log (l/l_0) = \log (t/t_0) - \log \frac{2b - t}{2b_0 - t_0}. \tag{12}$$

The log l/l_0 term of (12) can be eliminated by (2), and the final expression for the wall thickness, as a function of reduction, γ_0 and k_0, becomes

$$1 - \bar{b} = 1 - (\gamma_0/2)\bar{t} - (1 - \gamma_0/2)(\bar{t})^{\frac{1+k_0}{1-k_0}}, \tag{13}$$

where:

$$\bar{b} = b/b_0$$
$$\bar{t} = t/t_0$$
$$\gamma_0 = t_0/b_0.$$

Analysis of k_0

The foregoing analysis represents the status of the hollow tube drawing process from continuum dislocation theory concepts. For further evaluation, say of k_0, one must resort to experimental observations. The term k_0, given in (13), accounts for the parametric effects on the wall thickness changes. Since the influential parameters are the ratio of the initial wall thickness to initial outer radius (γ_0), die angle (β), and apparent work hardening capacity (n), one would expect that

$$k_0 = f(\gamma_0, \beta, n). \tag{14}$$

This relation can be simplified by holding β and n constant and considering the dimensional limits and extremum characteristics of a tube. For the limiting conditions it has been determined [2] that

$$\bar{b} = \bar{t} \text{ for rods } (\gamma_0 = 1) \text{ and}$$
$$\bar{b} = (\bar{t})^{-2} \text{ for infinitely thin-wall tubes } (\gamma_0 = 0). \tag{15}$$

An examination of (13) in view of these limits discloses that $k_0 = 0$ for $\gamma_0 = 1$ and $k_0 = 3$ for $\gamma_0 = 0$. In conjunction with these limits one has the extremum condition of constancy of wall, i.e., the wall thickness remains constant during the drawing process. If a linear function is assumed between k_0 and γ_0 within the above limits, the γ_0 for a tube whose wall remains virtually constant during reduction is $2/3$. This condition corresponds to $k_0 = 1$. Experiment has shown [2] that the γ_0 values observed for this behavior lies within 0.15 to 0.40, depending on the die-

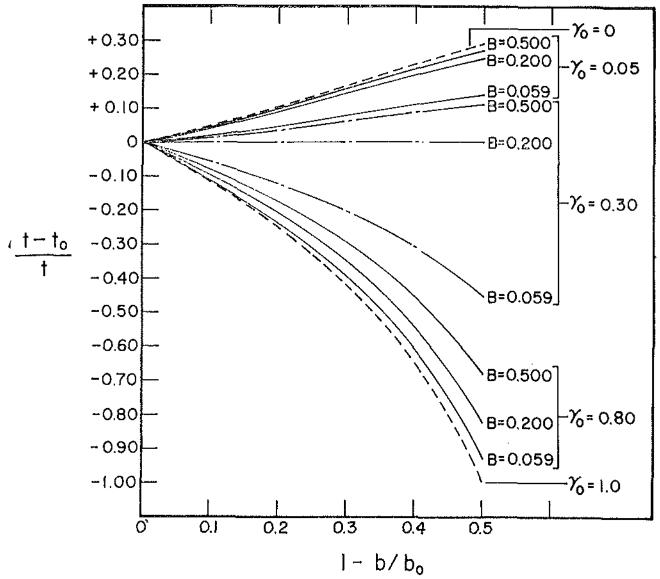

Fig. 2 The effect of B on the wall thickness changes with reduction for various γ_0 values

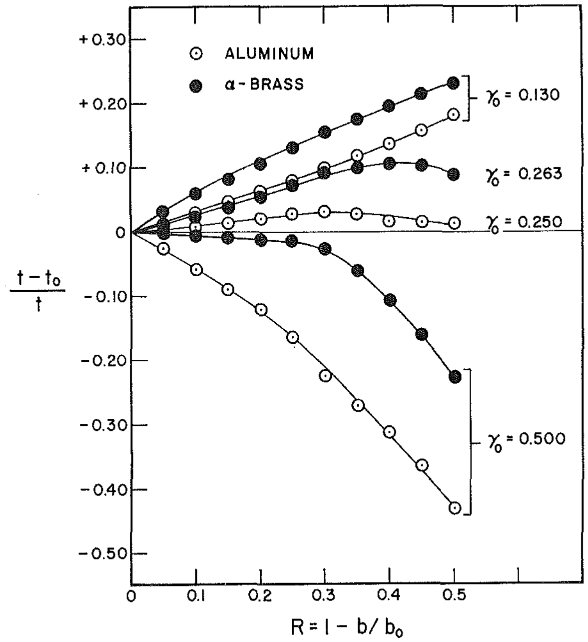

Fig. 4 The wall thickness changes as a function of reduction for 1100S aluminum and α-brass tubes

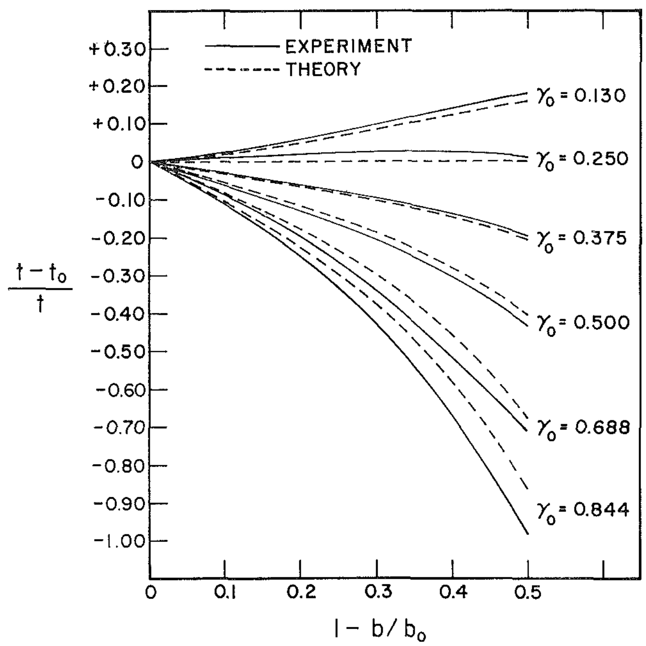

Fig. 3 Comparison of theory and experiment (1100S aluminum) for the wall thickness changes with reduction

angle and the tube material. As a result, the simplest envisage function for k_0 as function of γ_0 would be

$$k_0 = 3B\left(\frac{1 + B}{\gamma_0 + B} - 1\right). \tag{16}$$

The parameter, B, would then correspond to a tube with a γ_0 value for which the extremum condition (constancy of wall) is obeyed, and would account for the remaining parameters, β and n. For selected values of B, the change in wall thickness as a function of reduction is shown in Fig. 2 for a range of γ_0 values. It is apparent that as B increases there is a greater tendency for the tube wall to thicken.

Comparison With Experiment

As was mentioned previously, there are three parameters which affect wall thickness changes during reduction. These are γ_0, die-angle, and possibly the material's work hardening capability. The latter two parameters determine the value for B. Unfortunately, theory does not, at this time, provide a reasonable estimate of this parametric constant and one must turn to experiment for its determination.

A comparison between experiment and prediction from equation (13) is shown in Fig. 3. The experimental results are on 1100S Al for which a value of B $\approx$ 0.2 was observed [2]. If this value for B is used in equation (16) which in turn is substituted into equation (13), the dashed curves are obtained for equivalent γ_0 values. The agreement between the predicted and experimental results is satisfactory. Such agreement is not afforded from classical predictions.

Experimental observations showing the apparent effect of the material's work hardening capability on the wall thickness changes are shown in Fig. 4 for α-brass and 1100S aluminum tubing series. The flow curves for both materials can be empirically described by a power form and the work hardening exponent for α-brass is twice that for 1100S aluminum. From the theory the changes shown in Fig. 4 follow expectation. A material which exhibits a larger capacity to work harden would be expected to possess a higher dislocation density than one with less tendency for work hardening. From equation (10) a large dislocation density corresponds to a large k_0 value, and from (16) a corresponding large value of B results. Consequently, a material with a high work hardening capacity such as the α-brass series would be expected to exhibit a greater wall thickening tendency.

To examine the effect of work hardening and die-angle from continuum dislocation theory appears to require an analysis of the dislocation distribution along planes parallel (longitudinal planes) to the tube axis. The analysis would involve distributions associated with the bending (entering the die) and unbending (exit of the die) in conjunction with the work hardening occurring in the reduction process.

Discussion

The classical theories for hollow-tube drawing have resulted in an over-emphasis of the wall thickening and then thinning behavior that has been predicted for a reduction range of approximately 50 percent. This behavior has been experimentally observed over a narrow γ_0 range. The magnitude of the wall thickness changes for this range is small and nearly correspond to the extremum point, $\Delta t \approx 0$. The continuum dislocation model does not predict a thickening then thinning behavior, but in view of the experimental evidence for the wall thickness changes, the theory is sufficient.

The expression given by (13) will adequately describe the wall thickness changes once the value for the parameter, B, has been ascertained. Further experimental effort is needed for establishing empirical relationships between B and β or B and n. This would yield semiempirical relations for (16) and hence (13), which could prove useful in tube forming schedules where hollow drawing is an important part of the process.

Acknowledgment

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