Image Compression using Phase Diversity of Synthesis Filters

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Abstract

This paper presents enhancement of image reconstruction by comprehensively exploiting the phase diversity of adaptive synthesis filter bank. A novel approach of selection of synthesis delay filters in various reconstruction groups and possible placing configurations of optimal groups in adaptive synthesis filter bank have been explored. In contrast with linear phase synthesis filters, synthesis section is built with a set of filters, where each filter has different phase characteristics. Such adaptive formation of synthesis filter bank when employed in image compression shows optimal improvements in MSE/ PSNR over conventional filter bank.

Keywords: *Synthesis delay filters; reconstruction groups; phase diversity; placing configuration.*

1. Introduction

Filter banks and wavelets have attracted a lot of attention in recent years [1–2]. These systems are constructed from two filter banks: an analysis filter bank and a synthesis filter bank. The analysis filter bank divides an input signal into subband coefficients and, on the other hand, the synthesis filter bank produces an output signal from subband coefficients using sets of digital filters. They have many applications in audio/video compression, signal analysis, and communications. One of the significant applications is subband (or transform) coding of images. In subband image coding, these multirate systems are used for generating transform coefficients to be efficiently quantized and entropy coded. In the decoder, the reconstructed image is obtained via the inverse systems.

For image compression applications**,** quantization is usually applied to the subband signals, which causes quantization errors to occur in the decoded image. With linear filters, these errors are most visible around sharp edges, where they appear as ringing artifacts. Even with linear filters designed to reduce ringing, there is still significant noise around sharp edges [3].

An interesting variation of the analysis-synthesis filter bank was pioneered by Nayebi et al. [4], where the constituent filters were allowed to be adaptive. It was shown that exact reconstruction could be achieved in an adaptive environment by having the analysis filters and synthesis filters change in accordance with a set of reconstruction/design conditions. The formulation was later refined by Sodagar et al. [5], where a post filter was introduced that made design and implementation much simpler. One of the key issues of this approach, however, is that the synthesis filters must be changed in lock step with the analysis filters in order to perform reconstruction, which requires dynamic synchronization. This issue was explored by Arrowood et al. [6]. Both forward and adaptive methods of synchronous adaptive analysis-synthesis were reported.

Lettsome [7] developed an analysis-synthesis system with a non-adaptive analysis section and an adaptive synthesis section and eliminated the issue of maintaining synchrony, which further simplified the operation of the filter bank. The synthesis filters were used to exploit the phase diversity in the synthesis section by means of a combination of minimum, maximum and linear phase synthesis filters. These filters were used in only first level of subband structure by using all the possible reconstruction combinations. It was shown that exact reconstruction could be possible and at the same time reconstruction quality could be enhanced.

Adaptive synthesis filter banks [8] explored a comprehensive approach to exploit the phase diversity of the synthesis section, where a large number of synthesis filters were efficiently handled and a method to reduce the computation complexity of the synthesis filter bank was discussed. The synthesis delay filters were optimized in order to minimize the reconstruction error and deployed in first three levels of subband structure and linear phase filters were used in the remaining levels. The handling of large number of synthesis delay filters was discussed by grouping them along with linear phase filters $\mathbf{S_p}$ as

$$\mathbf{G_1 = S_1, S_m, S_p}$$

$$\mathbf{G_2 = S_2, S_{m-1}, S_p}$$

$$\mathbf{G_3 = S_3, S_{m-2}, S_p}$$

$$\vdots \qquad \vdots$$

$$\mathbf{G_{m/2} = S_{m/2}, S_{(m/2)+1}, S_p}$$

where, $\mathbf{S_1, S_2, S_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots S_{m-1}, S_m}$ were the delay filters. The placing of the delay filters was accomplished before the linear phase filters by keeping the delay filters of the groups

$$\mathbf{G_{m/2}, G_{(m/2)-1}, G_{(m/2)-2}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots G_1}$$

at $\mathbf{1^{st}, 2^{nd}, 3^{rd}, \ldots\ldots\ldots\ldots\ldots\ldots m/2}$ positions respectively in the synthesis filter bank. But how the selection of the two delay filters in each group was made and why the above placing configuration used, had not been explained.

In this paper, we discuss the selection of the delay filters in various reconstruction groups of adaptive synthesis filter bank in detail by using optimal phase diversity of synthesis filters. We consider a design example of an adaptive synthesis filter bank comprised of four delay filters along with linear phase filters, which can be extended to filter banks composed of large number of delay filters. The four delay filters are divided into six possible reconstruction groups, where each reconstruction group is composed of two delay filters along with linear phase filters. We also describe all the possible placing configurations of the optimal reconstruction groups and deploy them only in first three levels of subband structure.

2. Adaptive Synthesis Filter bank

In adaptive synthesis filter bank, we have designed four synthesis delay filters along with linear phase filters represented by $\mathbf{S_a, S_b, S_c, S_d}$ and $\mathbf{S_p}$ respectively to exploit phase diversity in synthesis section. Odd length filters have been considered because even length filters exhibit fractional delay in compression.

2.1 Block Diagram

The block diagram of the design example is shown in Fig 1. The analysis section is the conventional one, while the synthesis reconstruction filters are switched adaptively and selectively in the reconstruction process with minimum reconstruction error on pixel by pixel basis.

2.2 Design Procedure

If 'L' is the length of given analysis filters, we can design 'L − 2' synthesis filters by using the following time domain equation

$$\mathbf{A = SB}$$

where, **A** is a block Toeplitz matrix of analysis filter coefficients, **S** is a matrix of synthesis filter coefficients, and **B** is the reconstruction matrix. In a more expanded form, above equation can be expressed as

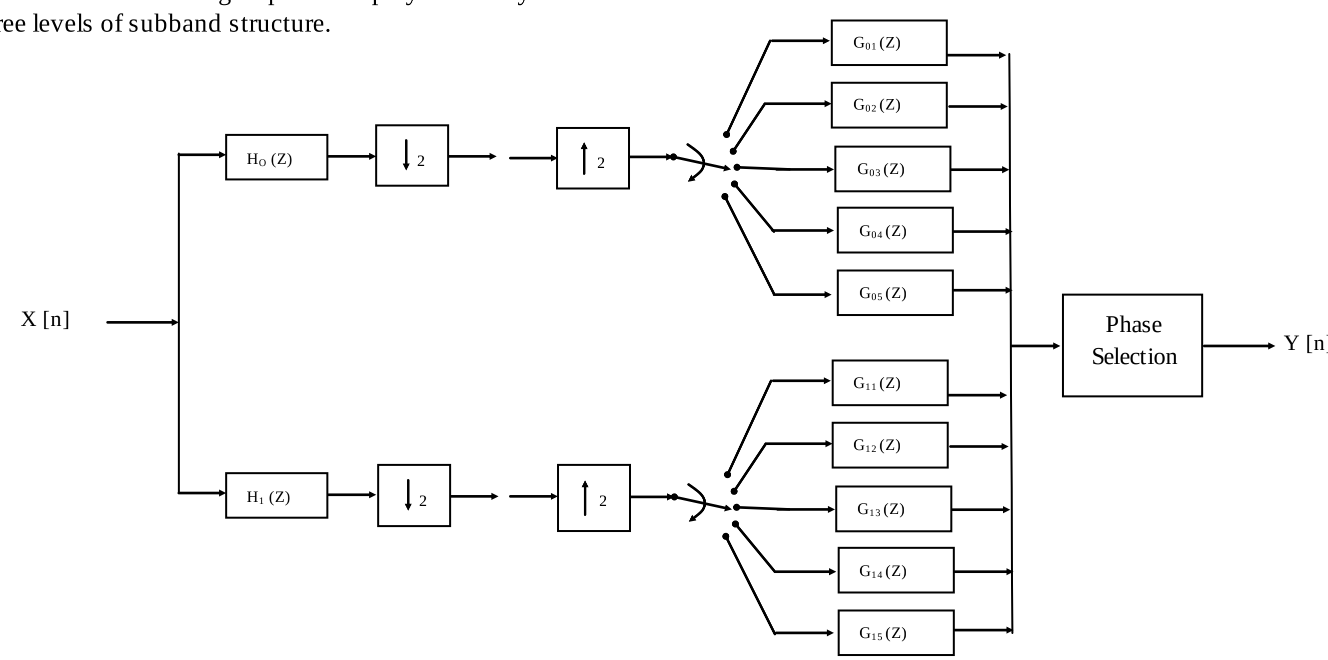

Figure 1. Block diagram conventional analysis and adaptive synthesis filter bank

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$$\underbrace{\begin{bmatrix} \mathbf{P}_0^T & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{P}_1^T & \mathbf{P}_0^T & \vdots & \mathbf{0} & \mathbf{0} \\ \mathbf{P}_2^T & \mathbf{P}_1^T & \vdots & \vdots & \\ \mathbf{P}_3^T & \mathbf{P}_2^T & \ddots & \ddots & \\ \vdots & \vdots & \ddots & \ddots & \\ \mathbf{P}_{K-1}^T & \mathbf{P}_{K-2}^T & \dots & \mathbf{P}_0^T & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{K-1}^T & \ddots & \mathbf{P}_1^T & h_0(0) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_{K-1}^T & h_0(L-1) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \mathbf{Q}_0 \\ \mathbf{Q}_1 \\ \vdots \\ \mathbf{Q}_{K-2} \\ \mathbf{Q}_{K-1} \end{bmatrix}}_{\mathbf{S}} = \underbrace{\begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{J}_R \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}}_{\mathbf{B}}. \tag{1}$$

where, **K** is the length of longer filter and shorter filter is adjusted to **K** by zero padding at the back end. The length parameter of **P** and **Q** in the above equation is set to **K**. So **A** and **S** are defined in terms of

$$\mathbf{P} = [\mathbf{P}_0|\mathbf{P}_1|\dots|\mathbf{P}_{K-1}]$$

and

$$\mathbf{Q} = [\mathbf{Q}_0|\mathbf{Q}_1|\dots|\mathbf{Q}_{K-1}].$$

The submatrices of **P** are defined as

$$\mathbf{P}_i^T = [h_0(i)h_1(i)]$$

where $h_0(i)$ and $h_1(i)$ represent the lowpass and highpass analysis filters and the **Q** matrices contain the lowpass and highpass synthesis filter coefficients. The **Q** matrices are given by

$$\mathbf{Q}_0 = [g_0(0)g_0(1)],$$

$$\mathbf{Q}_1 = [g_1(0)g_1(1)],$$

$$\mathbf{Q}_2 = [g_0(2)g_0(3)],$$

$$\mathbf{Q}_3 = [g_1(2)g_1(3)],$$

and so on until all the synthesis filter coefficients are included. Finally, $\mathbf{J_R}$ in reconstruction matrix **B** defined as

$$\mathbf{J}_R = \begin{bmatrix} 0 & 0 & \dots & 1 \\ 0 & & \cdot & 0 \\ \vdots & \cdot & & 0 \\ 1 & 0 & \dots & 0 \end{bmatrix}.$$

The position of $\mathbf{J_R}$ in the reconstruction matrix **B** controls the phase characteristics of the synthesis filters. Given a desired sample delay, "$\mathbf{J_R}$" is positioned in the (n −1) location of matrix **B** where "n" is the desired system delay.

For the general odd-length case, if **R** is decimation factor and **M** is number of bands then dimensions of matrices **A**, **S** and **B** will be 2K – R – (K mod R) rows and rnd(MK/R) columns, rnd(MK/R) rows and R columns and 2K – R – (K mod R) by R respectively.

Synthesis filter coefficients are computed by using equation (1) in the form

$$\mathbf{S} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{B}.$$

and reconstruction error is calculated by using equation

$$\epsilon_r = \|\mathbf{AS} - \mathbf{B}\|_F^2$$

This reconstruction error of these filters can be minimized by optimizing synthesis filter coefficients. For low pass filters, the sum of the odd coefficients and the sum of the even coefficients both are made approximately equal to 0.7071. Similarly, for high pass filters, the sum of the odd coefficients and the sum of the even coefficients are made approximately equal to 0.7071 and -0.7071 respectively.

The four delay filters are divided into six possible reconstruction groups along with linear phase filters as

$$\mathbf{G_1 = S_a, S_b, S_p}$$

$$\mathbf{G_2 = S_a, S_c, S_p}$$

$$\mathbf{G_3 = S_a, S_d, S_p}$$

$$\mathbf{G_4 = S_b, S_c, S_p}$$

$$\mathbf{G_5 = S_b, S_d, S_p}$$

$$\mathbf{G_6 = S_c, S_d, S_p}$$

where, $\mathbf{S_p}$ denotes linear phase synthesis filters.

In the reconstruction process synthesis filters are selected in an adaptive mechanism i.e. at edges delay filters are used and in the smooth regions, reconstruction is accomplished by linear phase filters. The selection is based on comparison of three outputs of each group on pixel by pixel basis. Let $y_o(i)$, $y_n(i)$ and $y_h(i)$ are the outputs of $\mathbf{S_a}$, $\mathbf{S_p}$ and $\mathbf{S_b}$ respectively, then if

$$\mathbf{y_o(i) \approx y_n(i)}$$

and $y_h(i)$ is approximately not equal to $y_n(i)$, then reconstruction is made by $\mathbf{S_a}$. Similarly, if

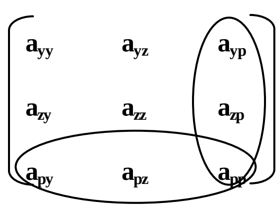
$$\mathbf{y_h(i) \approx y_n(i)}$$

Table 1: Contributions of six reconstruction groups in image enhancement for compression ratio = 0.5 bits per pixel.

Image	Conventional Synthesis Filters		Adaptive Synthesis Filters											
			$G_1 = S_a, S_b$		$G_2 = S_a, S_c$		$G_3 = S_a, S_d$		$G_4 = S_b, S_c$		$G_5 = S_b, S_d$		$G_6 = S_c, S_d$	
	MSE	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*
Lena	36.1644	32.55	36.1184	32.55	36.1213	32.55	**36.1156**	**32.55**	**36.0894**	**32.56**	36.1190	32.55	36.1555	32.55
Cameraman	57.6412	30.52	57.5369	30.53	57.6043	30.53	**57.4396**	**30.54**	**36.5055**	**30.53**	57.5071	30.53	57.6388	30.52
House	25.6653	34.04	25.6164	34.05	25.6691	34.04	25.6262	34.04	**25.5234**	**34.06**	**25.5389**	**34.06**	25.6485	34.04
Goldhill	37.8357	32.35	37.8129	32.35	37.8327	32.35	**37.7616**	**32.36**	**37.7647**	**32.36**	37.7812	32.36	37.8185	32.35
Peppers	43.4659	31.75	43.3753	31.76	43.4271	31.75	**43.3532**	**31.76**	**43.3627**	**31.76**	43.3653	31.76	43.4673	31.75
Clock	17.8484	35.61	17.8456	35.62	17.8605	35.61	**17.8173**	**35.62**	**17.7804**	**35.63**	17.8194	35.62	17.8339	35.62
Chemical Plant	87.6350	28.70	87.5539	28.71	87.5874	28.71	**87.3090**	**28.72**	**87.5359**	**28.71**	87.5482	28.71	87.6209	28.70

and $y_o(i)$ is approximately not equal to $y_n(i)$, then reconstruction is accomplished by $\mathbf{S_b}$. In case, if none of the delay filter of each group meet this criteria, then reconstruction is accomplished by linear phase filters.

Each reconstruction group of synthesis filters is comprised of three filters, so there will be nine possible reconstruction combinations. However, computational complexity of the each group can be reduced from 9 possible reconstruction combinations to 5, by using only the reconstruction combinations of the delay filters comprising of linear phase filters because other combinations do not significantly contribute in optimal image reconstruction. If "y", "z" and "p" represent $\mathbf{S_a}$, $\mathbf{S_b}$ and linear phase filters respectively, then suggested reconstruction combinations are encircled as



After analyzing the contribution of each reconstruction group in enhancement of image reconstruction, the best two groups are selected for image reconstruction. The placing of the two selected reconstruction groups is also analyzed by interchanging their positions before linear phase filters in the synthesis filter bank. Let $\mathbf{G_x}$ and $\mathbf{G_y}$ are selected for image reconstruction, then possible placing configurations will be

$$\mathbf{P_1} = \begin{bmatrix} \mathbf{G_x} \\ \mathbf{G_y} \\ \mathbf{S_p} \end{bmatrix}$$

and

$$\mathbf{P_2} = \begin{bmatrix} \mathbf{G_y} \\ \mathbf{G_x} \\ \mathbf{S_p} \end{bmatrix}$$

in the synthesis filter bank. The most appropriate placing configuration is then chosen and used for image reconstruction upto level 3 of subband tree. In the other subband levels, linear phase filters are employed.

3. Simulations and Results

Popular bi-orthogonal Daubechies 9/7 filters have been selected because of their superior performance. Since the length of Daubechies 9/7 analysis filters is 16, so we can design 14 different synthesis filters. The sizes of matrices **A, S** and **B** will be 15 x 9, 9 x 2 and 15 x 2 respectively.

We have incorporated the adaptive synthesis filter bank in conventional popular SPHIT (Set Partitioning in Hierarchical Trees) coder and modified the synthesis section so that all the delay and linear phase filters take part in optimal image reconstruction.

The contributions of the six possible reconstruction groups in the enhancement of a number of bench mark images are shown in Table 1 for compression ratio 0.5 bits per pixel.

Since, the contributions of groups $\mathbf{G_3}$ and $\mathbf{G_4}$ are quite significant in image enhancement (as shown in bold), so these two groups are selected for reconstruction. The results of possible placing configurations of groups $\mathbf{G_3}$ and $\mathbf{G_4}$ in synthesis filter bank are shown in Table 2. The placing configuration $\mathbf{P_2}$ has overall outperformed the $\mathbf{P_1}$ in the enhancement of all the test images. So, this configuration is most suitable for the adaptive synthesis filter bank in image reconstruction.

4. Conclusion

Selection procedure of delay filters in various reconstruction groups of synthesis filter bank and possible placing configurations of the optimal groups are discussed by a simple design example. By using this method, most appropriate groups and their optimal placing configuration are selected in adaptive synthesis filter bank and this method can be easily extended to adaptive synthesis filter banks comprised of a large number of delay filters. The marginal improvement in MSE/ PSNR is due to simple optimization technique of low pass and high pass synthesis filters coefficients. A slightly more sophisticated optimizing technique for the synthesis delay filters may result a significant improvement in objective performance of image compression.

Table 2: Contributions of placing configurations in image enhancement for compression ratio = 0.5 bits per pixel. compression ratio = 0.5 bits per pixel.

Image	Placing Configurations of proposed Adaptive Synthesis Filter Bank			
	$\mathbf{P_1}$		$\mathbf{P_2}$	
	MSE	*PSNR*	*MSE*	*PSNR*
Lena	36.0993	32.56	36.0142	32.57
Cameraman	57.3961	30.54	57.2908	30.55
House	25.5684	34.05	25.4791	34.07
Goldhill	37.7401	32.36	37.7060	32.37
Peppers	43.3066	31.77	43.2767	31.77
Clock	17.7997	35.63	17.7223	35.65
Chemical Plant	87.2838	28.72	87.3040	28.72

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