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The Effect of Structural Damping on the Forced Vibrations of Cylindrical Sandwich Shells[1]

This paper presents an examination of the effects of structural damping on the axisymmetric vibrations of a cylindrical sandwich shell. It is shown that the use of core materials with high damping properties can result in large reductions in resonant response over conventional materials. The radial vibration of the shell resulting from a time harmonic radial load is first calculated by an exact method. The radial vibration is then calculated by an approximate formula, which requires only a knowledge of the damping properties and the natural (undamped) modes. In numerical examples the resonant vibrations of two steel-faced cylinders are compared. One has a polymeric, the other an elastomeric core. The results indicate that for the assumed conditions they are both effective for suppressing resonant vibration, the polymeric core being generally more effective than the elastomeric core.

Introduction

This paper represents a continuation of an investigation of the forced vibrations of damped sandwich structures. In two previous publications [1, 2][2] the authors investigated the effect of structural damping on the forced vibrations of homogeneous and sandwich plates, vibrating both in vacuo and in fluid media. Various aspects of the problem of sandwich plates with structural damping have been investigated also by other authors; for example, Keer and Lazan [3] and Y. Y. Yu [4]. It was shown in [2] that a considerable reduction in resonant vibration could be realized by the use of special high-damping materials as cores in sandwich plates. This was indicated by reductions in the quality factors (and dynamic magnifications) of about two orders of magnitude over those of conventional sandwich plates. Such reductions are of particular practical benefit in design applications where excitation of the structure at frequencies that coincide with its natural frequencies cannot be avoided; as, for example, when a structure is subjected to intensive noise. The emphasis in this paper, as in the previous publications, is placed on suppressing resonant vibratory motion by the optimal use of structural damping.

The structure considered is an infinitely long circular cylindrical sandwich shell. The sandwich shell is made up of two thin face sheets, usually metal in practice, with a thick high-damping core between them. The damping is provided primarily by the core, which must be a double-purpose material with both shear stiffness and shear-damping properties. Both the facings and the core are elastic materials with linear structural damping. The sandwich configuration represents one of two configurations that were found to be most efficient for damping in the study on plates [1, 2]. The other was the sandwich plate with lightweight core, e.g., honeycomb, and with damping layers applied to the surfaces.

The problem considered in this paper can be stated formally as that of a damped cylindrical sandwich shell subjected to a time-harmonic radially symmetric pressure. The pressure is assumed to vary sinusoidally in the longitudinal direction. The primary objective is to determine quantitatively the damping effectiveness of various materials when used as cores in the sandwich shell and to show how their effectiveness varies with frequency, wavelength, and geometric parameters.

While the shell is assumed to be infinitely long, it will be seen that the results apply immediately to a finite-length shell simply supported at the ends. Also, the response to a pressure of any longitudinal variation can be obtained from the results of this problem by superposition.

[1] Work done under contract Nonr 4199(00) for the Office of Naval Research, U. S. Navy Department.

[2] Numbers in brackets designate References at end of paper.

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Nomenclature

x, θ = coordinates in longitudinal and circumferential directions, respectively
t = time
ω, Ω = angular frequency, frequency parameter
u, w, ψ = displacement at middle surface in longitudinal direction; displacement at middle surface in radial direction; rotation of normal in the longitudinal direction
a = radius of cylinder middle surface
l, λ = wavelength in longitudinal direction; wavelength parameter
p = pressure
h_1, h_2 = one-half core thickness, facing thickness
ρ_1, ρ_2 = core density, facing density
h = $h_1 + h_2$
E, ν, G = elastic constants
$\bar{E}, \bar{\nu}, \bar{G}$ = damping constants
r_ρ, r_h, r_1, r_2 = density, thickness, and elastic-property ratios
d = thickness ratio
η, δ = damping-loss factors
k = shear coefficient
x_j, X_j = displacement; amplitude of displacement
f_i = load
m_{ij}, k_{ij}, c_{ij} = inertia, elastic, and damping coefficients
M_r, C_{rs}, Γ_r = generalized mass, damping coefficient, and force
Φ = normal mode number
q_r, Q_r = normal coordinate, amplitude of normal coordinate
W = dimensionless radial displacement

Subscripts

1, 2 = core, facings
i, j = displacements, forces
r, s, p = modes

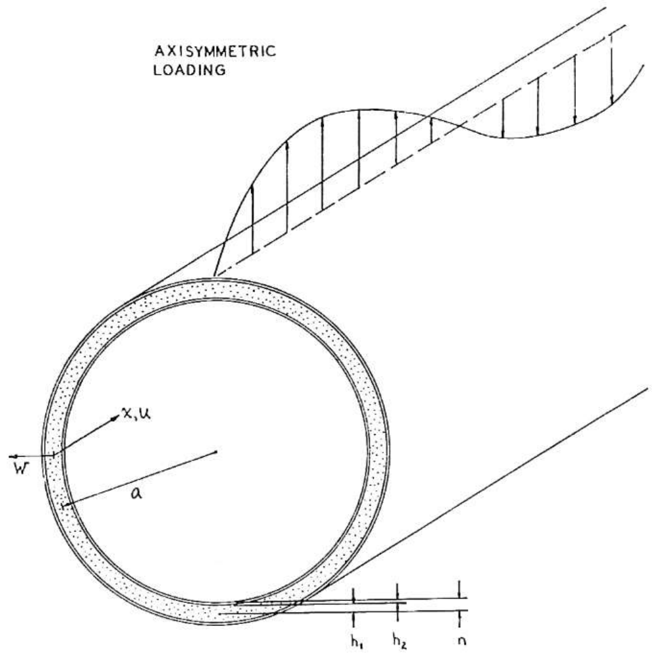

Fig. 1 Infinite cylindrical sandwich shell and loading

Equations of Motion

The equations of motion for the forced, axisymmetric vibrations of a damped sandwich cylinder, as shown in Fig. 1, have been obtained by modifying the displacement equations of motion for the free vibration of an undamped sandwich cylinder as given by Yi-Yuan Yu in [5]. These equations have been modified by adding a time-harmonic forcing function acting in the radial direction, and by replacing the real material constants with complex constants. The latter modification properly introduces linear structural damping in the case of forced vibrations under simple harmonic motion.

The pertinent equations for the shell with thin face sheets are equations (18) of [5]. The first modification requires the addition of a term $p(x, t) = p(x)e^{i\omega t}$ on the left-hand side of the second of equations (18). The terms of this equation represent force components per unit area in the radial direction.

The second modification is made as follows: The stress-strain equation for each layer of the damped sandwich cylinder can be written

$$\begin{aligned} \sigma_x &= (A_{11}\epsilon_x + A_{12}\epsilon_\theta) + i(B_{11}\epsilon_x + B_{12}\epsilon_\theta) \\ \sigma_\theta &= (A_{21}\epsilon_x + A_{22}\epsilon_\theta) + i(B_{21}\epsilon_x + B_{22}\epsilon_\theta) \\ \tau_{x\theta} &= A_{33}\gamma_{x\theta} + iB_{33}\gamma_{x\theta} \end{aligned} \quad (1)$$

In equations (1), both the A and B may be frequency and temperature-dependent quantities. The manner in which these quantities vary with temperature and frequency is quite different for different materials. Therefore, in the present derivation we shall assume for generality that they are constants, and for specific cases later, we use the appropriate values for the temperature and frequency ranges concerned. We shall let

$$\begin{aligned} A_{11} &= E & B_{11} &= \bar{E} \\ A_{12} &= E\nu & B_{12} &= \bar{E}\bar{\nu} \\ A_{33} &= G & B_{33} &= \bar{G} \end{aligned} \quad (2)$$

In equations (2), E is in the simplest cases Young's modulus divided by $(1 - \nu^2)$ where ν is Poisson's ratio. In some cases, however, it varies with frequency [6]. If the elastic constants of equations (18) of [5] are replaced by complex constants whose real and imaginary parts are given by equations (2), we obtain for the three governing equations of motion:

$$\begin{aligned} &2(E_1h_1 + E_2h_2)u'' + \left(E_1\frac{2h_1^3}{3} + 2E_2h_1h_2h\right)\frac{1}{a}\psi'' \\ &\quad + 2(E_1\nu_1h_1 + E_2\nu_2h_2)\frac{1}{a}w' + 2i(\bar{E}_1h_1 + \bar{E}_2h_2)u'' \\ &\quad + i\left(\bar{E}_1\frac{2h_1^3}{3} + 2\bar{E}_2h_1h_2h\right)\frac{1}{a}\psi'' + 2i(\bar{E}_1\bar{\nu}_1h_1 + \bar{E}_2\bar{\nu}_2h_2)\frac{1}{a}w' \\ &\quad = 2(\rho_1h_1 + \rho_2h_2)\ddot{u} + \left(\rho_1\frac{2h_1^3}{3} + 2\rho_2h_1h_2h\right)\frac{1}{a}\ddot{\psi} \end{aligned} \quad (3a)$$

$$\begin{aligned} &2k_1G_1h_1(\psi' + w'') - 2(E_1\nu_1h_1 + E_2\nu_2h_2)\frac{1}{a}u' \\ &\quad - 2(E_1h_1 + E_2h_2)\frac{1}{a^2}w + 2ik_1\bar{G}_1h_1(\psi' + w'') \\ &\quad - 2i(\bar{E}_1\bar{\nu}_1h_1 + \bar{E}_2\bar{\nu}_2h_2)\frac{1}{a}u' - 2i(\bar{E}_1h_1 + \bar{E}_2h_2)\frac{1}{a^2}w \\ &\quad + p(x)e^{i\omega t} = 2(\rho_1h_1 + \rho_2h_2)\ddot{w} \end{aligned} \quad (3b)$$

$$\begin{aligned} &\left(E_1\frac{2h_1^3}{3} + 2E_2h_1h_2h\right)\left(\frac{u''}{a} + \psi''\right) - 2k_1G_1h_1(\psi + w') \\ &\quad + i\left(\bar{E}_1\frac{2h_1^3}{3} + 2\bar{E}_2h_1h_2h\right)\left(\frac{u''}{a} + \psi''\right) - 2k_1\bar{G}_1h_1i(\psi + w') \\ &\quad = \left(\rho_1\frac{2h_1^3}{3} + 2\rho_2h_1h_2h\right)\left(\frac{\ddot{u}}{a} + \ddot{\psi}\right) \end{aligned} \quad (3c)$$

where dot (•) means derivative with respect to time and prime (′) means derivative with respect to the longitudinal coordinate x. Here, also, the subscript 1 refers to the core while the subscript 2 refers to the faces of the sandwich. The coefficient k is a shear coefficient whose exact value is determined in [5] by matching the simple thickness-shear frequency calculated from the sandwich-shell theory with that from the theory of elasticity. The value of k_1 is usually near unity; in fact it is shown in [5] that if $\rho_1h_1/\rho_2h_2 > 2$, we may take $k = 1$ without appreciable error.

We set

$$p(x) = p_0 \sin\frac{2\pi}{l}x \quad (4)$$

where l is the wavelength. Then the equations of motion can be reduced to ordinary differential equations by the following substitutions:

$$\begin{aligned} u(x, t) &= u(t)\cos\frac{2\pi}{l}x \\ w(x, t) &= w(t)\sin\frac{2\pi}{l}x \\ \psi(x, t) &= \psi(t)\cos\frac{2\pi}{l}x \end{aligned} \quad (5)$$

The equations of motion are more useful in dimensionless form. We therefore introduce the ratios

$$\begin{aligned} &r_\rho = \frac{\rho_2}{\rho_1} \qquad r_h = \frac{h_2}{h_1} \qquad r_1 = \frac{E_1}{G_1} \qquad r_2 = \frac{E_2}{G_1} \\ &\delta_1 = \frac{\bar{G}_1}{G_1} \qquad \eta_1 = \frac{\bar{E}_1}{E_1} \qquad \eta_2 = \frac{\bar{E}_2}{E_2} \\ &d = \frac{h_1}{a} \qquad \lambda = \frac{2\pi a}{l} \qquad \Omega = \frac{\rho_2}{E_2}a^2\omega^2 \end{aligned} \quad (6)$$

With the foregoing ratios substituted into equations (3), and after some manipulation, the equations can be written in the following form:

$$\sum_{j=1}^{3} m_{ij}\ddot{x}_j + \sum_{j=1}^{3} (k_{ij} + ic_{ij})x_j = f_i e^{i\omega t} \qquad i = 1, 2, 3 \quad (7)$$

where

$$m_{ji} = m_{ij}, \qquad k_{ji} = k_{ij}, \qquad c_{ji} = c_{ij}$$

The coefficients are given by the following expressions:

$$m_{11} = m_{22} = \frac{2\Omega}{r_\rho \omega^2}(1 + r_\rho r_h)d$$

$$m_{12} = m_{23} = 0 \qquad (8a)$$

$$m_{13} = m_{33} = \frac{2\Omega}{3r_\rho\omega^2}[1 + 3r_\rho r_h(1 + r_h)]d^3$$

$$k_{11} = 2(r_1 + r_2 r_h)\frac{\lambda^2 d}{r_2}$$

$$k_{12} = -2(r_1\nu_1 + r_2\nu_2 r_h)\frac{\lambda d}{r_2}$$

$$k_{13} = \frac{2}{3}[r_1 + 3r_2 r_h(1 + r_h)]\frac{\lambda^2 d^3}{r_2}$$

$$k_{22} = 2[k_1\lambda^2 + (r_1 + r_2 r_h)]\frac{d}{r_2} \qquad (8b)$$

$$k_{23} = 2k\frac{\lambda d}{r_2}$$

$$k_{33} = \frac{2}{3}[r_1 + 3r_2 r_h(1 + r_h)]\frac{\lambda^2 d^3}{r_2} + 2k_1\frac{d}{r_2}$$

$$c_{11} = 2(\eta_1 r_1 + \eta_2 r_2 r_h)\frac{\lambda^2 d}{r_2}$$

$$c_{12} = -2(\eta_1 r_1 \bar{\nu}_1 + \eta_2 r_2 \bar{\nu}_2 r_h)\frac{\lambda d}{r_a}$$

$$c_{13} = \frac{2}{3}\left[r_1 + 3\frac{\eta_2}{\eta_1}r_2 r_h(1 + r_h)\right]\frac{d^3\lambda^2\eta_1}{r_2}$$

$$c_{22} = [2k_1\lambda^2\delta_1 + 2(\eta_1 r_1 + \eta_2 r_2 r_h)]\frac{\bar{\omega}}{r_2} \qquad (8c)$$

$$c_{23} = 2k_1\frac{\lambda d}{r_2}\delta_1$$

$$c_{33} = [\tfrac{2}{3}\eta_1 r_1 + 2\eta_2 r_2 r_h(1 + r_h)]\frac{\lambda^2 d^3}{r_2} + 2k_1\frac{d\delta_1}{r_2}$$

The dimensionless displacements and loadings are

$$x_1 = \frac{u(t)E_2}{p_0 a} \qquad f_1 = 0$$

$$x_2 = \frac{w(t)E_2}{p_0 a} \qquad f_2 = 1 \qquad (8d)$$

$$x_3 = \frac{\psi(t)E_2}{p_0} \qquad f_3 = 0$$

The quantities m_{ij}, c_{ij}, and k_{ij} are thus functions of the elastic properties (r_1, r_2, ν_1, ν_2) as well as of the damping-loss coefficients (η_1, η_2, and δ_1), the damping-coupling coefficients ($\bar{\nu}_1$ and $\bar{\nu}_2$), the density parameter (r_ρ), the geometric parameters (r_h and d), the wavelength parameter (λ), the shear coefficient (k_1), and the frequency parameter (Ω).

Solution for Radial Displacement

Exact Solution

With the equations of motion reduced to ordinary differential equations, we can obtain an expression for the radial displacement in a straightforward manner. As a solution to equation (7) we let

$$x_j = X_j e^{i\omega t} \qquad (9)$$

where X_j is complex. Substitution into equation (7) yields

$$\sum_{j=1}^{3}\{(k_{ij} - \omega^2 m_{ij}) + ic_{ij}\}X_j = f_i, \qquad i = 1, 2, 3 \quad (10)$$

The solution can be developed more conveniently from this point using matrix algebra. Introducing the following matrices defined by their ith, jth-terms

$$k_{ij} - \omega^2 m_{ij} \rightarrow [B] \qquad X_j \rightarrow \{X^R\} + i\{X^I\}$$
$$c_{ij} \rightarrow [C] \qquad f_i \rightarrow \{f\} \qquad (11)$$

Equation (10) becomes

$$([B] + i[C])(\{X^R\} + i\{X^I\}) = \{f\} \qquad (12)$$

Expanding the left-hand side of equation (12) and equating real and imaginary parts, we can write

$$\left[\begin{array}{c|c} -C & B \\ \hline B & C \end{array}\right]\left[\begin{array}{c} X^I \\ \hline X^R \end{array}\right] = \left[\begin{array}{c} f \\ \hline 0 \end{array}\right] \qquad (13)$$

If we denote the inverse of the 6×6 matrix of real coefficients by

$$\left[\begin{array}{c|c} -C & B \\ \hline B & C \end{array}\right]^{-1} = \left[\begin{array}{c|c} A & G \\ \hline G & H \end{array}\right] \qquad (14)$$

we have for the solution for the ith-displacement

$$|X_i|^2 = (X_i^I)^2 + (X_i^R)^2 \qquad (15a)$$

or

$$|X_i|^2 = \left(\sum_{j=1}^{3} A_{ij}f_j\right)^2 + \left(\sum_{j=1}^{3} G_{ij}f_j\right)^2 \qquad (15b)$$

Using f_j from equations (8d), and denoting the amplitude of the radial displacement $|X_2|$ as W, we can write

$$W = \left|\frac{wE_2}{p_0 a}\right| = (A_{2,2}^2 + G_{2,2}^2)^{1/2} \qquad (16)$$

Approximate Solution

In many practical cases the normal modes are only slightly affected by damping. In such cases the response can be determined from a simpler calculation. If we express the displacements in terms of the normal mode numbers $\Phi_j^{(r)}$ and the normal coordinates $q_r(t)$ we can put the equations of motion, equations (7), in modal form by following the standard procedure. Thus we obtain the following three modal equations replacing equations (7):

$$M_r\ddot{q}_r + \omega_r^2 M_r q_r + i\sum_{s=1}^{3} C_{rs}q_s = \Gamma_r e^{i\omega t} \qquad r = 1, 2, 3$$

where

$$C_{rs} = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}\Phi_i^{(r)}\Phi_j^{(s)}$$
$$\Gamma_r = \sum_{i=1}^{3} f_i\Phi_i^{(r)} \qquad (17)$$

and M_r and ω_r are the rth generalized mass and natural frequency, respectively. As a solution to equations (17) we let

Table 1 Table of material properties

	Example A	Example B
Facing material	steel	steel
Young's modulus, E^*, psi	30×10^6	30×10^6
Extensional damping modulus, $\bar{E}$, psi	10^5	10^5
Extensional loss factor, η	0.0033	0.0033
Poisson's ratio, ν	0.3	0.3
Density, ρ, lb-sec²/in.⁴	7.8×10^{-4}	7.8×10^{-4}
Core material	Polymer	Elastomer
Shear modulus, G, psi	10^4	10^2
Shear damping modulus, $\bar{G}$, psi	10^3	50
Shear loss factor, δ	0.1	0.5
Poisson's ratio, ν	0.4	0.4
Density, ρ, lb-sec²/in.⁴	1.4×10^{-4}	1.1×10^{-4}

$$q_r = Q_r e^{i(\omega t - \phi_r)} \tag{18}$$

With equation (18) substituted into equation (17), and with ω equal to one of the natural frequencies, say, ω_p, the first two terms of the pth of equations (17) cancel. If, in addition, the motion is assumed to be entirely in the pth-mode, then $q_s = 0$ for $s \neq p$, and the pth-equation reduces to

$$iC_{pp}Q_p e^{i(\omega_p t - \phi_p)} = \Gamma_p e^{i\omega t} \tag{19}$$

Thus we find that $\phi_p = \pi/2$ and

$$Q_p = \frac{\sum_{i=1}^{3} f_i \Phi_i^{(p)}}{\sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} \Phi_i^{(p)} \Phi_j^{(p)}} = \frac{\Gamma_p}{C_{pp}} \tag{20}$$

If the normal modes are normalized with respect to the radial displacement x_2, then the radial displacement at the pth-resonance, W_p, becomes simply Q_p:

$$Q_p = W_p = \left| \frac{wE_2}{p_0 a} \right| \tag{21}$$

It has been shown [7] that if the damping matrix is a linear combination of the mass and stiffness matrices, then the normal modes of undamped vibration are unaffected by damping. In such cases the radial displacement given by equations (20) and (21) is exact.

For nonresonant vibration, structural damping usually has a negligible effect. Thus, if we neglect the damping terms of equations (17), we obtain the following solution using equation (18):

$$Q_r = \frac{\Gamma_r}{M_r(\omega_r^2 - \omega^2)}, \qquad r = 1, 2, 3 \tag{22}$$

With the modes normalized with respect to the radial displacement x_2, the nonresonant radial displacement is simply

$$W = \sum_{r=1}^{3} Q_r \tag{23}$$

Thus equations (21) and (23) form an alternate solution to equation (16) for calculating the radial displacement; equation (21) for resonant vibration and equation (23) for nonresonant vibration.

A common measure of damping effectiveness is the quality factor of the system. For the rth-mode, this factor is given by

$$\text{QF} = \frac{\omega_r^2 M_r}{C_{rr}} \tag{24}$$

Numerical Examples

Numerical calculations were performed for two sandwich cylinders with quite different high-damping materials as cores. These calculations illustrate the effectiveness of typical damping materials for controlling the resonant vibrations of a shell.

In the first case, labeled example A, the core was given properties that are typical of a high-damping polymeric material. These properties were obtained by taking representative experimental data from [8]. These properties approximate those for polystyrene and some methacrylates. The elastic and damping properties for the facings, which are steel, were also obtained from experimental data of [8].

In example B, the same steel facings were used but the core was given properties representative of an elastomeric, or rubbery material. These properties correspond to experimentally derived data on butyl rubber and Thiokol rubber. The significant material properties for both shells are summarized in Table 1. Notice that while the stiffness and damping moduli of the elastomer are much lower than those of the polymer, the loss factors of the elastomer are much higher. The significance of this is seen later in the results.

The geometric properties used were as follows:

$$\frac{h_1}{a} = 0.045 \qquad \frac{h_2}{h_1} = 0.10 \quad = \frac{2\pi a}{l} = 1.0$$

Thus the total shell thickness $2(h_1 + h_2)$ was one tenth of the radius and the core was ten times as thick as the facings. The half-wavelength $l/2$ was equal to π times the radius.

The radial displacement parameter W is plotted against the forcing frequency parameter Ω for example A in Fig. 2. Shown for comparison (as a dashed line) is the response curve for the shell with the damping moduli set equal to zero. The curve for example B is of similar shape. Several observations regarding the response can be made from these curves:

(a) While the sandwich shell under a longitudinally sinusoidal loading has three degrees of freedom and three natural modes, there are only two peaks in the response curve. This is because only two natural modes, the lower two, involve the radial displacement. This is clear from an examination of the natural modes, which are shown in Fig. 3 for example A. The natural modes for B are quite similar.

(b) The undamped natural frequency parameters (Ω_n) for the steel/polymer shell (example A) and the steel-elastomer shell (example B) are shown in the following:

	$\Omega_n \times 10^2$	
Mode No.	A	B
1	23.60	22.53
2	53.14	52.64
3	148.8	64.43

It is seen that only the third natural frequencies are substantially different. Examination of the associated mode shapes gives an explanation for this. In the first two modes there is relatively little shear deformation while the third mode is predominantly shear. Thus the shell with the stiffer core, shell A, has the higher third natural frequency.

(c) The peak values of radial displacement are as follows:

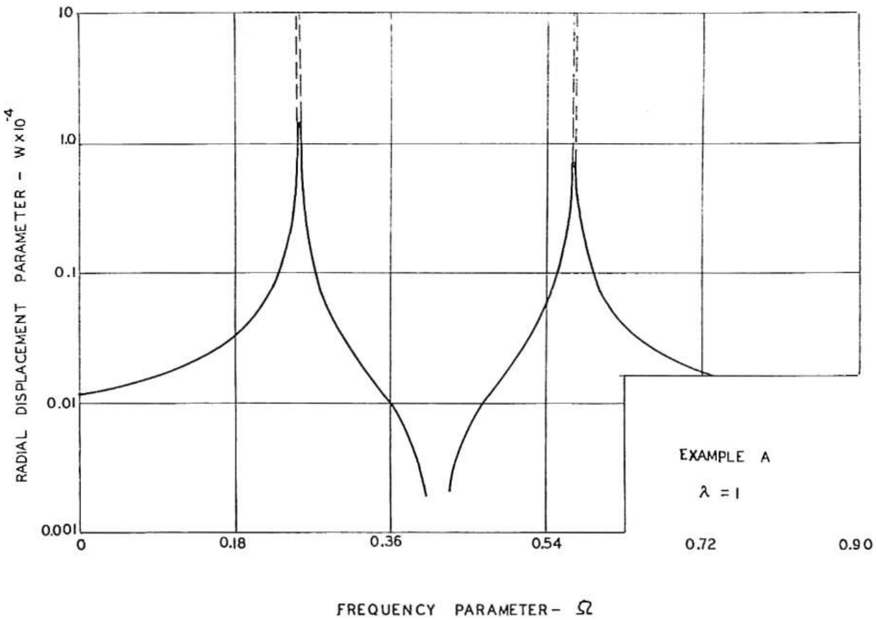

Fig. 2 Axisymmetric radial response of an infinite cylindrical sandwich shell

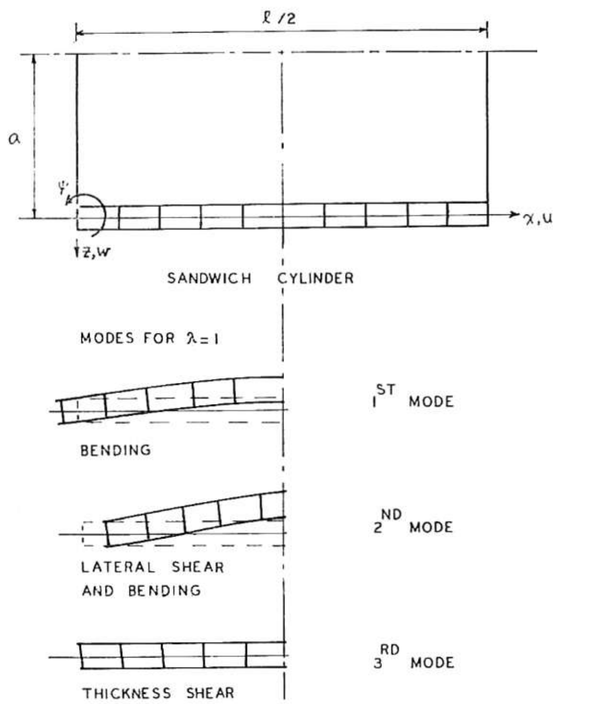

Fig. 3 Axisymmetric modes of vibration for an infinite cylindrical sandwich shell

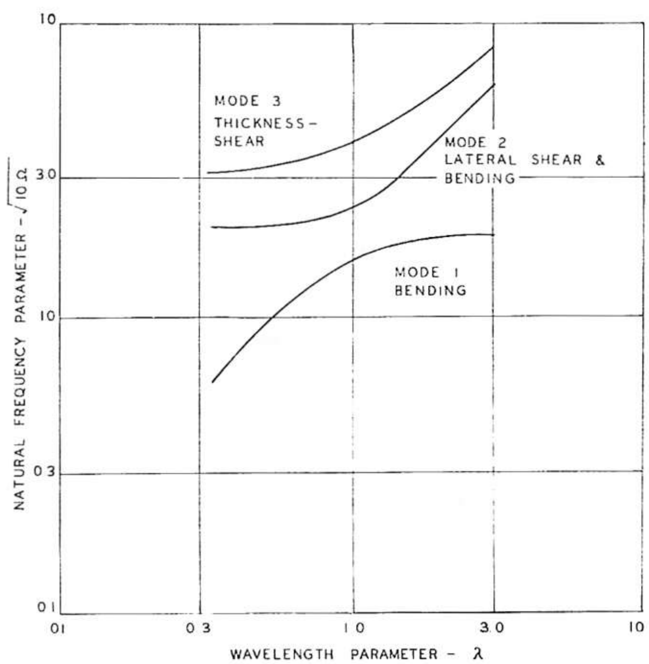

Fig. 4 Variation of natural frequency with wavelength for an infinite cylindrical sandwich shell—example A

W_{max}	A	B
W_1	14220	24240
W_2	6920	12900

The results indicate that core A (polymeric) is more effective for suppressing vibration in this particular example for wavelength $\lambda = 1$.

(*d*) A comparison of the response of the structure with damping and without damping shows that damping becomes significant only near resonance. This indicates that for nonresonant vibrations, damping will not reduce the vibration amplitudes significantly.

(*e*) The circles in Fig. 2 give the peak response as calculated by the approximate formula, equation (20). These values are practically the same as the exact values.

The response curve in Fig. 2, the mode shapes in Fig. 3, and the foregoing discussion are all associated with wavelength parameter $\lambda = 1$. Figs. 4 through 6 show how the response varies with wavelength of applied load. Figs. 4 and 5 give the variation in natural frequency with wavelength and Fig. 6 gives the variation in the damping effectiveness with wavelength as measured by the ratio of W_{max} to the static value W_{st}. This ratio is numerically equal to the quality factor of the system, which is given by equation (24).

Some points worth noting regarding these results are as follows:

(*a*) For example B, the curves for natural frequencies corre-

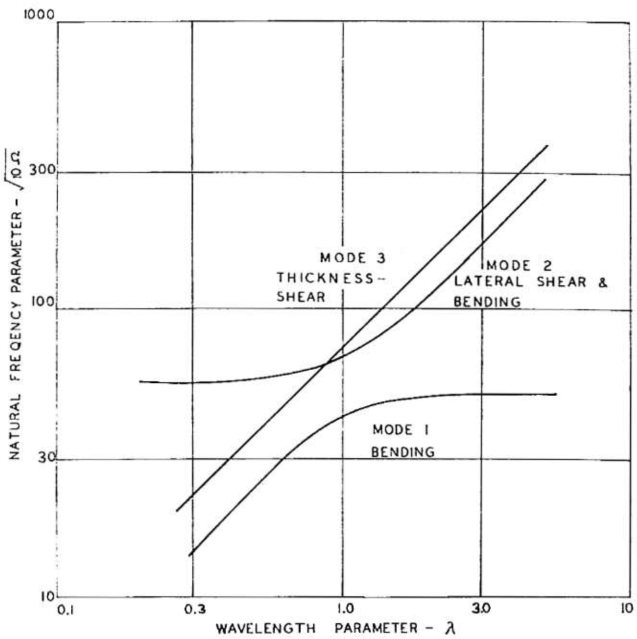

Fig. 5 Variation of natural frequency with wavelength for an infinite cylindrical sandwich shell—example B

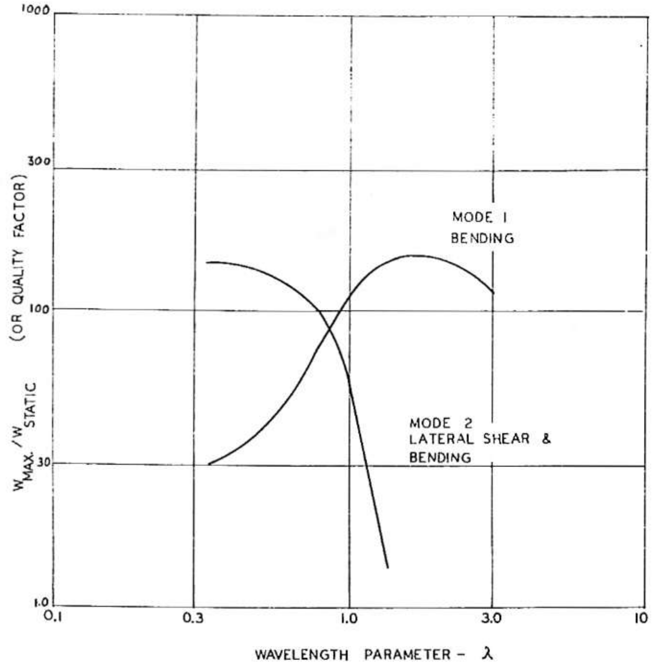

Fig. 6 Quality factor versus wavelength for an infinite cylindrical sandwich shell

sponding to the modes identified in Fig. 3 as 2 and 3 actually cross, so that for long wavelengths ($\lambda < 0.85$) the thickness shear mode, formerly the third mode, becomes the second. The possibility of this occurring was pointed out previously by Yi-Yuan Yu [5]. It can be accounted for in this case by the low shear modulus of the elastomeric core in shell B. Note also that at $\lambda = 0.85$ the two modes have the same natural frequency.

(*b*) The quality-factor curves for the two shells are quite similar in shape. The curves show that shell A is superior for suppression of resonant vibration for any wavelength.

(*c*) The response curves show, as expected, that mode 2 becomes highly suppressed at short wavelengths; $(QF)_2$ small for large λ. As shown in Fig. 3, this mode is largely lateral shear deformation. Owing to the large shear-damping capacity of the core, this deformation is heavily damped.

(*d*) It is well known that the resonant amplitudes of vibration (or quality factors) of actual structures cannot be predicted accurately by a theory such as the present one which considers only material damping, since an additional amount of damping of possibly the same order of magnitude as the material damping can result from friction in bolted or welded joints, shear in the adhesive between the core and facings, and so on. However, the importance of the present results is that they indicate the reductions in resonant vibration (for example, as measured by the QF, or quality factors) that are possible resulting from material damping alone. By comparing the QF for the damped sandwich shell, Fig. 6, with the theoretical QF for conventional structures not especially designed for damping (which have been calculated to be of the order of 10^4 for homogeneous shells and 10^3 for honeycomb sandwiches), we can measure the effectiveness of the present design concept. Such a comparison shows that the present configuration, with QF of about 10^2, gives at least an order-of-magnitude reduction over conventional structures.

The discrepancy between theoretical values such as those obtained here, based only on material damping, and measured QF seems to be from one to two orders of magnitude, depending on the nature and amount of friction in the structure. If the present results are to be used in connection with design, this fact should be taken into account on the basis of experience or tests pending analytical examination of the problem.

Conclusions

This paper has accomplished the following:

It has presented a solution for the steady-state response of a damped cylindrical sandwich shell under a radially symmetric time-harmonic pressure.

It has given a formula for the approximate calculation of resonant peak responses, based on the classical mode shapes, and demonstrated its accuracy in some typical cases.

It has illustrated, by way of examples, the relative responses of cylinders with polymeric and elastomeric cores, also including the effects of these core materials on natural frequencies and modes.

With the reduction of radial displacement by means of damping established for the axisymmetric modes, the authors intend to extend the analysis to the study of lobar modes and ultimately to the determination of the reduction resulting from damping of sound radiation from sandwich cylinders submerged in fluid media.

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