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Biased Procurement Auctions

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Abstract In a complex procurement a buyer may consider biasing the auction rules in order to account for differences in product characteristics offered by the sellers. This paper studies the gathering, disclosure and use of information about this bias. While we also describe the optimal procurement auction in our setting, the main focus of the paper is on the case where the buyer does not have commitment power. We find that without commitment full disclosure of the buyer's preferences is optimal. Furthermore, lack of commitment distorts the buyer's incentives to learn about its preferences: unlike the commitment case, without commitment the value of this information can be negative.

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1 Introduction

Comparing sellers is a complex task in a procurement auction. Bid proposals are lengthy documents, that differ in many dimensions besides price, such as technical specifications and aesthetic properties of the item to be supplied, time of delivery, payment conditions, amount and quality of service, and supplier reliability.

Such complexity introduces several important strategic issues that do not exist in a simple auction where the winner is always the lowest bidder. This paper presents a model that focuses on two of those issues: the fact that the preferences of the buyer may not be known in advance by the sellers, and the fact that the procurement process involves extensive negotiations.

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Procurement practices in both the private and public sectors allow for considering other dimensions besides price in the evaluation of bids, and for ex-post negotiations. Gene Richter, former Chief Purchasing Officer of IBM, argues that simple lowest-bid auctions should not be used even for the simplest procurements:

> "There is nothing that a company buys that I can think of where only the price is important. There is a price, quality, delivering and technology issue in everything. So the purest use of auctions where the lowest bidder gets the business no matter what is terrible." ISM

The Federal Acquisition Regulation (FAR), the set of rules that govern U.S. Federal Procurement, allow for "procurement by negotiation". In this protocol, bidders are evaluated according to a weighted average of pre-specified criteria. However, the weights do not have to be announced before bidding. This effectively allows a federal agency to procure without revealing its preferences. Furthermore, according to the FAR the buyer can engage in bargaining with a selected group of sellers over all aspects of their proposal, including price.

In environments where it may not be clear what are the buyer's preferences across proposals, several interesting questions arise: is it in the buyer's interest to reveal these preferences to the sellers? Should the buyer always follow its preferences in selecting the winner, or can he do better by distorting its choice? Are there circumstances where the buyer will prefer not to know about its own preferences?

On the other hand, procurement of complex items typically involves extensive negotiations and some degree of flexibility in the auction protocol. Often there is communication and negotiation both before and after the formal bidding stage. In such environment, the specifics of the auctioning process are less important than recognizing the possibility of further negotiation afterward, since it can greatly affect bidder behavior in the auction. Taking into account this possibility in the analysis leads to predictions that differ sharply from the ones obtained in the standard optimal auction framework, where the buyer is assumed to have full commitment power.

We obtain two main results regarding the treatment of information under no commitment. The first is that full disclosure is optimal: if an uncommitted buyer is known to be withholding information about his preferences, then the auction breaks down, in the sense that equilibrium bidding leads to very high prices independently of the sellers costs.

The second result is that the value of information about the buyer's preference for the buyer is negative under no commitment in our framework. While information is needed to select the best seller, and thus is socially beneficial, in equilibrium prices change in response to this knowledge in a way that more than compensates for this gain.

When the buyer can commit to the optimal auction, the value of information will be positive, but he will implement an allocation that is not socially efficient. Thus, the model suggests there is a tension between the incentives to gather information and to utilize it efficiently.

The importance of incorporating dimensions besides price in complex procurement auctions has been recognized in the literature. These other dimensions have either been modeled a choice variable of the supplier (Che 1993; Burguet and Che 2004) or as exogenous, private information on the supplier's side (Zheng 2000; Lamping 2006). The current paper focuses instead on a third possibility: that the selection criteria depend on information the buyer possesses about its preferences.

In our model, product characteristics besides price are represented by an exogenous random variable that can be observed by the buyer, but not the seller. While in reality product specifications are determined by a combination of seller capabilities, buyer desires and decisions made in the negotiation process,[1] we decided to focus on private information on the buyer side to highlight the aspect that is novel in the literature.

This is not a paper about corruption in procurement, although the analysis here is complementary to the study of this important topic. The natural way to model explicit corruption is to introduce an agency problem: the buyer is required to hire a self-interested agent to run the auction (Burguet and Perry 2007; Burguet and Che 2004; Kosenok and Lambert-Mogiliansky 2004; Lambert-Mogiliansky and Sonin 2006). In this paper, there is no corruption *per se*, since the buyer is allowed to make decisions directly in the auction.

However, this paper is agnostic about the reasons behind the buyer's preferences over sellers. These may be legitimate reasons, such as differences in technical specifications or aesthetic properties of the products, or may be illegitimate, such as favoritism or corruption. As such, this paper may contribute to the literature on corruption as an analysis of the subgame that happens once such bias exists.

A second way in which the paper complements the literature on corruption is to provide a benchmark. In complex procurement environments the outcome of an auction may seem suboptimal for a variety of reasons — perhaps the buyer may seem to focus too little or too much on price; perhaps negotiations have been carried out for too long, or not long enough; perhaps the rules of the auction, and particularly the criteria to select the winner, having been changing along the way. Before we can judge if these patterns indicate corruption, we should investigate whether they could be made by honest buyers. This task is done in this paper.

The paper is organized as follows. Section 2 describes the setting and contrasts it with other settings studied in the literature. Section 3 describes the optimal procurement mechanism that is feasible under commitment. Section 4 presents a model of a procurement auction without commitment and characterizes an equilibrium. Section 5 discusses the incentives for gathering information. Section 6 provides some final remarks.

2 Environment and Related Literature

A buyer needs to procure a single, indivisible item that can be acquired from one of two potential suppliers. Products from different suppliers have different costs and different values for the buyer. The costs are private information of the suppliers; the values are private information of the buyer.

Let c_i denote the cost of producing the item by supplier i and θ_i the value of this good to the buyer. The costs c_1 and c_2 are independent random variables, with the same absolutely continuous distribution F, with positive density f over a compact support $[\underline{c}, \bar{c}]$. Throughout the paper we shall assume that F is *regular*, in the sense that

$$\vartheta(x) = x + \frac{F(x)}{f(x)}$$

[1] An important issue in product design is bundling (Stigler 1968; Adams and Yellen 1976; Dassiou and Glycopantis 2007).

is a monotone increasing function. For simplicity we assume that the gains from trade are large enough so that it is always optimal for the buyer to acquire the good: $\theta_i > \vartheta(\bar{c})$, for both $i = 1, 2$.

The distribution of (θ_1, θ_2) is independent of c_1 and c_2. Before the auction begins, each seller i observes c_i. The buyer may directly observe (θ_1, θ_2), if he so wishes. It is convenient to reparameterize (θ_1, θ_2) as the valuation difference

$$\Delta = \theta_1 - \theta_2$$

and the average valuation $\bar{\theta} = (\theta_1 + \theta_2)/2$.

The novel aspect of the environment is the presence and nature of θ_i. If $\theta_1 = \theta_2$, then the model would be a standard independent private values procurement auction. Procurement auctions for goods with differential values have been investigated in the literature, but these differences have been conceptualized as either private information of the suppliers, or as the outcome of decisions of the suppliers about specifications of their products. Che (1993) studies a procurement model in which the attributes other than price are a choice variable of the sellers, and the auction should be designed in a way that does not distort the incentives to select the most beneficial bundle of characteristics.[2] If the θ_i are exogenous but private information of the sellers, one would obtain a mechanism design model with multidimensional types similar to the ones studied by Laffont et al. (1987), McAfee and McMillan (1988), Armstrong (1996) and Zheng (2000). Here, it will be assumed that the differential values are exogenous and private information of the *buyer*. Assuming that (θ_1, θ_2) is independent of the sellers' private information allows us to focus on the aspect of the problem that is novel in the literature.

Wang (2000) investigates a procurement model with lack of commitment and private information on the buyer side, but the buyer has a single valuation for the item that does not depend on the identity of the seller. Thus, Wang's model does not apply to the investigation of the effects of differences in product characteristics.

The present framework is also different from the scale auction studied by Athey and Levin (2001). In a scale auction, the criterion for selecting the winner is an average of bids across different dimensions. That is similar to the present hypothesis that the criterion is a combination of price and other aspects. However, two crucial distinctions exist. First, in the current model, information about perceived quality is on the buyer's side. Second, in a scale auction, bids are on a per-unit basis, and the final price depends also in the quantity traded, which is determined after the auction. This gives rise to the possibility of strategic skewing of the bids. This possibility is not present in the current model, since prices considered here are the total amounts paid upfront.

As the subsequent analysis will reveal, the model is also related to the asymmetric auctions literature. Since Griesmer et al. (1967), most of this literature focuses on the existence and characterization of first-price auction equilibria (Lebrun 1996; Bajari 2001; Maskin and Riley 2000b). Some work has been done on the comparative statics of revenues across auction rules, and results have been obtained for specific forms of asymmetry (McAfee and McMillan 1989; Maskin and Riley 2000a; Arozamena and Cantillon 2004). In the current framework, not only is there a natural form of asymmetry, but this form also allows for sharp conclusions regarding how the buyer's information should be gathered, disclosed and utilized in the auction.

[2] Rezende (2000) provides another example of a model in which product specifications are chosen by the suppliers.

3 Procurement under commitment

We begin by considering the problem of a buyer with commitment power to use any auction rule to procure the item. Following the approach of Myerson (1981) and McAfee and McMillan (1989), it can be shown that an auction that maximizes the buyer's revenue is as follows: i) the buyer always acquires information about Δ; ii) the buyer implements a direct revelation mechanism, where sellers are asked to provides reports $\hat{c}_1$ and $\hat{c}_2$ of their costs; iii) the winner is selected as follows: seller 1 is the winner if $\hat{c}_1 < \hat{c}_2 + \phi(\hat{c}_2, \Delta)$ and to seller 2 if $\hat{c}_1 > \hat{c}_2 + \phi(\hat{c}_2, \Delta)$,[3] where $\phi(x, \Delta)$ is the optimal bias function that implicitly solves

$$\Delta = \phi + \frac{F(x+\phi)}{f(x+\phi)} - \frac{F(x)}{f(x)};$$

iii) If 1 wins, he gets paid $\hat{c}_2 + \phi(\hat{c}_2, \Delta)$; and if 2 wins, he gets paid $\hat{c}_1 - \phi(\hat{c}_2, \Delta)$.

The optimal auction calls for introducing a bias ϕ in favor of the preferred seller, but this bias is smaller than the "honest" bias Δ:

Proposition 1 *If F is regular, ϕ has the same sign as Δ at all points.*
If F is log-concave,[4] $|\phi| \leq |\Delta|$ *at all points.*

Proof See Appendix.

As in the context of an asymmetric auction (see, for example, McAfee and McMillan (1989)), the optimal mechanism introduces an inefficiency in favor of the disadvantaged bidder. Here, the optimal bias is determined by trading off the two effects in the buyer's profits:

The efficiency effect: Moving the bias toward Δ leads to an improvement in the value of the trade, since it makes the best supplier more likely to win;
The competition effect: Moving the bias away from zero reduces competition between the suppliers, increases their mark-ups, and increases the price to be payed.

In order to maximize efficiency, the bias should be equal to Δ; in order to maximize competition, it should be zero. In general the optimal bias lays between these two targets.[5]

4 Procurement under no commitment

In this section we study procurement when the buyer lacks commitment power to implement the optimal mechanism. For concreteness we study how procurement takes

[3] We will not explicitly discuss the problem of how to break a tie, since ties in costs occur with probability zero in this setting. We could assume that an endogenous tie-breaking rule is in place: in the event of a tie the sellers are given an opportunity to break the tie voluntarily. Endogenous tie-breaking rules are discussed in Milgrom (1989); Simon and Zame (1990); Jackson et al. (2002).

[4] A distribution F is *log-concave* if $\log F$ is concave or, equivalently, if $\frac{F(x)}{f(x)}$ is increasing in x.

[5] Working with the more restricted set of mechanisms that involve a linear scoring rule, Burguet and Che (2004) have found that the same property holds in their model. However, the logic behind underplaying quality differences is not the same: there, the purpose was to reduce the scope for corruption; here, it is to magnify the effect of competition.

place after the buyer learns the value of Δ; if Δ is not known, the same analysis will apply, substituting $E[\Delta|I]$ for Δ in what follows, where I is the information available to the buyer at the start of the auction.

In an environment without commitment, standard mechanism design tools cannot be used. Vartiainen (2007) proposes a setting to study mechanism design under no commitment, and applying his methods to a standard auction setting he finds that without external commitment devices, the only mechanism that can be used is the English auction.

We investigate a specific auction protocol that generalizes the standard English auction to the biased procurement setting: an open descending price auction with alternating bids where the price of each seller is allowed to be different. In view of the result in Vartiainen (2007), we believe specializing the analysis to this particular auction rule is without loss of generality. For example, the optimal auction under commitment discussed in the previous section can be implemented using the auction protocol described in this section. [6]

In addition, and consistently with the approach of Vartiainen (2007), we assume the buyer can engage in further negotiations after the auction is over with the prospective winner. Thus, during the auction the bids made by the sellers are promises to deliver the good at those prices, but the buyer is not committed to accept the auction price necessarily.

4.1 The auction protocol

We study an open auction with alternating bids. Each bidder has his own ask price, represented by p_1 and p_2. Initially $p_1 = p_2 = \bar{c}$, the highest possible cost.

At each round, the buyer has the option of disclosing information about Δ. We model communication as *event disclosure*, in the spirit of Grossman (1981) and Milgrom (1981): the buyer announces an event that has happened (i.e., a set that contains the true realization of Δ).

After the information disclosure, the current bidder (1 in the odd rounds, 2 in the even rounds) then decides whether to drop his current ask price by a fixed bid decrement ι or not — we call the first action "drop", and the second "stop".

Formally, we represent the auction as follows: let $t = 1, 2, \ldots$ index the round of the auction. At the start of each round, the buyer discloses a measurable set $E_t \subset \mathbb{R}$, with the restriction that $\Delta \in E_t$. This disclosure is public: both sellers observe E_t. After this the seller whose turn it is (1 if t is odd, 2 if even) takes an action $a_t \in \{$drop, stop$\}$.

The auction stops when both bidders play "stop" consecutively. At this point the buyer selects one of the suppliers. He can close the deal at the winner's ask price, or engage in further bargaining with the winner.

The bargaining is modeled as follows. The buyer makes a sequence of price offers, and the selected seller accepts or rejects them. We assume players face a time discount factor across stages in the bargaining process, and are interested in characterizing equilibrium behavior when this factor converges to one.

This part of the game corresponds to a model extensively studied in the literature of bargaining with one-sided offers and durable good monopolies (Coase 1972; Gul

[6] With the caveat that if the bid decrements used in the descending price auction are large, then it implements an approximation of the optimal allocation.

et al. 1986; Sobel and Takahashi 1983; Fudenberg et al. 1985). We do not explicitly discuss actions taken in the bargaining stage, but rather apply results found in the literature.[7]

Let p_i be the final ask price of seller i in the auction. At this point, the buyer has a belief about c_i, given by the history and the equilibrium strategy played by i. Let T_i be the support of this belief, and let $x = \sup T_i$. As long as bidding below cost never happens in equilibrium, $x \leq p_i$.

Since $\theta_i > \bar{c} \geq p_i \geq x$, we are in what the literature refers to as the "gap case": there is a gap between the price at which all types of the seller accept to trade and the maximum price the buyer is willing to pay. In the gap case, Fudenberg et al. (1985) have shown that there is a generically unique equilibrium of the bargaining game, and as the time discount factor goes to one, the Coase conjecture applies: trade occurs with probability 1 at the price x. Rather than replicating the analysis of the bargaining phase, we focus on the actions taken in the auction phase, assuming that the final transaction price payed by an uncommitted buyer is $y = x = \sup T_i$.

We investigate the properties of perfect Bayesian equilibria in pure strategies of the auction game. Let a history H_t be a list of all the actions taken up to round t: $H_t = (E_1, a_1, \ldots, E_{t-1}, a_{t-1})$. Define similarly $H_t^1 = (E_1, a_1, \ldots, E_t)$, for t odd and $H_t^2 = (E_1, a_1, \ldots, E_t)$, for t even; these are the possible histories before sellers 1 and 2 take an action.

A strategy s_1 for seller 1 (resp. s_2 for seller 2) is a mapping between pairs of odd-indexed (resp. even-indexed) histories and cost types to actions: $s_i(H_t^i, c_i) = a_t \in \{\text{stop,drop}\}$. A strategy $s_B = (\mathcal{E}, w, y)$ for the buyer is a mapping that assigns: i) for every history and realization of Δ, a set $\mathcal{E}(H_t|\Delta) \ni \Delta$; ii) for every history where bidding stopped (that is, any history H_t where $a_{t-1} = a_{t-2} = \text{stop}$), a choice of winner $w(H_t, \Delta) \in \{1, 2\}$ and a price $y(H_t, \Delta)$ payed to the winner.

During the auction, as actions are taken players update beliefs about the other players' private information. Since actions are public and priors are identical, both sellers hold the same belief $\mu_B(H_t^i)$ about the buyer's information given the current history (formally, μ_B is a mapping between histories and distributions over the real line). Let $P_i(H_t^i, \mu_B, s_B)$ be the probability of seller i winning if the auction ends at this point.[8]

Let $\mu_i(H_t)$ be the belief the buyer (and the other seller) have about i's private information c_i, given the history H_t. Let $x_i(\mu_i(H_t))$ be the supremum of the support of $\mu_i(H_t)$.

An *equilibrium* is a profile $(s_B, s_1, s_2, \mu_B, \mu_1, \mu_2)$ such that: i) given (s_B, s_1, s_2), beliefs μ_B, μ_1 and μ_2 satisfy Bayes law for every history that occurs with positive probability in equilibrium; ii) for every history H_t where the auction ends $s_B = (\mathcal{E}, w, y)$ assigns a payment $y(H_t, \Delta) = x_i(\mu_i(H_t))$, where $i = w(H_t)$; iii) actions of the sellers, information disclosure and the winner selection specified by s_1, s_2 and s_B are sequentially rational. In other words, we study a perfect Bayesian equilibrium where the final transaction price is $x_i(\mu_i(H_t))$.

[7] A difference between the current framework and the standard durable good monopoly model is that in our game in some cases both sides are privately informed. However since it is common knowledge that the buyer has strictly positive profits at any price at or below the auction price, his private information does not fundamentally affect the analysis.

[8] If the last two actions taken by sellers in H_t^i were not stop, $P_i(H_t^i, \mu_B, s_B)$ is the probability conditional on the next two actions being stop (for all sellers).

4.2 Full disclosure of information

We now describe an equilibrium that involves full disclosure of information by the buyer.

Let s_1^* be the following strategy for bidder 1: $s_1^*(H_t^1, c_1) =$ drop if $p_1 - \iota > c_1$ and $\Pr(p_1 < p_2 + \Delta | H_t^1) = 0$, otherwise stop. Similarly, define s_2^* as: $s_2^*(H_t^2, c_2) =$ drop if $p_2 - \iota > c_2$ and $\Pr(p_1 > p_2 + \Delta | H_t^2) = 0$, otherwise stop.

If s_i^* is played, i only drops his price at a history where enough information is revealed to make him believe he will lose for sure at the current prices. Let $\mathcal{H}$ be the set of histories where every seller has acted this way in the past.[9]

Let $s_B^* = (\mathcal{E}^*, w^*, y^*)$ be the following strategy for the buyer: i) For every H_t, $\mathcal{E}^*(H_t, \Delta) = \{\Delta\}$; ii) if the auction ends at $H_t \in \mathcal{H}$, if $p_1 < p_2 + \Delta$ then $w^*(H_t, \Delta) = 1$ and $y^*(H_t, \Delta) = p_1$; if $p_1 > p_2 + \Delta$ then $w^*(H_t, \Delta) = 2$ and $y^*(H_t, \Delta) = p_2$; iii) if the auction ends at $H_t \notin \mathcal{H}$, then the buyer acts as if the cost of the seller that dropped unexpectedly is $\underline{c}$, and selects the winner accordingly.[10]

Let $\mu_B^*(H_t^i)$ be any belief compatible with s_B^*. Let $\mu_i(H_t)$ as follows: if $H_t \notin \mathcal{H}$ and i is the seller that dropped unexpectedly, $\mu_i(H_t)$ assigns probability 1 to $c_i = \underline{c}$; otherwise, let $\mu_i(H_t)$ be the distribution of c_i conditional on $c_i < p_i$, where p_i is the current price at history H_t.[11]

Proposition 2 *$(s_B^*, s_1^*, s_2^*, \mu_B^*, \mu_1^*, \mu_2^*)$ is an equilibrium.*

Proof See Appendix. □

A key property of optimal behavior of the sellers in this equilibrium is a reluctance to bid: sellers only lower their ask prices when they believe they will lose the auction for sure otherwise. The next result shows that this is a general property of equilibrium bidding in the absence of commitment:

Lemma 1 (Secrecy) *Let $(s_B, s_1, s_2, \mu_B, \mu_1, \mu_2)$ be any equilibrium. Consider a history H_t^i, and let $p_i 0$ be i's current price. If i) $x_i(\mu_i((H_t^i, stop))) = p_i$ and ii) $P_i(H_t^i, \mu_B, s_B) > 0$, then s_i calls for bidder i is to stop, for* all *types c_i.*

Proof Suppose that there are some types of seller i that prefer to drop in equilibrium, and let x be the supremum of the types that prefer to drop. Since all types with costs above $p_i - \iota$ will have negative profits if they drop the price, we know $x \leq p_i - \iota$.

Take a type $c_i = x - \epsilon$, $\epsilon \geq 0$. If this type drops, in equilibrium he reveals that his type is (weakly) below x, and as a result anticipates the bargaining negotiation price will be x. Therefore his expected profits by dropping are at most $(x - c_i) = \epsilon$. On the other hand, since by assumption the supremum of the cost types that would have

[9] That is, $H_t \in \mathcal{H}$, if, for any odd period k where $a_k =$ drop, $\Pr(p_1 < p_2 + \Delta | H_k^1) = 0$, and for every even period k where $a_k =$ drop, $\Pr(p_1 > p_2 + \Delta | H_k^1) = 0$.

[10] Formally, if 1 is the seller that played $a_k =$ drop with $\Pr(p_1 < p_2 + \Delta | H_k^1) > 0$, $\underline{c} < p_2 + \Delta$ then $w^*(H_t, \Delta) = 1$ and $y^*(H_t, \Delta) = \underline{c}$.

[11] In histories out of the equilibrium path, μ_i is extremely optimistic, assigning probability one to $c_i = \underline{c}$. These beliefs are justified by the following argument, in the spirit of universal divinity (Banks and Sobel 1987): suppose a seller with cost c_i decides to drop when $P(H_t^i, \mu_B^*, s_B^*) > 0$. If there is a set of future circumstances that makes this deviation profitable by increasing i's probability of winning, then the same set of circumstances make the same deviation even more profitable to a seller with cost $\underline{c} < c_i$. As a result, $\underline{c}$ is the type most likely to be the one that deviated.

stopped is p_i, by stopping he obtains $(p_i - c_i)P(H_t^i, \mu_B, s_B) \geq (\iota + \epsilon)P(H_t^i, \mu_B, s_B) > 0$. For all values of ϵ arbitrarily close to zero stopping is optimal, which contradicts the definition of x. □

In making their decisions during the bidding process, sellers anticipate that the final transaction price depends on the buyer's beliefs about their costs. As a result, they strive to be secretive, by refraining from dropping their bid (and thus revealing their eagerness to win the contract) unless when strictly necessary.

A consequence of the secrecy lemma is the following:

Proposition 3 *If the buyer does not reveal who is the prospective winner at beginning of the auction, the unique best response for both sellers is to stop.*

Proof Suppose that at the beginning of the auction there is uncertainty about who is the seller with an advantage and the buyer refuses to disclose this piece of information. From the Secrecy lemma we know that both sellers will stop in equilibrium, no matter how low their actual costs are. □

Thus, if the buyer does not disclose information about Δ, the auction breaks down, in the sense that sellers implicitly refuse to compete and settle for extremely high prices in any equilibrium.

4.3 Discussion

In this game, the buyer fully discloses information about Δ because otherwise sellers refuse to bid down their prices. The possible breakdown of the auction is perhaps surprising, and deserves further discussion.

The breakdown depends on two ingredients: some degree of uncertainty about the winning criterion and lack of commitment by the buyer.

If there was uncertainty about the decision criterion of the buyer but commitment to honor the final auction price, equilibrium prices would be similar to the ones obtained in duopoly model: final prices would be above costs even for the losing seller, and mark-ups would depend on the the available information about Δ; disclosure of information would tend to reduce mark-ups, as it makes demand more elastic. In particular, if the uncertainty about Δ is small, the price outcomes would be near the ones obtained with full disclosure.

On the other hand, if the buyer is not committed to the auction prices, then by the arguments in the previous section even a small amount of uncertainty about Δ leads to very high prices. In other words, the only mechanism that can be successfully implemented by an uncommitted buyer is an English auction without uncertainty. This is because this is the only mechanism that elicits information from the bidders in a way that selects the efficient allocation while revealing minimal information about the winner's costs. If there is uncertainty about Δ, this property no longer holds.

In this paper we investigated only the polar cases where the buyer is either fully committed or fully uncommitted. The case of partial commitment is beyond the scope of the paper for two reasons: first, there is no single natural way of introducing partial commitment in the model, and this choice should probably be guided by features of specific applications; second, for at least some choices the model becomes significantly less tractable.

For example, suppose that the buyer is free to engage in one round of negotiation with the selected seller, but can commit not to engage in a second round if his offer is rejected (that is, he has commitment power to make a single take-it-or-leave-it offer after the auction).[12] In this case, the final transaction price will be the auction price minus a mark-down that depends of the buyer's beliefs about the seller's cost at that time. Since the sellers anticipate the mark-down, they will not only refrain from bidding down to cost but will also avoid to reveal information that increases the mark-down. It is thus possible that the secrecy lemma generalizes, but a formal analysis is difficult since it involves a dynamic game where the pay-off relevant state is a pair of belief probability measures that may evolve over time in nontrivial ways.

5 Information Gathering

We have shown that a buyer without commitment power will fully disclose the information he has about Δ prior to the auction. A final question remains: should the buyer invest in learning Δ in the first place, given that he anticipates that future course of action? Intuition tells that this information benefit is likely to be lower in this circumstance, since the outcome is less than optimal. This section establishes that in fact the benefit of learning Δ can be *negative*: the buyer may be *worse-off* by doing so.

Consider the following simple example: suppose that $c_1 = c_2 = 0$, and the value of one of the sellers is 11, while the other is 9 (that is, $\bar{\theta} = 10$, $\Delta = 2$ or -2). Information about the identity of the preferred seller is socially beneficial: with it, gains from trade are 11-0 = 11; without it, the expected gains are 10.

The expected profits of the buyer without information are also 10, as sellers ask a price of $p_i = c_i = 0$. However, if the identity of the preferred seller is known, that changes his behavior in the auction: if for example seller 1 is the preferred seller, he will only lower his price to $p_1 = c_2 + \Delta = 2$; similarly, if 2 is the preferred seller, $p_2 = c_1 - \Delta = 2$. As a result, the profits of the buyer with the information are $11 - 2 = 9 < 10$!

This example shows that information about Δ may hurt the buyer. We next show that this may be true as well in expectation.

The argument hinges on the shape of the buyer's expected profit as a function of Δ. To simplify the discussion, we assume that the bid decrements are negligible in this section.[13]

We impose an inequality on the cost distribution. Let $f_{c_1-c_2}$ represent the density of the random variable $c_1 - c_2$. The next proposition requires that, for all values d in the support of Δ,

$$f_{c_1-c_2}(d) \geq f(\bar{c} - |d|);$$

In other words, we need that the event that $c_1 - c_2 = \Delta$ is more likely than the event $c_2 = \bar{c} - \Delta$ (or $c_1 = \bar{c} + \Delta$, if $\Delta < 0$).

[12] I thank George Deltas for suggesting me this alternative.

[13] The outcome of the auction with non-negligible decrements is as follows: let $\lfloor \Delta \rfloor$ and $\lceil \Delta \rceil$ be the multiples of ι below and above Δ; similarly, let $\lceil c_i \rceil = \min_k \{\bar{c} - k\iota | \bar{c} - k\iota > c_i, k \in \{0, 1, \ldots\}\}$, $i = 1, 2$. Then in equilibrium if $c_1 < \lceil c_2 \rceil + \lfloor \Delta \rfloor$, 1 wins and pays $\lceil c_2 \rceil + \lfloor \Delta \rfloor$; if $c_2 < \lceil c_1 \rceil - \lceil \Delta \rceil$, 2 wins and pays $\lceil c_1 \rceil + \lceil \Delta \rceil$. As $\iota \to 0$, $\lceil c_i \rceil \to c_i$ and $\lfloor \Delta \rfloor, \lceil \Delta \rceil \to \Delta$, and the outcome becomes: if $c_1 < c_2 + \Delta$, 1 wins and pays $c_2 + \Delta$; if $c1 > c_2 + \Delta$, 2 wins and pays $c_1 - \Delta$.

Proposition 4 *If* $f_{c_1-c_2}(d) \geq f(\bar{c}-|d|)$ *for all* d *in the support of* Δ*, then it is optimal for the buyer not to acquire information about* Δ*.*

Proof See Appendix. □

The buyer's expected profits are the difference between expected value and expected price to be paid. A realization of Δ one unit higher than expected (holding $\bar{\theta}$ fixed) means that θ_1 is up by half unit and θ_2 is down by half unit. So the expected effect is $1/2\,\Pr(1 \text{ wins}) - 1/2\,\Pr(2 \text{ wins})$, and the expected value is convex in Δ.

However, the effect through expected price is stronger. A one-unit-higher Δ leads to an increase in p_1 by one unit (as long as $p_1 < \bar{c}$), and a decrease in p_2 by one unit. So ignoring the cases where $p_i = \bar{c}$, we find that the curvature of the expected price is twice as strong as the expected value, and as a result the expected profit is concave. The condition in the proposition guarantees that the effect of the region where $p_i = \bar{c}$ is not strong enough to upset this conclusion.

6 Concluding Remarks

A buyer that contemplates the possibility of using a procurement auction to meet a particular demand does so because the demand is indeed particular: the good in question is often complex, and requires substantial effort to produce, design or even describe. While this complexity certainly makes costs and therefore prices uncertain, it also makes products offered by different suppliers different along other dimensions.

For a buyer of such products, a matter of great importance is how he should act on perceived differences along these other dimensions. This paper has provided answers to some natural questions about the treatment of information on these differences: what are the incentives to collect, disclose, and act on this information. We have have obtained three main results.

The first result is that in the optimal auction the buyer should commit to bias its choice of supplier toward the preferred one, but the bias should be less then the true perceived difference in value. Thus, from an efficiency perspective, the optimal allocation favors the weaker supplier. The reason is that the optimal bias is a compromise between two conflicting objectives: to maximize the efficiency of the allocation, that would ask for full bias, and to maximize the degree of competition between suppliers, that would ask for no bias.

The two other results hold when the buyer lacks commitment power to implement the optimal auction. One of them may be translated in practical terms to the following piece of advice for the buyer: "It is a bad idea to let the suppliers bid without knowing how you will choose the winner". The reason for that is that with this uncertainty, open bidding no longer has the property of revealing the suppliers' costs in a way that is immune to opportunism by the buyer. Without this feature, the resulting equilibria have very high prices as outcomes, as suppliers bid cautiously in order not to reveal information that can be used against them in the future.

The last result is that learning about differences in suppliers' products may *decrease* the buyer's expected profits if he lacks commitment power. The reason is that while learning about this difference allows the buyer to pick the best supplier, it also implicitly places this supplier in a stronger bargaining position. So the gain in efficiency with this information is more than fully captured by the suppliers through higher equilibrium prices.

In a complex procurement, elicitation of information on the buyer's preferences is not a matter of personal introspection: proposals are lengthy documents, that requires costly specialized knowledge to read and evaluate. In this context, it is feasible for a buyer to publicly decide not to learn about his preferences: it is a matter of not investing in proposal evaluation.

In practice, of course, many other considerations arise that are absent in the model and make information about preferences valuable. First, in the model we have assumed that the buyer will necessarily acquire the item, thus making information about average quality $\bar{\theta}$ worthless; in practice, a buyer may opt for not acquiring the item from any seller, and in this case it is worth learning about $\bar{\theta}$. Second, in the model there are only two suppliers; in a model with many suppliers, the value of information depends on the suppliers ranking.[14]

Therefore, the finding should not be taken as a general recommendation, but rather as a remark that it may be optimal to invest less in proposal evaluation for uncommitted buyers, as it may place the the favorite supplier in a strong bargaining position and result a higher price.

Also, from a theoretical perspective the finding creates an interesting trade-off between allocational and informational efficiency: increasing the buyer's ability to commit allows him to exert monopsonistic power (and therefore introduces allocational inefficiency) but provides the right incentives to collect information, while on the other hand without commitment one would have allocational efficiency, but incorrect incentives to collect information.

A Appendix: Proofs

Proof (of Proposition 1) For the first claim, we can write the equation that defines ϕ as

$$\Delta = \left(x + \phi(x,\Delta) + \frac{F(x+\phi(x,\Delta))}{f(x+\phi(x,\Delta))}\right) - \left(x + \frac{F(x)}{f(x)}\right).$$

Since by regularity $x + \frac{F(x)}{f(x)}$ is monotone increasing, ϕ should have the same sign as Δ.

For the second claim, it is enough to rewrite the definition of ϕ as

$$\Delta - \phi(x,\Delta) = \frac{F(x+\phi(x,\Delta))}{f(x+\phi(x,\Delta))} - \frac{F(x)}{f(x)}$$

and use the definition of log-concavity. □

Proof (of Proposition 2) By construction, μ_B^* is compatible with Bayes Law along the equilibrium path, given the strategies. We next show that μ_i^* is also compatible with Bayes Law. Let $W_1 = \{H_t^i | P_1(H_t^i, \mu_B^*, s_B^*) > 0\}$; this is the set of histories in which seller 1 (and 2) believes that if the auction ends at current prices, 1 will be selected winner with positive probability. Likewise, let $W_2 = \{H_t^i | P_2(H_t^i, \mu_B^*, s_B^*) > 0\}$.

We show by finite induction that during the bidding process, the supremum of the set of costs c_i consistent with the history so far is p_i, the seller's current price. As a result, μ_i^* satisfies Bayes Law.

Initially the claim is valid, since $p_1 = p_2 = \bar{c}$. Suppose the claim is true for all previous histories. If the current history is in $W_1 \cap W_2$, all types of both sellers stop, and the buyer does not learn any new information by observing bidding. If the history is in $W_1 \setminus W_2$, all

[14] In a model with many bidders, information about the difference in values between the winner and the second best supplier has negative value, but information on the difference in values between the second and third best supplier have a positive value.

types of seller 1 stop, and all types of seller 2 with cost below $p_2 - \iota$ drop. As a result, the buyer does not update his beliefs about seller 1, and if seller 2 drops, the buyer only learns that $c_2 < p_2 - \iota = p_2'$, the new price. The argument is analogous for a history in $W_2 \setminus W_1$. We thus verify the claim.

Next, we verify rationality of w^*. Consider the winner selection problem for a history where price reached (p_1, p_2) and both bidders stopped. If the buyer selects seller i, he will obtain at the bargaining stage price p_i; thus his optimal choice is to pick 1 if $p_1 < p_2 + \Delta$ and 2 if $p_1 > p_2 + \Delta$.

Third, $\mathcal{E}^*$ is also rational: it is always optimal for the buyer to disclose information about Δ to convince one of the sellers that he will lose; if not, both players will stop and the auction will end, in which case he will obtain a price p_1 or p_2. If he instead reveals the prospective winner, bidding continues, which is always weakly better.

We now consider the incentives for the sellers. Suppose seller 2 follows s_2^* and the buyer follows s_B^*. Suppose, by the hypothesis of induction, that seller 1 has followed s_1^* in the past, and is considering a deviation at a history with prices p_1 and p_2.

If the current history is not in W_1, and $p_1 - \iota > c_1$, strategy s_1^* calls for seller 1 to drop. The expected profits if he follows s_1^* and drops are (weakly) positive. Suppose 1 deviates and stops in the current period. In all future histories after this deviation in which he auction ends before 1 drops his price, the probability of 1 winning is zero, since the 1's ask price is the same and 2's ask price is weakly lower. For all these histories, 1's expected profits are 0. Suppose then that 1 deviates from s_1 by stopping in the current period and dropping in the future; this delay cannot be profitable, since even if the auction does not end during the period, 1's price will be the same (namely, $p_1 - \iota$) and 2's price will be lower, which makes the probability of 1 winning lower. We conclude it is not profitable to not drop in these conditions.

If the current history is in W_1 and $p_1 - \iota > c_1$, by stopping seller 1 obtains (weakly) positive profits. If seller 1 drops, the buyer updates his belief and from now on assumes 1's cost is $\underline{c}$. As a result, expected profits from a deviation are zero. Thus, is its optimal not to drop.

Finally, dropping p_1 below c_1 does not increase profits, since lower prices lead to a higher probability of winning. We conclude that s_1^* is a best response. The argument for seller 2 is symmetric. □

Proof (of Proposition 4)

Suppose for concreteness that $\Delta \geq 0$. (The case $\Delta < 0$ is symmetric.) The realized profit for the buyer, given a realization (Δ, c_1, c_2), is

$$\pi(\bar{\theta}, \Delta, c_1, c_2) = \bar{\theta} + \begin{cases} -\Delta/2 - (c_1 - \Delta), & \text{if } c_2 + \Delta < c_1; \\ +\Delta/2 - (c_2 + \Delta), & \text{if } c_1 < c_2 + \Delta < \bar{c}; \\ +\Delta/2 - (\bar{c}), & \text{if } c_2 + \Delta \geq \bar{c}. \end{cases}$$

The clauses represent the profits when 2 wins, when 1 wins and is payed $c_2 + \Delta$ and when 1 wins and is payed $\bar{c}$, respectively. Notice that π is a continuous function.

Let $\Pi(\bar{\theta}, \Delta) = \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} \pi(\bar{\theta}, \Delta, c_1, c_2) dF(c_1) dF(c_2)$ be the buyer's expected profits given $(\bar{\theta}, \Delta)$. Then

$$\begin{aligned} \Pi(\bar{\theta}, \Delta) = \bar{\theta} &+ \int_{\underline{c}}^{\bar{c}-\Delta} \int_{c_2+\Delta}^{\bar{c}} (\Delta/2 - c_1) dF(c_1) dF(c_2) \\ &+ \int_{\underline{c}}^{\bar{c}-\Delta} \int_{\underline{c}}^{c_2+\Delta} (-\Delta/2 - c_2) dF(c_1) dF(c_2) + \int_{\bar{c}-\Delta}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} (\Delta/2 - \bar{c}) dF(c_1) dF(c_2). \end{aligned}$$

Applying Leibniz's rule,

$$\begin{aligned} \frac{\partial}{\partial \Delta} \Pi(\bar{\theta}, \Delta) = {} & 1/2 \Pr(c_2 + \Delta < c_1) - 1/2 \Pr(c_1 < c_2 + \Delta < \bar{c}) + 1/2 \Pr(c_2 + \Delta \geq \bar{c}) \\ & - \int_{\bar{c}}^{\bar{c}} (\Delta/2 - c_1) dF(c_1) f(\bar{c} - \Delta) - \int_{\underline{c}}^{\bar{c}-\Delta} (-\Delta/2 - c_2) f(c_2 + \Delta) dF(c_2) \\ & - \int_{\underline{c}}^{\bar{c}} (\Delta/2 - \bar{c}) dF(c_1) f(\bar{c} - \Delta) + \int_{\underline{c}}^{\bar{c}-\Delta} (-\Delta/2 - c_2) f(c_2 + \Delta) dF(c_2) \end{aligned}$$

$$+\int_{\underline{c}}^{\bar{c}}(\Delta/2-\bar{c})f(\bar{c}-\Delta)dF(c_2)$$
$$=1/2-\Pr(c_1<c_2+\Delta<\bar{c}).$$

(Since π is continuous, all the terms involving integrals cancel out).

We thus obtain

$$\frac{\partial^2}{\partial\Delta^2}\Pi(\bar{\theta},\Delta)=-[f_{c_1-c_2}(\Delta)-f(\bar{c}-\Delta)].$$

By assumption, this expression is negative. Therefore, Π is concave. By the Jensen inequality, expected profits are higher when the buyer does not learn Δ. □

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