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Study of Thermally Induced Vibration of Non-Homogeneous Trapezoidal Plate with Varying Thickness and Density

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Abstract The goal of present investigation is to study the effect of thermal gradient on the vibrations of non-homogeneous trapezoidal plate whose thickness varies parabolically and density varies linearly. Effect of other plate parameters such as non-homogeneity constant, taper constant and aspect ratios have also been studied. C-S-C-S boundary condition with two term deflection is taken into consideration. Rayleigh-Ritz method is used to find the solution of the problem. Results are calculated with great accuracy and presented in tabular form. Comparison of the results with published data has also shown.

Keywords Vibration, Trapezoidal Plate, Thermal Gradient, Non-Homogeneity, Parabolically Varying Thickness

1. Introduction

Thermal effect on vibration of non-homogenous plates are of great interest in the field of engineering such as for better designing of gas turbines, jet engine, space craft and nuclear power projects. Such structures are exposed to high intensity heat fluxes and thus material properties undergo significant changes, in particular the thermal effect on the modules of elasticity of the material can not be taken as negligible.

Plates of variable thickness are frequently used in order to economize the plate material or to lighten the plates, especially when it is used in the wings of high-speed and high performance aircrafts. The study of vibration of plates has acquired great importance in the field of research, engineering and space technology. In the engineering we cannot move without considering the effect of vibration because almost machines and engineering structuresexperie nces vibrations. Structures of plates have wide applications in ships, bridges etc. In the aeronautical field, analysis of thermally induced vibrations in non-homogeneous plates of variable thickness has a great interest due to their utility in aircraft wings.

Many analyses show that plate vibrations are based on non-homogeneity of materials. Non-homogeneity can be natural or artificial. Non-homogeneous materials such as plywood, delta wood, fibre-reinforced plastic etc, are used in engineering design and technology to strengthen the construction. Study of the effect of vibration cannot be restricted only in the field of science but, our day to day life is also affected by it. Vibration of plates of various shapes, homogeneous or non-homogeneous, orthotropic or isotropic, with or without variation in thickness, have been studied by various authors, with or without considering the effect of temperature.

A large number of researchers have reported about the vibration analysis of different types of plates. Some of them are mentioned here.

Sharma *et al.*[1] studied the free transverse vibrations of non-homogeneous circular plates of linearly varying thickness. Gupta *et al.*[2] worked on the vibrations of non-homogeneous rectangular plate of variable thickness in both directions with thermal gradient effect. Gutierrez and Laura[3] calculated the fundamental frequency of vibrating rectangular, non-homogeneous plates. Gupta and Kumar[4] studied the effect of thermal gradient on free vibration of non-homogeneous visco-elastic rectangular plate of parabolically varying thickness.

Gupta *et al.*[5] observed the transverse vibration of non-homogeneous orthotropic visco-elastic circular plate of varying parabolic thickness. Lal et *al* [6] studied the transverse vibrations of non-homogeneous rectangular plates of uniform thickness using boundary characteristic orthogonal polynomials. Gupta *et al.*[7] observed the effect of non-homogeneity on the free vibrations of orthotropic visco-elastic rectangular plate of parabolic varying thickness. Gupta and Kumar[8] studied the thermal effect on vibration

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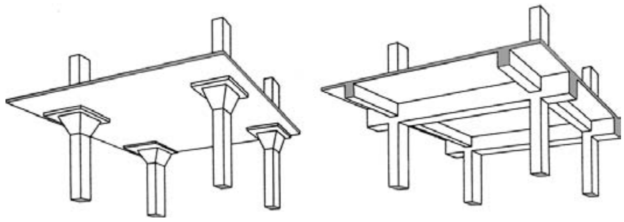
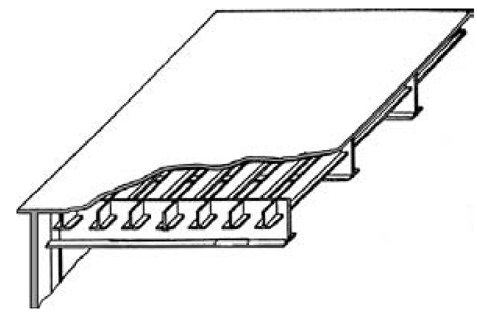
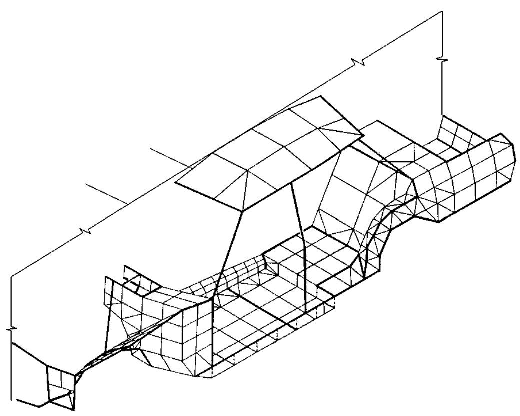
of non-homogenous visco-elastic rectangular plate of linear varying thickness. Gupta *et al*.[9] worked on the vibration of visco-elastic orthotropic parallelogram plate with parabolically thickness variation. El-Sayad and Ghazy[10] studied the Rayleigh-Ritz method for free vibration of midline trapezoidal plates. Gupta and Kumar[11] did the free vibration analysis of non-homogeneous visco-elastic circular plate with varying thickness subjected to thermal gradient. Lal and Dhanpati[12] worked on the transverse vibrations of non-homogeneous orthotropic rectangular plates of variable thickness. Gupta et *al*.[13] did the vibration analysis of non-homogeneous circular plate of nonlinear thickness variation by differential quadrature method. Chakraverty et *al*.[14] noticed the effect of non-homogeneity on natural frequencies of vibration of elliptic plates. Chen et *al*[15] studied on free vibration of non-homogeneous transversely isotropic magneto-electro elastic plates. Gupta *et al*



Reinforced concrete slabs in buildings



Steel bridge deck



Automobile industry

Figure 1. Use of plates

[16] observedthe effect of non-homogeneity on vibration of orthotropic visco-elastic rectangular plate of linearly varying thickness.

Gupta and Sharma[17] observed the effect of thermal gradient on transverse vibration of non-homogeneous orthotropic trapezoidal plate of parabolically varying thickness. Chen et *al*[18] studied the free vibration of cantilevered symmetrically laminated thick trapezoidal plates. Bambill et *al*.[19] studied the transverse vibrations of rectangular, trapezoidal and triangular orthotropic, cantilever plates. Gupta and Sharma[20] observed the thermally induced vibration of orthotropic trapezoidal plate of linearly varying thickness. Gurses *et al*.[21] analysed the shear deformable laminated composite trapezoidal plates. Kitipornchai et *al*.[22] discussed a global approach for vibration of thick trapezoidal plates. Gupta and Sharma[23] studied the thermal gradient effect on frequencies of a trapezoidal plate of linearly varying thickness. Gupta and Sharma[24] studied the thermal effect on vibration of non-homogeneous trapezoidal plate of linearly varying thickness.

The authors have so far not come across any paper dealing with parabolically varying thickness and linearly varying density. In the present work the effect of non-homogeneity, taper constant, thermal gradient and aspect ratios has been studied. The frequencies for the first two modes of vibration are obtained for C-S-C-S non-homogeneous trapezoidal plate by Rayleigh-Ritz method. The authenticity and accuracy of numerical results of the present work has been verified with the authors published paper[25]. Results are presented in tabular form. Comparison of results has also been presented.

2. Theoretical Formulation

2.1. Geometry of the Plate

The plate under consideration is shown in figure 2 which is thin, symmetric and non-homogeneous trapezoidal plate. Here h_0 is the maximum plate thickness occurring at the left edge and αh_0 is the minimum plate thickness occurring at the right edge.

2.2. Thickness and Density

The thickness of the plate is parabolic in x direction and is of the form

$$h(\xi) = h_0[1-(1-\alpha)(\xi+\frac{1}{2})^2] \tag{1}$$

where h_0 is thickness along the edge ξ = x/a= -1/2.

Non-homogeneity of plate arises due to variation in density which is linear in x direction and is of the form

$$\rho = \rho_0[1-(1-\alpha_1)(\xi+\frac{1}{2})] \tag{2}$$

It is assumed that the plate considered here is subjected to a steady one-dimensional temperature distribution along the length i.e. in the x direction,

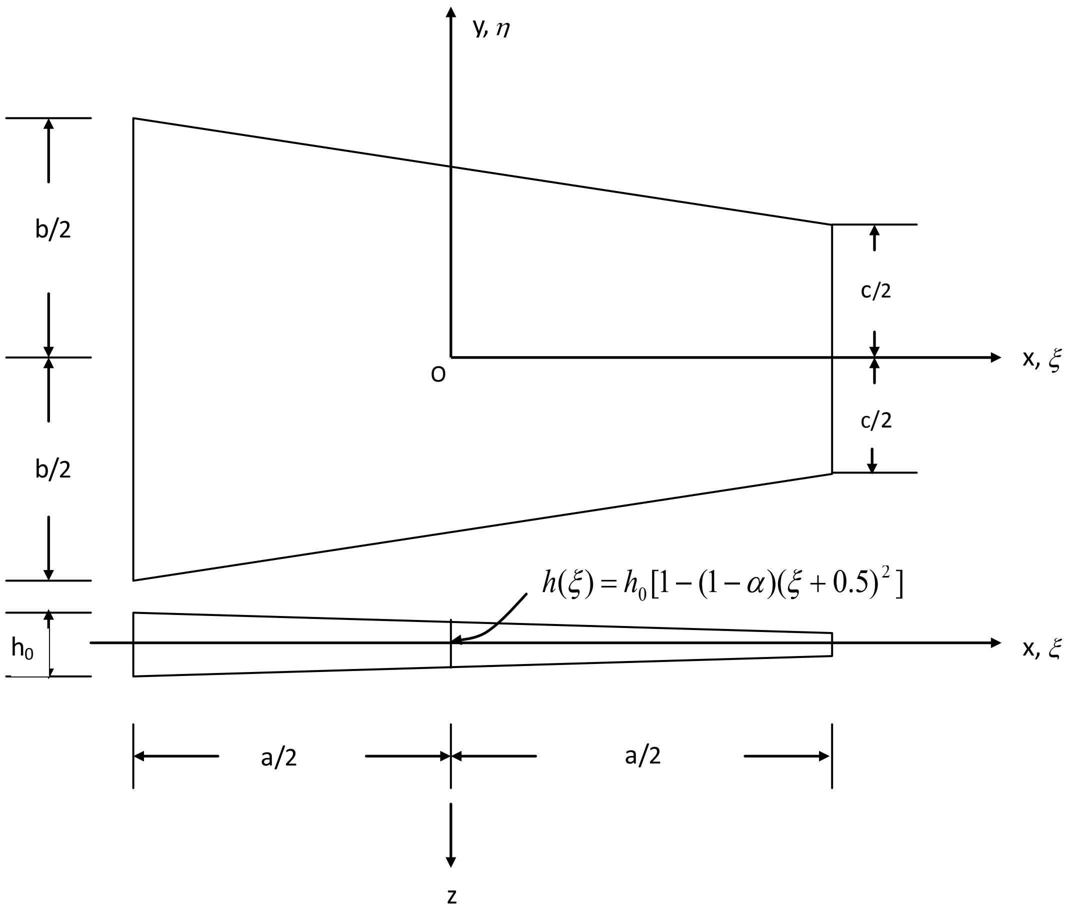


Figure 2. Geometry of trapezoidal plate with variable thickness

Nomenclature of used Symbols

a	length of the plate	ξ	non-dimensional co-ordinate = $\frac{x}{a}$
b	width of the plate at left edge	η	non-dimensional co-ordinate = $\frac{y}{b}$
ω	angular frequency	T	kinetic energy
c	width of the plate at right edge	V	strain energy
x	longitudinal co-ordinate	λ	frequency parameter
y	vertical co-ordinate	z	transverse co-ordinate
α_1	non-homogeneity constant	w	deflection function
h	plate thickness	$D(\xi)$	flexural rigidity
E	Young's modulus	β	thermal gradient
υ	Poisson's ratio	α	taper constant
τ	temperature distribution		
β	thermal gradient		
α	taper constant		
ρ	mass density per unit volume of the plate		

$$\tau = \tau_0\left(\frac{1}{2} - \xi\right) \tag{3}$$

where τ denotes the temperature excess above the reference temperature at a distance $\xi = \frac{x}{a}$ and τ_0 denotes the temperature excess above the reference temperature at the end $\xi = -\frac{1}{2}$.

For most of the elastic materials, modulus of elasticity is described as

$$E = E_0(1 - \gamma\tau) \tag{4}$$

where E_0 is the value of Young's modulus along the reference temperature i.e. at $\tau = 0$ and γ is the slope of the variation of E with τ.

On substituting value of τ from equation (3) into (4)

$$E = E_0\left(1 - \beta\left(\frac{1}{2} - \xi\right)\right) \tag{5}$$

where $\beta = \gamma\tau_0$ $(0 \le \beta \le 1)$.

2.3. Equation of Motion

The governing differential equation for kinetic energy T and strain energy V is given by

$$T = \frac{ab}{2}\omega^2 \int_A h(\xi)\rho w^2 dA \tag{6}$$

and

$$V = \frac{ab}{2}\int_A D(\xi)\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial \xi^2} + \frac{1}{b^2}\frac{\partial^2 w}{\partial \eta^2}\right)^2 - 2(1-\upsilon)\left[\frac{1}{a^2 b^2}\frac{\partial^2 w}{\partial \xi^2}\frac{\partial^2 w}{\partial \eta^2} - \left(\frac{1}{ab}\frac{\partial^2 w}{\partial \xi \partial \eta}\right)^2\right]\right\}dA \tag{7}$$

in which $D(\xi)$ is the flexural rigidity of the plate, which is given by

$$D(\xi) = D_0\left[1 - (1-\alpha)\left(\xi + \frac{1}{2}\right)^2\right]^3 \tag{8}$$

Also

$$D_0 = \frac{E h_0^3}{12(1-\upsilon^2)} \tag{9}$$

is the flexural rigidity of the plate at the side $\xi = -\frac{1}{2}$, A is the area of the plate and ρ is the mass density per unit area of the plate.

Using equation (9) and (5) in (8) flexural rigidity is given by

$$D(\xi) = \frac{E_0 h_0^3}{12(1-\upsilon^2)}\left[1 - (1-\alpha)\left(\xi + \frac{1}{2}\right)^2\right]^3\left[1 - \beta\left(\frac{1}{2} - \xi\right)\right] \tag{10}$$

Using equation (10) in (7) & (1), (2) in (6)

$$V = \frac{ab}{2}\frac{E_0 h_0^3}{12(1-\upsilon^2)}\int_A \left[1 - (1-\alpha)\left(\xi + \frac{1}{2}\right)^2\right]^3\left[1 - \beta\left(\frac{1}{2} - \xi\right)\right]\left\{\begin{aligned}&\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial \xi^2} + \frac{1}{b^2}\frac{\partial^2 w}{\partial \eta^2}\right)^2 - \\ &2(1-\upsilon)\left[\frac{1}{a^2 b^2}\frac{\partial^2 w}{\partial \xi^2}\frac{\partial^2 w}{\partial \eta^2} - \left(\frac{1}{ab}\frac{\partial^2 w}{\partial \xi \partial \eta}\right)^2\right]\end{aligned}\right\}dA \tag{11}$$

$$T = \frac{ab}{2}\rho_0 h_0 \omega^2 \int_A [1 - (1-\alpha)(\xi + \frac{1}{2})^2][1 - (1-\alpha_1)(\xi + \frac{1}{2})\ ]w^2 dA \tag{12}$$

2.4. Deflection Function and Boundary Condition

A two term deflection function is taken as

$$w = A_1\left\{\left(\xi+\frac{1}{2}\right)\left(\xi-\frac{1}{2}\right)\right\}^2\left\{\eta-\left(\frac{b-c}{2}\right)\xi+\frac{b+c}{4}\right\}\left\{\eta+\left(\frac{b-c}{2}\right)\xi-\frac{b+c}{4}\right\}$$
$$+A_2\left\{\left(\xi+\frac{1}{2}\right)\left(\xi-\frac{1}{2}\right)\right\}^3\left\{\eta-\left(\frac{b-c}{2}\right)\xi+\frac{b+c}{4}\right\}^2\left\{\eta+\left(\frac{b-c}{2}\right)\xi-\frac{b+c}{4}\right\}^2 \quad (13)$$

where A_1 and A_2 are constants to be evaluated. Eq. (13) satisfy boundary conditions and provide a good estimation to the frequency. Clamped-Simply supported-Clamped-Simply supported plate is taken into consideration as shown in figure.

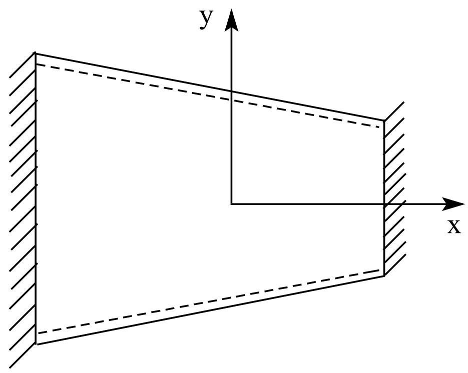


Figure 3. C-S-C-S boundary condition of a trapezoidal plate

Also for the plate considered here boundaries are defined by four straight lines

$$\eta = \frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}......................$$
$$\eta = -\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}....................$$
$$\xi = -\frac{1}{2}..$$
$$\xi = \frac{1}{2}.. \quad (14)$$

3. Method of Solution

Rayleigh-Ritz technique is used to find the solution of the problem. According to it maximum kinetic energy must be equal to maximum strain energy, so it is necessary for the problem under consideration that

$$\delta(V-T)=0 \quad (15)$$

Using equation (14) in (11) and (12)

$$.V= \frac{ab}{2}\frac{E_0\, h0^3}{12(1-\upsilon^2)}\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{\eta_1}^{\eta_2}\left[1-(1-\alpha)\left(\xi+\frac{1}{2}\right)^2\right]^3\left[1-\beta\left(\frac{1}{2}-\xi\right)\right]\left\{\begin{array}{l}\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)^2\\ -2(1-\upsilon)\left[\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right]\end{array}\right\}d\eta d\xi \quad (16)$$

$$\mathrm{T} = \frac{ab}{2}\omega^2 h_0\rho_0\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{\eta_1}^{\eta_2}([1-(1-\alpha)(\xi+\frac{1}{2})^2][1-(1-\alpha_1)(\xi+\frac{1}{2})])w^2\,d\eta d\xi \quad (17)$$

Now (15) becomes

$$\delta(V_1-\lambda^2 T_1)=0 \quad (18)$$

where

$$V_1=\frac{ab}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{\eta_1}^{\eta_2}\left[1-(1-\alpha)\left(\xi+\frac{1}{2}\right)^2\right]^3\left[1-\beta\left(\frac{1}{2}-\xi\right)\right]\left\{\begin{aligned}&\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)^2\\&-2(1-\upsilon)\left[\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right]\end{aligned}\right\}d\eta d\xi \quad (19)$$

$$T_1=\frac{ab}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{\eta_1}^{\eta_2}([1-(1-\alpha)(\xi+\frac{1}{2})^2][1-(1-\alpha_1)(\xi+\frac{1}{2})])w^2 d\eta d\xi \quad (20)$$

and $\lambda^2=\dfrac{12\omega^2\rho_0 a^4(1-\nu^2)}{E_0 h_0^{\,2}}$ is a frequency parameter.

Equation (18) involves the unknown A_1 and A_2 arising due to the substitution of w from eq (13). These two constants are to be determined from eq. (18), as follows

$$\frac{\partial}{\partial A_1}\left(V_1-\lambda^2 T_1\right)=0$$

$$\frac{\partial}{\partial A_2}\left(V_1-\lambda^2 T_1\right)=0 \quad (21)$$

On simplifying (21), One gets

$$b_{m1}A_1+b_{m2}A_2=0,\ m=1,2 \quad (22)$$

where $b_{m1},b_{m2}(m=1,2)$ involve parametric constant and the frequency parameter.

For a non-zero solution, it is desired that co-efficient of eq. (22) must be zero. So one gets the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12}\\ b_{21} & b_{22}\end{vmatrix}=0 \quad (23)$$

From eq. (23), one can obtain a quadratic equation in λ^2 from which the two values can found which constitutes first and second mode of vibration.

4. Results and Discussion

Frequencies for the first two modes of vibrations are computed for non-homogeneous trapezoidal plate whose thickness varies parabolically and density varies linearly. Different values of non-homogeneity constant (α_1), taper constant (α), thermal gradient (β) and aspect ratios (a/b, c/b) has been considered. All results are presented in tabular form.

Table 1 and 2: These tables shows the effect of taper constant α (0.0 to 1.0) on the frequency parameter λ for β = 0.0, 0.4, α_1 = 0.4, 1.0, a/b = 1.0 and c/b = 0.5, 1.0.

Table 3 and 4: These tables shows the behaviour of the frequency parameter λ with thermal gradient β (0.0 to 1.0), for α = 0.0, 0.4, α_1 = 0.4, 1.0, a/b = 1.0 and c/b = 0.5, 1.0.

Table 5, 6, 7 and 8: These tables show the effects of the frequency parameter λ with aspect ratio c/b for different combinations of α and β as follows.

(i) α = 0.0, β = 0.0

(ii) α = 0.0, β = 0.4

(iii) α = 0.4, β = 0.0

(iv) α = 0.4, β = 0.4.

Non-homogeneity constant α_1 varies from 0.0 to 0.4, aspect ratio a/b varies from 0 .75 to 1.0 and aspect ratio c/b varies from 0.25 to 1.0.

Table 9 and 10: In these tables effect of non-homogeneity constant α_1 (0.0 to 1.0) on frequency parameter has been shown. Four combinations of α and β (as in table 5 to 8), two values of aspect ratio c/b (0.5, 1.0) and one value of aspect ratio a/b (1.0) has been taken.

In **table 1 and 2** it is observed that the frequency parameter λ increases with increasing values of taper constant α for both the modes of vibration. The rate of increase as well as the value of frequency parameter λ is higher in second mode in comparison to the first mode.

Further on comparing the results of **table 1** and **2** it is found that on increasing the value of aspect ratio c/b from 0.5 to 1.0, frequency parameter decreases for both the modes of vibration.

In **table 3** and **4** it is seen that frequency parameter λ decreases with increasing values of thermal gradient β whatever be the values of other plate parameters. The rate of decrease as well as the value of frequency parameter λ is higher in second mode in comparison to the first mode. Also on increasing aspect ratio c/b from 0.5 to 1.0, frequency parameter decreases for both the modes of vibration.

It is noticed from **table 5**, **6**, **7** and **8** that the frequency parameter λ decreases with increasing values of aspect ratio c/b. Further if we compare **table 5** and **6** (a/b = 0.75) it is found that the frequency parameter decreases with increase in non-homogeneity constant α_1 i.e. from 0.0 to 0.4. Similar pattern is observed on comparing **table 7** and **8** (a/b = 1.0).

Now if we compare **table 5** and **7** in which aspect ratio a/b increased from 0.75 to 1.0 and non-homogeneity constant

α_1 is 0.0. It is observed that frequency parameter increases for both the mode of vibration. Similar pattern is observed on comparing **table 6** and **8** in which non-homogeneity constant α_1 is 0.4.

Table 9 and **10** clearly shows that frequency parameter λ decreases with increasing value of non-homogeneity constant α_1. Also when aspect ratio c/b increased from 0.5 to 1.0, frequency parameter decreases for both the mode of vibration

Table 1. Frequency parameter (λ) for a trapezoidal plate for different values of taper constant (α), thermal gradient ($\beta = 0.0, 0.4$), non-homogeneity constant (α_1 = 0.4, 1.0) and aspect ratios (a/b = 1.0), (c/b = 0.5)

α	α_1=0.4				α_1=1.0			
	$\beta = 0.0$		$\beta = 0.4$		$\beta = 0.0$		$\beta = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.0	40.256534	212.145227	38.399268	197.390839	35.065970 *35.065961	182.752281 *182.746902	33.450533 *33.450525	170.030136 *170.025133
0.2	40.694632	217.649163	38.690549	201.779358	35.374329 *35.374321	186.667554 *186.662068	33.634739 *33.634732	173.043975 *173.038890
0.4	41.315380	224.100543	39.126969	206.984913	35.846034 *35.846026	191.470899 *191.465274	33.949773 *33.949766	176.834596 *176.829402
0.6	41.135120	231.429879	39.719495	212.936786	36.494082 *36.494075	197.078225 *197.072435	34.404108 *34.404101	181.318257 *181.312931
0.8	43.162560	239.564593	40.474435	219.566355	37.324937 *37.324931	203.409607 *203.403628	35.002260 *35.002254	186.419448 *186.413969
1.0	44.399415	248.432258	41.393677	226.808352	38.339109 *38.339103	210.390192 *210.384004	35.745014 *35.745009	192.070139 *192.664490

*Comparison with author's paper[25]

Table 2. Frequency parameter (λ) for a trapezoidal plate for different values of taper constant (α), thermal gradient ($\beta = 0.0, 0.4$), non-homogeneity constant (α_1 = 0.4, 1.0) and aspect ratios (a/b = 1.0), (c/b = 1.0)

α	α_1=0.4				α_1=1.0			
	$\beta = 0.0$		$\beta = 0.4$		$\beta = 0.0$		$\beta = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.0	27.137212	129.818486	25.582627	120.041241	22.991546 *22.991536	110.916591 *110.910143	21.672240 *21.672232	102.573392 *102.567428
0.2	27.776268	135.027269	26.022172	124.151877	23.459693 *23.459684	114.739371 *114.732949	21.976609 *21.976661	105.505327 *105.499419
0.4	28.810372	141.525176	26.755336	129.293254	24.265689 *24.265681	119.698898 *119.692419	22.533901 *22.533893	109.357883 *109.351961
0.6	30.310361	149.380763	27.832078	135.501410	25.465902 *25.465893	125.829673 *125.823062	23.383283 *23.383276	114.140639 *114.134641
0.8	32.291094	158.567660	29.269554	142.751267	27.070086 *27.070079	133.090534 *133.083727	24.536971 *24.536965	119.815929 *119.809800
1.0	34.723315	169.000947	31.056561	150.979092	29.051609 *29.051602	141.395696 *141.388637	25.983783 *25.983776	126.317599 *126.311293

*Comparison with author's paper[25]

Table 3. Frequency parameter (λ) for a trapezoidal plate for different values of thermal gradient β , taper constant ($\alpha = 0.0, 0.4$), non-homogeneity constant (α_1 = 0.4, 1.0) and aspect ratios (a/b = 1.0), (c/b = 0.5)

β	α_1=0.4				α_1=1.0			
	$\alpha = 0.0$		$\alpha = 0.4$		$\alpha = 0.0$		$\alpha = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.0	40.256534	212.145227	41.315380	224.100543	35.065970 *35.065961	182.752281 *182.746902	35.846034 *35.846026	191.47089 191.465274
0.2	39.340063	204.900649	40.236848	215.712401	34.268787 *34.268778	176.505672 *176.500477	34.911436 *34.911429	184.29796 184.292563
0.4	38.399268	197.390839	39.126969	206.984913	33.450533 *33.450525	170.030136 *170.025133	33.949773 *33.949766	176.83456 176.829402
0.6	37.431669	189.584409	37.982515	197.873273	32.609076 *32.609068	163.298552 *163.293747	32.958278 *33.949766	169.04239 *169.037405
0.8	36.434222	181.443259	36.799556	188.321868	31.741798 *31.741790	156.277992 *156.273393	31.933586 *31.933579	160.87363 *160.868909
1.0	35.403079	172.920390	35.573177	178.260238	30.845400 *30.845392	148.927826 *148.923444	30.871501 *30.845392	152.26796 *152.263514

*Comparison with author's paper[25]

Table 4. Frequency parameter (λ) for a trapezoidal plate for different values of thermal gradient β , taper constant ($\alpha = 0.0, 0.4$), non-homogeneity constant (α_1 = 0.4, 1.0) and aspect ratios (a/b = 1.0), (c/b = 1.0)

β	α_1=0.4				α_1=1.0			
	$\alpha = 0.0$		$\alpha = 0.4$		$\alpha = 0.0$		$\alpha = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.0	27.137212	129.818486	28.810372	141.525176	22.991546 *22.991536	110.916591 *110.910143	24.265689 *24.265681	119.698898 *119.692419
0.2	26.373790	125.024966	27.803427	135.546939	22.343699 *22.343690	106.826042 *106.819831	23.417148 *23.417140	114.644772 *114.638565
0.4	25.582627	120.041241	26.755336	129.293254	21.672240 *21.672232	102.573392 *102.567423	22.533901 *22.533893	109.357883 *109.351961
0.6	24.759615	114.842863	25.659917	122.722257	20.973665 *20.973657	98.137837 *98.132128	21.610729 *21.610722	103.802872 *103.797250
0.8	23.899346	109.399700	24.509039	115.780313	20.243358 *20.243350	93.493737 *93.488298	20.640763 *20.640756	97.934444 *97.929139
1.0	22.994423	103.673916	23.291528	108.396812	19.475003 *19.474995	88.608913 *88.603757	19.614563 *19.614556	91.692977 *91.688009

*Comparison with author's paper[25]

Table 5. Frequency parameter (λ) for a trapezoidal plate for different combinations of thermal gradient (β), taper constant (α) and fixed value of non-homogeneity constant (α_1 = 0.0) & aspect ratio (a/b = 0.75)

c/b	$\alpha = 0.0, \beta = 0.0$		$\alpha = 0.0, \beta = 0.4$		$\alpha = 0.4, \beta = 0.0$		$\alpha = 0.4, \beta = 0.4$	
	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode
.25	50.781420	220.637068	48.970291	208.536396	51.363098	228.773031	49.334318	214.886448
.50	40.246760	173.580809	38.626246	163.554573	40.994438	181.617518	39.104539	169.717936
.75	32.961185	136.473316	31.447791	128.221708	34.081575	144.831410	32.205874	134.581089
1.0	28.261857	110.048152	26.787805	103.254464	29.970990	119.141099	27.989856	110.094693

Table 6. Frequency parameter (λ) for a trapezoidal plate for different combinations of thermal gradient (β), taper constant (α) and fixed value of non-homogeneity constant (α_1 = 0.4) and aspect ratio (a/b = 0.75)

c/b	$\alpha = 0.0, \beta = 0.0$		$\alpha = 0.0, \beta = 0.4$		$\alpha = 0.4, \beta = 0.0$		$\alpha = 0.4, \beta = 0.4$	
	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode
.25	45.885197	194.970902	44.254760	184.252605	46.275063	200.752785	44.453262	188.541551
.50	35.727952	152.047763	34.294279	143.244861	36.226379	157.472069	34.562012	147.130155
.75	28.757349	119.005493	27.437708	111.807045	29.552361	124.792020	27.928466	115.949431
1.0	24.278734	95.475791	23.007455	89.601078	25.555152	102.099955	23.863578	94.356716

Table 7. Frequency parameter (λ) for a trapezoidal plate for different combinations of thermal gradient (β), taper constant (α) and fixed value of non-homogeneity constant (α_1 = 0.0) and aspect ratio (a/b = 1.0)

c/b	$\alpha = 0.0, \beta = 0.0$		$\alpha = 0.0, \beta = 0.4$		$\alpha = 0.4, \beta = 0.0$		$\alpha = 0.4, \beta = 0.4$	
	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode
.25	56.818289	301.922778	54.464421	282.420325	58.100262	318.155592	55.415783	295.877891
.50	45.347669	242.191642	43.252070	225.365538	46.747405	258.494530	44.267564	238.772041
.75	37.139193	190.213245	35.228027	176.195015	38.835808	206.264773	36.451631	189.317531
1.0	31.590430	149.627140	29.784542	138.340338	33.791695	165.132010	31.382998	150.851802

Table 8. Frequency parameter (λ) for a trapezoidal plate for different combinations of thermal gradient (β), taper constant (α) and fixed value of non-homogeneity constant (α_1 = 0.4) and aspect ratio (a/b = 1.0)

c/b	$\alpha = 0.0, \beta = 0.0$		$\alpha = 0.0, \beta = 0.4$		$\alpha = 0.4, \beta = 0.0$		$\alpha = 0.4, \beta = 0.4$	
	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second Mode
.25	51.345711	266.771196	49.222122	249.521280	52.356914	279.123355	49.940913	259.562574
.50	40.256534	212.145227	38.399268	197.390839	41.315380	224.100543	39.126969	206.984913
.75	32.402442	165.867294	30.733758	153.640922	33.676618	177.715586	31.610679	163.106233
1.0	27.137212	129.818486	25.582627	120.041241	28.810372	141.525176	26.755336	129.293254

Table 9. Frequency parameter (λ) for a trapezoidal plate for different value of non-homogeneity constant with different combinations of thermal gradient (β) and taper constant (α) and aspect ratios (a/b = 1.0), (c/b = 0.5)

α_1	$\alpha = 0.0, \beta = 0.0$		$\alpha = 0.0, \beta = 0.4$		$\alpha = 0.4, \beta = 0.0$		$\alpha = 0.4, \beta = 0.4$	
	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode
0.0	45.347669	242.191642	43.252070	225.365538	46.747405	258.494530	44.267564	238.772041
0.2	42.575611	225.662425	40.609931	209.975285	43.780876	239.427415	41.46036	221.149257
0.4	40.256534	212.145227	38.399268	197.390839	41.315380	224.100543	39.126969	206.984913
0.6	38.279188	200.817857	36.514175	186.846016	39.224179	211.420472	37.147611	195.267622
0.8	36.567153	191.143002	34.881886	177.840178	37.421284	200.697373	35.440999	185.359413
1.0	35.065970	182.752281	33.450533	170.030136	35.846034	191.470899	33.949773	176.834596

Table 10. Frequency parameter (λ) for a trapezoidal plate for different value of non-homogeneity constant with different combinations of thermal gradient (β) and taper constant (α) and aspect ratios (a/b = 1.0), (c/b = 1.0)

α_1	$\alpha = 0.0, \beta = 0.0$		$\alpha = 0.0, \beta = 0.4$		$\alpha = 0.4, \beta = 0.0$		$\alpha = 0.4, \beta = 0.4$	
	First mode	Second mode	First mode	Second mode	First mode	Second mode	First mode	Second mode
0.0	31.590430	149.627140	29.784542	138.340338	33.791695	165.132010	31.382998	150.851802
0.2	29.111485	138.664324	27.445292	128.213892	31.004872	151.967636	28.793942	138.830092
0.4	27.137212	129.818486	25.582627	120.041241	28.810372	141.525176	26.755336	129.293254
0.6	25.516778	122.482397	24.054015	113.262403	27.024306	132.978728	25.096244	121.487524
0.8	24.155738	116.268659	22.770243	107.519977	25.533910	125.815731	23.711866	114.945038
1.0	22.991546	110.916591	21.672240	102.573392	24.265689	119.698898	22.533901	109.357883

5. Confirmation of Results

The accuracy of the present computations is compared with the published results[25] for C-S-C-S non-homogeneo us trapezoidal plate with non-homogeneity constant α_1 = 1.0, aspect ratio a/b = 1.0, c/b = 0.5, 1.0 and thermal gradient β = 0.0 to 1.0 and taper constant α = 0.0 to 1.0.

Table 1, 2, 3 and 4 shows a comparison of the values of frequency parameter obtained in the present problem and published paper of the authors[25]. A very close agreement is seen between the present results and of the published paper in which homogeneous plate with parabolically varying thickness (density is constant) has been considered.

6. Conclusions

The main purpose of the present work is to study the effect of thermal gradient on the frequencies of C-S-C-S non-homogeneous trapezoidal plate with other plate parameters as taper constant, non-homogeneity constant and aspect ratios. Thickness of the plate varies parabolically and density varies linearly. Rayleigh-Ritz technique is used to find frequencies for first two modes of vibration. Study shows that frequency parameters increases with increasing value of taper constant and aspect ratio a/b whereas it decreases with increasing value of non-homogeneity constant, thermal gradient and aspect ratio c/b.

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