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Analyses of Quantization Noise Spectrum for Multiresolution Quantization of Harmonic Signal

M. Kamenský, K. Kováč, G. Valky

Department of Measurement, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology, Ilkovičova 3, 81219 Bratislava, Slovak Republic, +421 2 60291393
Email: miroslav.kamensky@stuba.sk

Abstract. *An analysis of quantisation noise of multiresolution structure of analog-to digital converters (ADC) is performed. It shows how the noise level as well as the resulting noise spectrum is changing with a level of sampled signal. The theoretical formulas are approved by numerical simulation and experimental measurements performed by PC measuring card emulating two channels ADC structure. The results confirm that resulting quantization noise level is strongly dependent upon the amplitude of measured signal.*

Keywords: Spectrum of Quantization Noise, Multiresolution Quantization

1. Introduction

Time domain spectrum analysis becomes popular technique even in the area of electromagnetic interference (EMI) measurement. It offers time effective alternative to conventional measuring receivers. To ensure necessary high dynamic range at very fast sampling it uses multiresolution structure of sufficiently fast analog-to-digital converters (ADC). After ADCs processing subsystem performs DFT and continuously calculates resulting spectra. Recent systems are able to operate at sampling frequency of 250 MHz [1].

As EMI measurements require high dynamic range at mentioned sampling frequency multiresolution structure of ADCs becomes the main part determining the metrological properties of the system. Multiresolution divides whole input range of system into several intervals. In each interval uniform quantization is used, but the quantization step is different for each interval - multiresolution quantization (MQ). Concept of MQ, realised by lower resolution fast ADCs, allows to improve dynamic range of spectrum measurement but brings specific behaviour of quantization error and quantization noise spectrum. To verify impact of MQ on fidelity of spectrum measurements analysis of spectral properties of quantization noise is presented in the paper for harmonic signal intersecting two intervals of MQ.

2. Multiresolution Structure for Spectrum Measurement

In described system the measured signal is sampled in two or three channels simultaneously. In the Fig. 1 a structure of a multiresolution system is depicted. The input signal is split by a power splitter (PWS) into all channels. Each channel consists of limiter (LM), low-noise amplifier (LNA) and an ADC - usually 8-bit. Using digital signal processing the signal is reconstructed by extracting each sampled value from the ADC where the signal shows the maximum nonclipped value. Principle of MQ offers new quality measurements. But like every new method it requires sufficient analysis of its properties. Coming out of theory of uniform quantization and using simulation we try to point out some problems in spectral behaviour of quantization error in MQ. For simplicity harmonic input signal will be assumed on the input of system which input range is divided by MQ into two intervals.

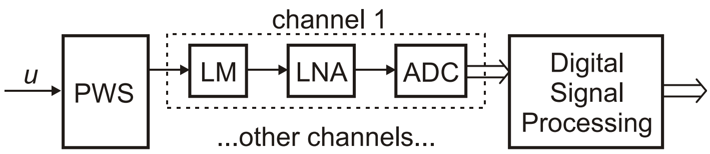


Fig. 1. Block structure of multiresolution quantization in spectral measurements.

3. Quantization Error of Harmonic Input

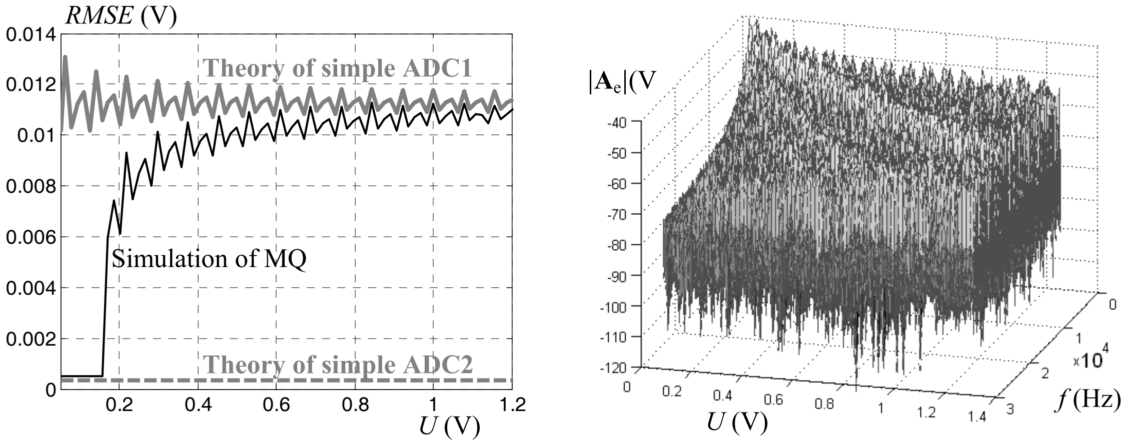
Consider simple harmonic signal with amplitude U and frequency f_u on the input of a measurement system

$$u(t) = U \sin(2\pi f_u t) \tag{1}$$

For evaluation of overall quantization error root mean square error (RMSE) is suitable. If amplitude U is large compared to quantization step q mean squared error (MSE) approaches good known value of $q^2/12$. But if the ratio U/q is low, better theoretical estimation should be used like in [2]. For assumed zero offset deterministic quantizer input (Eq. 1) and mid-tread quantizer the MSE is

$$MSE = \frac{q^2}{12} + \frac{q^2}{\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} J_0\left(\frac{k 2\pi U}{q}\right) \tag{2}$$

where J_0 (.) is the ordinary Bessel function of order zero.



a) b)

Fig. 2. Dependency of quantization error on harmonic signal amplitude U:
a) RMSE of multiresolution quatization compared with theoretical error of simple quantization
b) Simulation spectrum of quantization error for simple quantization

Simulated dependency of theoretical RMSE on signal amplitude is depicted in the Fig.2a. Solid gray line corresponds to 8-bit bipolar ADC (ADC1) with the range of ±5 V (q = 0.0391 V) and dashed gray line to similar ADC (ADC2) with the range 2^5 times smaller (±0.1563 V, q = 0.0012 V) than of the first one. Naturally better accuracy is achieved for lower quantization step q. As could be seen for considered harmonic input the change of overall quantization error is not critical with changing U. For ADC1 oscillations caused by the nature of J_0 (.) and by presence of $(-1)^k$ in Eq. 2 are higher for lower U. However for two-channel MQ one may expect RMSE close to the theory of ADC2. This was tested by simulation of harmonic signal (with little noise) for signal frequency f_u=100 Hz, sampling rate 512 time higher and for N = 4096 samples. Only for small signal amplitude the quantization noise level of MQ corresponds to better resolution ADC2 as shown in Fig.2a (black line).

Unfortunately there is quite steep slope in the error curve near to the end of ADC2 range. But RMSE dependency does not show in details the impact of error increase to spectrum estimated by Discrete Fourier Transform (DFT). Analysis of quantization noise spectrum should be done to unveil possible spurious spectral components.

If spectrum of quantization error is of interest, the analyses become slightly more complicated. Several approaches are suggested in [3]. Fortunately for a single ADC quite simple approximation is achieved by [4] based on modulation principle. From that power spectral density of quantization noise could be theoretically approximated by

$$S_{\mathrm{e}}(f)=\frac{q^3}{2\pi^3}\sum_{k=1}^{\infty}\frac{1}{k^2}\frac{1}{kU2\pi f_u\sqrt{1-\left(\frac{qf}{kU2\pi f_u}\right)^2}} \quad \text{for} \quad |f|<2\pi f_u\frac{U}{q}k \tag{3}$$

Amplitude spectrum of quantization error $|\mathbf{A}_{\mathrm{e}}|$ could be estimated from S_{e}. As could be seen from the equation it depends on ratio of signal amplitude to quantization step U/q. In accordance with expectation also for every spectrum component the theoretical error decreases with decreasing q like for total RMSE. But according to Eq. 3 the spectral distribution of quantization error is not flat. Error peaks occur at frequencies (for integer k)

$$f_{p,k}=k.2\pi\frac{U}{q}f_u \tag{4}$$

Exactly this effect could be observed in the Fig.2b where amplitude spectrum of quantization error $|\mathbf{A}_{\mathrm{e}}|$ obtained from simulation of single ADC1 is depicted in dependence on U. One can suspect that after addition of ADC2 the first peak near to $f_{\mathrm{p},1}$ ($f_{\mathrm{p},1}=1609\ U$) can still significantly disturb measurement for signal amplitude just above ADC2 range.

4. Results

The simulations of MQ were realized using parameters from previous section. For some amplitude U of harmonic signal also experiments were added. Data acquisition card was used as a multiresolution quantizer.

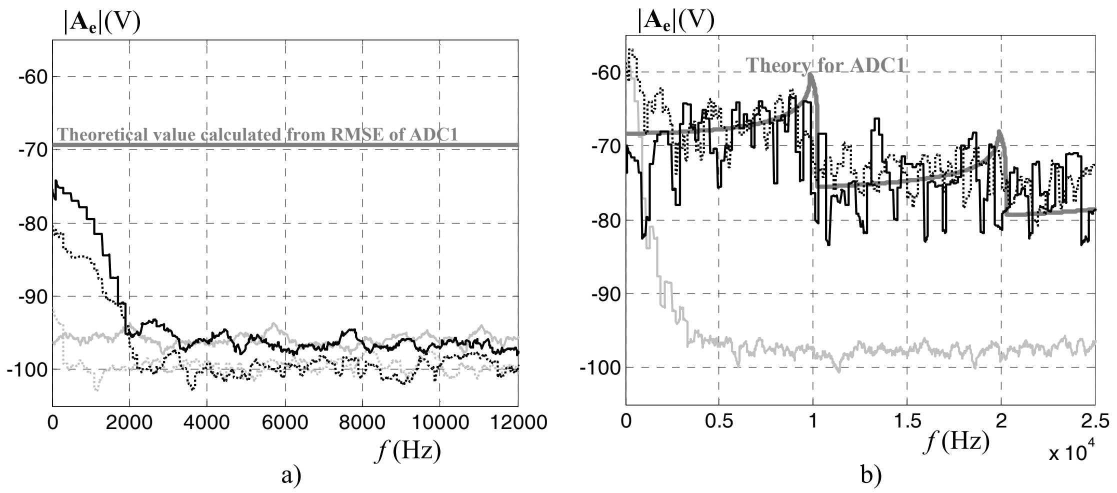


Fig. 3. Spectrum of quantization noise of MQ for several amplitudes U :
a) Part of spectrum for U= 0.1550 V (gray) and U= 0.1600 V (black): simulated—, experimental····
b) Spectrum for U= 0.1700 V (light gray) and U= 0.3125 V (black: simulated—, experimental····)

Smoothed spectrum of quantization error obtained from simulation and experiments is depicted in the Fig. 3a for U near to the end of ADC2 range (0.1563 V). If the amplitude is lower than range (U = 0.1550 V) both simulation and experimental spectrum is flat as fine quantization step of ADC2 is used for all samples. Experimental values are even lower than simulated ones because simulated input noise was higher than real. The results are close to theoretical value of -99.24 dB calculated from RMSE of ADC2. But if the amplitude U was increased only by cca 0.3 % extensive increase of 20dB occurred in low-frequency part of spectrum despite lower growth of RMSE (cca 7 dB). The peak of quantization noise can be even stronger if slightly higher amplitude is used. For simulated U = 0.1700 V (light gray line in the Fig. 3b) maximal spectral component is more than 10dB above theoretical value of -69.15 dB calculated from RMSE of ADC1. However later the spectrum becomes flatter with rising U approaching theoretical behaviour according to Eq. 3 like results for U = 0.3125 V depicted in the Fig. 3b.

5. Conclusions

Two channel of multiresolution structure has been used to analyse spectrum of quantization error for multiresolution quantization (MQ). The theoretically estimated spectrum of quantization noise is not flat as is generally supposed for simplification. It contains increased parts, even peaks at some frequencies, which are not inherent for measured signal but represent spurious components caused by restricted resolution of one stage of multiresolution structure. In some cases this disturbance leads to significant reduction of signal-to-noise ration (SNR) and dynamic range. The difference in distribution of power spectrum of quantization noise could be much higher for MQ than for simple uniform quantization. As presented for harmonic input signal critical input amplitude is just above the range of higher resolution ADC. For two 8-bit ADCs if ratio of ranges is 2^5 a difference between spectral components of quantization noise can reach 40 dB. Hereby a spurious component can exceed theoretical RMSE of the lower resolution ADC by more than 10 dB. So a ratio of maximal to minimal signal amplitude which is distinguishable for analysed MQ could correspond to 13-bit ADC. But SNR still corresponds only to 8-bit resolution.

Acknowledgements

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