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RECURRENCE QUANTIFICATION ANALYSIS OF SYSTEM SIGNALS FOR DETECTING TOOL WEAR IN A LATHE

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ABSTRACT

The work investigates applicability of recurrence quantification analysis (RQA) in metal cutting with an objective to detect tool wear. The effectiveness of applying a system input signal; the drive motor current, in relation to a system output signal; the tool vibration, for the analysis is also explored. The work establishes conclusively that three of the RQA variables, percent determinism, percent recurrence and entropy are sensitive to tool wear.

Keywords: Time series analysis, Tool flank wear, Recurrence quantification analysis, Chaos;

INTRODUCTION

In precision machining processes, one of the key factors that affect production optimization and surface quality of the work piece is the state of the cutting tool, characterized by the tool wear. Tool condition monitoring systems (TCMs) are, therefore, needed to ensure better quality of machining jobs and to obtain a reduction in the downtime of machine tools due to catastrophic tool failures. In addition, a successful online TCM results in significant savings in cost for manufacturers because of increased productivity and process reliability.

Significant progresses have already been made in tool developments, and in-process real time monitoring and control of tools by various methods have been proposed [1, 2]. Dan Li [3] has published a detailed review of tool wear and failure monitoring techniques for turning. Dimla[4] has conducted an evaluation of the suitability and sensitivity of the most widely used process parameters to tool wear and their potential applicability for successful online TCMs. It has been discovered by Bukk et al [5] that metal cutting shows low dimensional chaos. This realization has a significant impact on the classical views concerning the dynamics of metal cutting. On the basis of the chaos theory, a method for discriminating sensor signals generated under different conditions of metal cutting using the correlation dimension has been reported by Rajesh et al. [6].

However, this method is sensitive to noise and is therefore developed on the assumption of stationarity of the dataset. In this work, the powerful method of recurrence quantification analysis (RQA) is used to study the sensor signals generated during the cutting process, which are simultaneously recorded with the objective of characterizing them on the basis of the calculated RQA variables. The study concludes that this extracted information contained in the signal can be used for describing a tool used in cutting as fresh or worn tool. The signals generated during the machining process using cutting tools with different degrees of flank wear on them; fresh tools with flank wear=0mm and worn tools with flank wear=0.3mm, were analyzed for the purpose. The study also explored the effectiveness of applying a system input signal, the drive motor current, in relation to a system output signal, the tool vibration, for the analysis. This has particular practical importance in signal measurements and analysis as the current signal is simpler and easier to measure compared with any system output signal such as feed force or tool vibrations.

EXPERIMENTAL SETUP AND DATA-ACQUISITION SYSTEM

Experiments are conducted on a three phase, 3.7kW, 1400rpm PSG heavy duty lathe using CNMG 120408PM carbide inserts with a standard tool holder. The work pieces are made of 30mm diameter and 120mm long mild steel roads. The machining job involves reducing the diameter of the workpiece to 29.6 mm in the lathe. The cutting factors; speed (560 rpm), feed per revolution (0.06 mm), and depth of cut (0.2mm) are

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maintained constant during machining. Two arrays of experiments, one with the fresh tool and the other with the worn tool, containing 7 trials in each array are conducted. A total of 14 numbers of experiments are conducted and simultaneous recordings of sensor signals representing the time history of the lathe drive motor current as well as tool vibrations during cutting are done, resulting in 14 datasets in each of the two arrays. In the experiment, a fresh tool is characterized by flank wear = 0 mm and a worn tool has a flank wear = 0.3 mm.

The data acquisition system for drive motor current uses a 3phase line current sensor to measure the current drawn by the lathe drive motor. The sensor consists of a current transformer (CT) having an output range of ± 5 volts. The analog voltage signal from the output of the CT is sent to DAQ NI PCI 6221 through NI SHC68-68-EPM and SCB 68 for converting it to the digital domain. The sampling rate for this signal is fixed at 500Hz. The digitized data is recorded in the PC hard drive using the NI LabVIEW. Continuous data is recorded for a cut of 10sec duration during every trial of the experiments and from each of which 1000 data points representing a 2sec duration cut is randomly selected and analyzed.

An ADXL-150 accelerometer sensor, is used to pick up the vibration of the cutting tool. It is placed on the tool holder near its tail end to measure the vibration in the feed direction. The resulting output voltage signal, due to vibration, is amplified and passed through a low pass filter having a cut off frequency of 1kHz. Here the sampling rate is fixed at 10kHz. The data acquisition system for the vibration signals used a different channel of the same DAC NI PCI 6221 and similarly other peripherals along with the NI-LabVIEW. Continuous data is recorded for a cut of 10sec duration during every trials of the experiments and from each of which 1000 data points representing a 0.1sec cut is randomly selected and analyzed.

NONLINEAR ANALYSIS AND METHODS

Most often, in signal analysis, the amplitude distribution of the signal is analyzed and various statistical moments are used as characteristics. The nonlinear time series analysis (NTSA) approach is basically different from the statistical one, in the respect that it can overcome inherent limits of the traditional linear and statistical tools. Despite its wide range of applications, NTSA suffers from the problems of nonstationarity of the measured time series data, which may lead to pitfalls that may invalidate the analysis. These limitations can be overcome by the quantification of recurrence plots through RQA. In 1987, Eckmann et al. [7] introduced the concept of recurrence plot (RPs) that can visualize the recurrence behavior of the phase space trajectory of dynamical systems. Subsequently, the RQA was developed by Zbilut and Webber Jr. [8,9] and further extended with new measures of complexity by Marwan et al. [10]. The basic idea behind RQA is the identification of recurrence of local data points in a reconstructed phase space. Because RQA simply tallies recurrences, critical issues such as signal stationarity, noise, and statistical distribution of data are precluded. Thus, it is ideally suited for analyzing experimental signals that are generally characterized by nonstationarity and noise. In this section, our approach is described, based on phase space reconstruction, the recurrence plot, and the recurrence quantification analysis.

PHASE SPACE RECONSTRUCTION

Takens [11] proved a theorem that is the firm basis of the methodology of delays. Since one variable only is measured (the usual case in an experiment), the delay coordinate approach is used in the present analysis. Given a time series $x(1),\ x(2),\ x(3),.........x(N)$, we define points $X(i)$ in an m-dimensional state space as

$$X(i)=[x(i),\ x(i+\tau),\ x(i+2\tau),.......x(i+(m-1)\tau)], \quad (1)$$

for $i=1,2,3,....,N-(m-1)\tau$, where i represents time indices, τ indicates the time lag, and m is called the embedding dimension. Time evolution of $X(i)$ is called a trajectory of the system, and the space, which this trajectory evolves in, is called the phase space.

SELECTING THE MINIMUM EMBEDDING DIMENSION

The embedding dimension is the minimum dimension at which the reconstructed attractor can be considered completely unfolded. This parameter is usually estimated by the method of false nearest neighbors (FNN), proposed by Abarbanel [12].

By checking the neighborhood of points embedded in the projection manifolds of increasing dimension, the algorithm eliminates 'false neighbors'[13]. A natural criterion for detecting embedding errors is that the increase in distance between two neighboring points is large when proceeding from dimension m to *(m+1)*. This criterion is stated by designating any neighbor for which the following equation is valid as a false nearest neighbor.

$$\left[\frac{R_{m+1}^2(i,i_r)-R_m^2(i,i_r)}{R_m^2(i,i_r)}\right]^{1/2}=\frac{|x(i+m\tau)-x(i_r+m\tau)|}{R_m(i,i_r)}>R_{tol}, \quad (2)$$

Here, i and i_r are the time points corresponding to the neighbor and the reference point, respectively. R_m and R_{m+1} denote the distance in phase space with the embedding dimensions m and $(m+1)$ respectively, and R_{tol} is the tolerance threshold. For the present analysis the embedding dimension corresponding to the lowest value of FNN is selected.

SELECTING THE TIME LAG

To choose the time lag, τ, we use the nonlinear correlation function of average mutual information (AMI). Fraser et. al

[14] have established that delay corresponds to the first local minimum of the AMI function $I(\tau)$, which is defined as follows:

$$I(\tau) = \sum P(x(i), x(i+\tau)) \log_2 \left[\frac{P(x(i), x(i+\tau))}{P(x(i))P(x(i+\tau))} \right] \quad (3)$$

where $P(x(i))$ is the probability of the measure $x(i)$, $P(x(i+\tau))$ is the probability of the measure $x(i+\tau)$, and $P(x(i), x(i+\tau))$ is the joint probability of the measure of $x(i)$ and $x(i+\tau)$. Plotting $I(\tau)$ versus τ makes it possible to identify the best value for the time delay, and this is related to the first local minimum.

The values for time lag, τ, and embedding dimension, m, for the fresh tool and worn tool have been calculated following the AMI and FNN methods and are shown in Table 1. The time lag values for both the sensor signals for the two types of tools matches whereas their embedding dimensions differ. Since the work is aimed at monitoring of tool wear that takes place progressively as cutting takes place, phase space reconstruction using two different sets of values is avoided here. Instead, they are chosen from the representative values of the worn tool, which demands higher embedding dimension.

Table 1. Phase space reconstruction parameters obtained by AMI and FNN

Tool Type	Time lag, τ		Embedding dimension, m	
	Current signal	Vibration Signal	Current signal	Vibration Signal
Fresh Tool	3	6	3	5
Worn Tool	3	6	5	7

ANALYSIS BASED ON RECURRENCE PLOTS

A recurrence plot is a way to visually investigate the multidimensional phase space trajectory through a two dimensional representation [8]. Recurrence of states of the system, in the meaning that states are arbitrarily close after some time, is a well known property of deterministic dynamical systems and is typical for nonlinear or chaotic systems. A recurrence plot is derived from the distance plot, which is a symmetric [NxN]matrix, where a point (i, j) represents the distance between the coordinates $X(i)$ and $X(j)$ on the phase space trajectory. Thresholding the distance plot at a certain cutoff value transforms it into a RP which shows all the recurrent points as black spots.

$$RP(i, j) = \Theta(\varepsilon - \|X(i) - X(j)\|), \quad (4)$$

where $i, j = 1,.,.,., N$, ε is the cutoff distance, $\|\bullet\|$ is some norm, and $\Theta(\bullet)$ is the Heaviside function [15].

In the present analysis, recurrence plots are constructed applying the L_2 norm in distance calculations. The threshold ε is chosen by analyzing the measure of recurrence point density[16] as the percentage of maximum distance (Table 2) . Again, as followed and due to reasons assumed in phase space reconstruction, we use the threshold ε values obtained for the worn tool (11 for current, 9 for vibration) as representative values for RQA estimation.

Table 2. Calculated values for the threshold, ε

Tool Type	Current signal	Vibration Signal
Fresh Tool	10	7
Worn Tool	11	9

The recurrence plots constructed with the above chosen parameter values for a fresh tool and a worn tool are shown in Fig. 1. The RP axes are in time units. Since the RPs itself does not contain any visually appreciable quantitative information we utilize the RQA approach for the purpose in the present study.

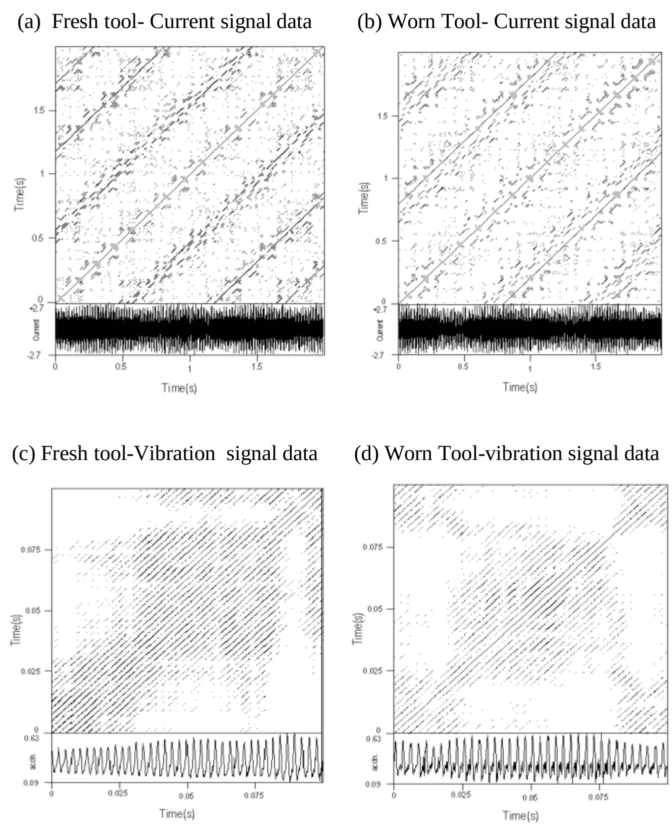

Figure 1. Recurrence plots of current and vibration signal data for system using fresh and worn tools.

RECURRENCE QUANTIFICATION ANALYSIS

The RQA is a tool based on the statistical description of the parallel lines distribution among the RP [9]. Measures of complexity are defined using the recurrence point density and diagonal line structures in the recurrence plot. These measures provide a qualitative description of the dynamics underlying the time series that is studied. In the original definition Eckman et al [7] used a fixed number of neighbours for determining recurrences. In the present analysis we use a fixed value for the threshold ε due to which the RP is symmetric across the central diagonal, called the line of identity (LOI). Attention is focused on the diagonal and vertical structures in the RP since from those stem the recurrence variables or quantifications. As the recurrence plot is symmetrical across the central diagonal, all quantitative feature extractions take place within the upper triangle in the RP [16], excluding the long diagonal (which provides no unique information) and lower triangle (which provides only redundant information).

We can derive eight statistical values from a RP using RQA. The first value is percent recurrence, quantifies the percentage of recurrent points falling within the specified radius. For a given window size W, where W refers to the recurrence window size after accounting for embedding and delay,

$$\text{Percent recurrence} = \frac{\text{Number of recurrent points in triangle} * 100}{(W(W-1)/2)} \quad (5)$$

The second variable is percent determinism and measures the percentage of recurrent points that are contained in lines parallel to the main diagonal of the RP, which are known as deterministic lines. A deterministic line is defined if it contains a predefined minimum number of recurrence points. It represents a measure of predictability of the system.

$$\text{Percent determinism} = \frac{\text{Number of points in diagonal lines} * 100}{\text{Number of recurrence points}} \quad (6)$$

The third recurrence variable is Linemax, which is simply the length of the longest diagonal line segment in the plot, excluding the main diagonal line of identity (where i = j). This is a very important recurrence variable because it inversely scales with the most positive Lyapunov exponent [7, 17]. Positive Lyapunov exponents gauge the rate at which trajectories diverge, and are the hallmark for dynamic chaos.

$$\text{Linemax} = \text{length of longest diagonal line in recurrence plot} \quad (7)$$

The fourth variable value is called entropy and it refers to the Shannon entropy of the distribution probability of the diagonal lines length. Entropy is a measure of signal complexity and is calibrated in units of bits/bin and is calculated by binning the deterministic lines according to their length. Individual histogram bin probabilities (P_{bin}) are computed for each non zero bin and then summed according to Shannon's equation.

$$\text{Entropy} = -\sum (P_{bin}) \log_2 (P_{bin}) \quad (8)$$

The fifth statistical value is the Trend which is used to detect non stationarity in the data. The trend essentially measures how quickly the RP pales away from the main diagonal and can be utilized as a measure of stationarity. If recurrent points are homogeneously distributed across the recurrence plot, Trend values will hover near zero units. If recurrent points are heterogeneously distributed across the recurrence plot, Trend values will deviate from zero units. Trend is computed as the slope of the least squares regression of percent local recurrence as a function of the orthogonal displacement from the central diagonal. Multiplying by 1000 increases the gain of the Trend variable.

$$\text{Trend} = 1000(\text{slope of percent local recurrence vs. displacement}) \quad (9)$$

For the detection of chaos-chaos transitions, Marwan et al. [10] introduced other two additional RQA variables, the Percent Laminarity and Trapping time, in which attention is focused on vertical line structures and black patches. Percent Laminarity is analogous to percent determinism except that it measures the percentage of recurrent points comprising vertical line structures rather than diagonal line structures. The line parameter still governs the minimum length of vertical lines to be included.

$$\text{Percent Laminarity} = \frac{\text{Number of points in vertical lines} * 100}{\text{Number of recurrent points}} \quad (10)$$

Trapping time on the other hand is the average length of vertical line structures. It represents the average time in which the system is "trapped" in a specific state.

$$\text{Trapping time} = \text{average length of vertical lines} \geq \text{parameter line} \quad (11)$$

The eighth recurrence variable is Vmax, which is simply the length of the longest diagonal line segment in the plot. This variable is analogous to the standard measure Linemax

$$\text{Vmax} = \text{length of longest vertical line in recurrence plot} \quad (12)$$

SURROGATE DATA TEST

The actual intact data RQA results for the fresh tool and the worn tool are compared with the RQA results obtained after randomizing their data points. It is found that this randomization effectively destroyed all the structures revealed under the input parameter values chosen earlier as shown in Table 3. For example, with other recurrence variables, percentage recurrence and percentage determinism dropped to

Table 3- RQA values of current and vibration signals for system using fresh and worn tool using both intact and randomized data sets

Fresh Tool	**Current sensor signals**	**Actual data, ε=11**	**Randomized data, ε=11**
	Percent recurrence (%)	3.491	0.113
	Percent determinism (%)	80.447	0.362
	Linemax	406	2
	Entropy	3.208	0
	Trend	1.028	0.06
	Percent laminarity (%)	0.00	0.725
	Vmax	--	2
	Trap time	--	--
	Vibration sensor signal	**Actual data, ε=9**	**Randomized data, ε=9**
	Percent recurrence (%)	2.625	0.007
	Percent determinism (%)	82.572	0.00
	Linemax	179	--
	Entropy	2.434	1
	Trend	-6.390	0.003
	Percent laminarity (%)	52.445	0.00
	Vmax	4	1
	Traptime	2.116	--
Worn Tool	**Current sensor signals**	**Actual data, ε=11**	**Randomized data, ε=11**
	Percent recurrence (%)	2.386	0.162
	Percent determinism (%)	73.827	0.00
	Linemax	597	--
	Entropy	2.823	--
	Trend	0.978	-0.007
	Percent laminarity (%)	0	0.506
	Vmax	--	2
	Trap time	--	2
	Vibration sensor signal	**Actual data, ε=9**	**Randomized data, ε=9**
	Percent recurrence (%)	1.033	0.014
	Percent determinism (%)	69.462	0.00
	Linemax	98	--
	Entropy	1.967	-1.00
	Trend	0.498	-0.006
	Percent laminarity (%)	10.828	0.00
	Vmax	3	--
	Traptime	2.004	--

around 0–5%. Furthermore, recurrence plots showed a homogeneous typology and did not resemble those for the actual intact data. Thus, it may be concluded that the results obtained under the present parameterization reflect true properties of the temporal evolution of cutting dynamics and contain a degree of deterministic structure.

MANN-WHITNEY U TEST

Now, to ascertain whether the differences seen in the calculated RQA variable values due to tool change, i.e. when the fresh tool is replaced by the worn tool, is by chance or otherwise, the non parametric Mann-Whitney U test [18] is carried out to find their statistical significance. The test results

TABLE 4. RQA Variables - Mann-Whitney U test results

Recurrence Variable	Fresh Tool	Worn Tool	P(Two tailed)	Significance Level
Current Signal				
Percent recurrence	3.491	2.386	0.014	Marginally significant
Percent determinism	80.447	73.827	0.0004	Highly significant
Entropy	3.208	2.823	0.0045	Significantly different
Vibration Signal				
Percent recurrence	2.625	1.033	0.013	Marginally significant
Percent determinism	82.572	69.462	0.0016	Significantly different
Entropy	2.434	1.967	0.0035	Significantly different

shows that the difference in values between different sets can arise by chance is by about five per cent or less only in case of percent recurrence, percent determinism and entropy whereas the remaining five parameters are found to be not significantly different statistically, as indicated by the respective P (two tailed) values in Table 4. This is in contrast to the recent theoretical simulation studied by Litak et al. [19] where the finding points to Linemax as the most conclusive variable in cutting dynamics, but partially in agreement with sensitiveness to Shannon's information entropy.

EPISODIC TEST

An episodic test conducted on the full length of sample datasets (Figure 2) shows a constancy of the significant RQA variables within the whole length of data. Here an epoch is designed to have a width of 400 data points and is made moving by giving a 2 point data shift. Also, the figures show wide separations between the means of the values of the recurrence variables suggesting of two distinct dynamics. This is explained by the source of the data: the upper graph in each is from a system using a fresh tool, whereas the lower graph in each is from a system using a worn tool. This result corroborates and reiterates the Mann-Whitney U test outputs.

Finally, it is to be noted here that the above tests are conducted with constant input parameter values for both the datasets: fresh as well as worn tool. As a check, we have examined the effects on RQA variables if the calculated input parameter values were used in RQA. Since the representative values of worn tool test data have been used as the constant input parameters, it is sufficient to analyze the fresh tool signal data only, but with the calculated input parameter values for it; ie. for current signal $\tau = 3$, for vibration signal $\tau = 6$, and the corresponding values for m and ε are 3, 5 and 10,7 respectively. The RQA results with these input parameters indicate a 40—50% increase in percent recurrence value while other significant RQA variables; the percent determinism and entropy, change by only 1—2%. The change in percent recurrence is attributed to the fact that, as the embedding dimension decreases from 5 to 3 along with a reduction in the threshold radius, the mean distance is found to decrease by over 20%. This result justifies the assumption to use constant input parameter values for online detection, as it is found to have no trade-off in using instantly calculated values for the input parameters epoch by epoch.

CONCLUSION

The study has been initiated with an assumption that system output signals (eg. vibration etc.) are much superior to the system input signals (eg. current drawn by the system) for analyzing the system dynamics. But, the present study could not establish such a distinction, implying that either of the signals can reliably transmit information.

The surrogate data test shows that the results obtained are the true properties of the temporal evolution of cutting process dynamics and contain a degree of deterministic structure. Moreover, the Mann Whitney U test reveals that the results obtained are not due to some form of chance occurrence.

The wide separation between the mean values of RQA variables representing the two conditions under study suggest that RQA can be an efficient tool in analyzing the time series related to tool wear. The advantage being that these features can be derived very easily from a noisy or non stationary time series signals which often is a challenge in mechanical systems signal processing. Also, since the data size and computational resource requirements are considerably not demanding in comparison to the existing TCM methods, the RQA based approach proves to be a cost effective alternative. These factors make RQA an attractive feature extraction methodology, suitable for deployment in real time cutting processes.

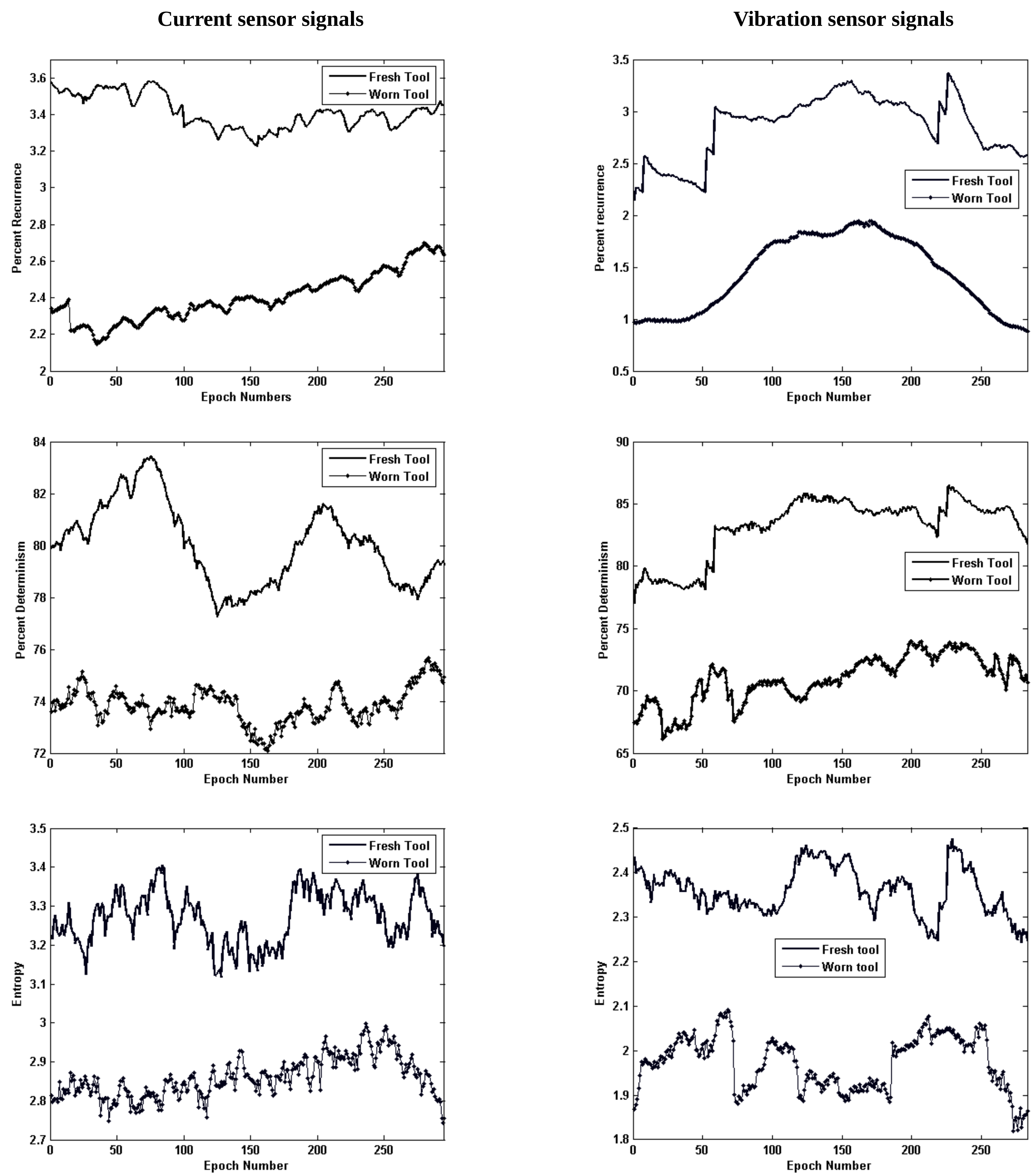

Figure 2. Episodic recurrence analysis of test results

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REFERENCES

[1] Kang, M.C., Kim, J.S., and Kim, J.H., 2001, "A monitoring technique using a multi sensor in high speed machining‖," J. Mater. Process. Technol., 113, pp. 331–336.

[2] Sokolowski, A., and Kosmol, J., 2001, "Selected examples of cutting process monitoring and diagnostics‖, " J. Mater. Process. Technol., 113, pp. 322–330.

[3] Dan Li, and Mathew, J, 1990, "Tool wear and failure monitoring techniques for turning- A review," Int. J. Mach. Tools Manuf., 30(4), pp. 579-598

[4] Dimla, E., and Dimla Snr., 2000, "Sensor signals for tool wear monitoring in metal cutting operations-a review of methods," Int. J. Mach. Tools Manuf., 40, pp. 1073-1098.

[5] Bukkapatnam, S.T.S, Lakhtakia, A., and Kumara, S.R.T., 1995, "Analysis of sensor signals shows turning on a lathe exhibits low dimensional chaos," Phys. Rev. E:, 52(3), pp. 2375-2387

[6] Rajesh, V.G., and Narayanan Namboothiri, V.N., 2007, "Application of the correlation dimension for tool condition monitoring," Proc. of the National Conf. on ETME, SVNIT, Surat, India.

[7] Eckmann, J.P., Kamphorst, S.O., and Ruelle, D., 1987, "Recurrence Plots of Dynamical Systems, "Europhysics Letters, 5, pp. 973–977

[8] Zbilut, J.P., and Webber Jr., C.L., 1992, "Embeddings and delays as derived from quantification of recurrence plots," Phys. Lett. A, 171 (3–4), pp. 199–203.

[9] Webber Jr., C. L, and Zbilut, J.P., 1994, "Dynamical assessment of physiological systems and states using recurrence plot strategies," J. Appl. Physiol. 76(2), pp. 965-973

[10] Marwan, N., Wessel, N., Meyerfeldt, U., Schirdewan, A., and Kurths, J., 2002, "Recurrence plot based measures of complexity and its application to heart rate variability data," Phys. Rev. E, 66(2), pp. 026702.

[11] Takens, F., 1981, “Detecting Strange Attractors in Turbulence-Dynamical Systems and Turbulence (Lecture Notes in Mathematics),” Springer-Verlag, Heidelberg, pp. 366-381.

[12] Henry, D. I., and Abarbanel, 1996, "Analysis of the observed chaotic data," Springer-Verlag Telos.

[13] Kennel, M.B., Brown, R., and Abarbanel, H.D.I., 1992, "Determining embedding Dimension from phase-space reconstruction using a geometrical construction," Phys. Rev. A, 25, pp. 3403-3411

[14] Fraser, A.M., and Swinney, H.L., 1986, "Independent coordinates for strange attractors from mutual information," Phys. Rev. A, 33, pp. 1134-1140

[15] Marwan, N., 2003, "Encounters with neighbors - current developments of concepts based on recurrence plots and their applications," Ph.D. Thesis, University of Potsdam, ISBN 3-00-012347-4.

[16] Webber Jr., C.L., and Zbilut, J.P., 2004, "Recurrence quantification analysis of nonlinear dynamical systems," Tutorials in contemporary nonlinear methods for the behavioral sciences, (Chapter 2, pp. 26-94), M.A. Riley, G. Van Orden, eds. Retrieved December 1.

[17] Trulla, L.L., Giuliani, A., Zbilut, J.P., and Webber, C.L., 1996, "Recurrence quantification analysis of the logistic equation with transients", Phys. Lett. A., 223, pp. 255-260

[18] Mann, H.B., and Whitney, D.R., 1947, "On a test of whether one of two random variables is stochastically larger than the other," Ann. Math. Statist. 18(1), pp. 50-60.

[19] Litak, G., Syta, A., Marwan, N., and Kurths, J., 2007, "Dynamics of a cutting process by recurrence plots," Second international workshop on recurrence plots, Siena, Italy.