Proceedings of WTC2005
World Tribology Congress III
September 12-16, 2005, Washington, D.C., USA

WTC2005-64171

LOBATTO POINT QUADRATURE FOR THERMAL LUBRICATION PROBLEMS INVOLVING COMPRESSIBLE LUBRICANTS

L. Moraru
Mechanical Engineering
The University of Toledo, Toledo, Ohio 43606

T.G. Keith, Jr.
NASA Glenn Research Center
Cleveland, Ohio 44135

ABSTRACT

Refined solutions of thermal lubrication problems generally require fine mesh and many iteration steps. To resolve these difficulties, Elrod and Brewe *(1)* proposed an efficient algorithm based on the use of Lobatto point quadrature. Within this approach, the unknown temperature across the film is written in a series of Legendre polynomials. This paper presents a Lobatto point quadrature algorithm which is applicable for thermal lubrication problems with compressible lubricants. In this case both density and viscosity of the lubricant are taken to be temperature and pressure dependent. The transverse velocity is obtained from the continuity equation. Use of the Labatto point calculation method has resulted in greater accuracy without the use of a large number of grid points

INTRODUCTION

Accurate solutions of thermal lubrication problems employ discretization both across the film thickness and along the other directions of the fluid domain. Intensive numerical integration across the film is usually required. Improving the calculation of this integration can further reduce the computational time. One of the fastest procedures is Gauss open integration, which employs the optimal collocation of the integration points (usually at the zeros of various types of Jacobi polynomials). Moreover, function representation in series of orthogonal polynomials with collocation over these grid points offers improved interpolation *(2)*. However, the classical Gaussian quadrature is "open", i.e., the ends of the integrals are not used. For differential equations, the end points correspond to the boundary points, so they have to be included in the quadrature. Therefore, the application of Gaussian integration within the solution of PDEs requires the use of the Lobatto point version of the Gauss scheme

ANALYSIS, RESULTS AND DISCUSSION

Elrod and Brewe *(1)* presented a Lobatto-Galerkin solution of the thermal hydrodynamic lubrication problem for incompressible lubricants. Elrod *(2)* further improved the algorithm by replacing the Galerkin solution with a finite difference scheme. The difference formulas across the film were obtained by use of a series of Legendre polynomials. Up-wind discretization was employed for the convective terms in the energy equation. This newer Lobatto methodology has been implemented in a number of papers dealing with various aspects of the incompressible lubrication. This paper presents the extension of the Lobatto point quadrature algorithm *(2)* for the case of compressible lubricants. Viscosity and density are pressure and temperature dependent.

The physical domain (x,y,z) with $0 \le z \le h$ is mapped into the computational domain (ξ_x, ξ_y, ζ) with $-1 \le \zeta \le 1$. The vertical coordinate $\zeta = (2/h)z - 1$, while $\xi_x = x$ and $\xi_y = y$. The unknown functions are represented as a series of Legendre polynomials $f(\zeta) = \sum \underline{f_k} P_k(\zeta)$ where $\underline{f_k}$ are constants. All the integrals and derivatives are deduced in terms of the nodal values (f_i) of the functions at the Lobatto points.

It can be proven that, irrespective of the order of the series expansions, the thermal Reynolds equation can be written as, Ref. *(4)*

$$\frac{\partial}{\partial \xi_x}\left\{\frac{\widetilde{\rho}_x h^3}{12}\left[\left(\underline{\xi_F}\right)_0 + \frac{2}{5}\left(\underline{\xi_F}\right)_2 - \frac{[(\underline{\xi_F})_1]^2}{3(\underline{\xi_F})_0}\right]\cdot\frac{\partial p}{\partial \xi_x}\right\} +$$
$$\frac{\partial}{\partial \xi_y}\left\{\frac{\widetilde{\rho}_y h^3}{12}\left[\left(\underline{\xi_F}\right)_0 + \frac{2}{5}\left(\underline{\xi_F}\right)_2 - \frac{[(\underline{\xi_F})_1]^2}{3(\underline{\xi_F})_0}\right]\cdot\frac{\partial p}{\partial \xi_y}\right\} = \qquad [1]$$
$$\frac{u_L + u_U}{2}\cdot\frac{\partial(\widetilde{\rho}_x h)}{\partial \xi_x} + \frac{\partial\left(\overline{\rho} h\right)}{\partial t} - \frac{u_U - u_L}{6}\cdot\frac{\partial}{\partial \xi_x}\left[\widetilde{\rho}_x h\frac{(\underline{\xi_F})_1}{(\underline{\xi_F})_0}\right]$$

Copyright © 2005 by ASME

Downloaded From: https://proceedings.asmedigitalcollection.asme.org on 06/30/2019 Terms of Use: http://www.asme.org/about-asme/terms-of-use

where $\xi_F \equiv 1/\mu = \sum (\xi_F)_k P_k$ is the "fluidity" function and $\bar{\rho} = \frac{1}{2}\int_{-1}^{1} \rho\, d\zeta$, $\tilde{\rho}_x = \left(\int_{-1}^{1} \rho u d\zeta\right) / \left(\int_{-1}^{1} u d\zeta\right)$ while $\tilde{\rho}_y$ is computed using the *v* velocity instead of *u*.

Only one-dimensional applications are presented within this paper, so the sub-index *x* is dropped.

In the computational domain, the energy equation for a one-dimensional film is, *(4)*,

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial \xi} + \frac{2}{h}\frac{\partial T}{\partial \zeta}\left[w - u(\zeta+1)\frac{\partial (h/2)}{\partial \xi}\right] = \frac{4\kappa}{h^2}\frac{\partial^2 T}{\partial \zeta^2} + \frac{\beta T}{\rho C_p}\left(\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial \xi}\right) + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial \zeta}\right)^2 \quad [2]$$

The unknown temperature is calculated at each location along the bearing and across the oil film. The horizontal speed *u* is obtained by double integrating the *x* fluid momentum equation, as indicated in Ref. *(2)*. The vertical speed *w* can be obtained by integrating the continuity equation across the film, in the computational domain. This yields, *(4)*,

$$\rho w = (\rho w)_L + \frac{\partial h}{\partial \xi}\cdot\frac{\zeta+1}{2}\rho u - \frac{1}{2}\cdot\frac{\partial}{\partial \xi}\left[h\int_{-1}^{\zeta}\rho u d\zeta\right] - \frac{\partial}{\partial t}\left[\frac{h}{2}\int_{-1}^{\zeta}\rho\, d\zeta\right] \quad [3]$$

The subindex *L* specifies the bottom surface. The densities and the viscosities are computed at each (ξ, ζ) location using the actual temperatures and pressures.

Computation of elastohydrodynamic lubrication (EHL) cases also requires the computation of the surface deformation. This was performed using the Coefficients of Influence Method, described in Ref. *(3)*.

The resulting system of equations, Eqs. [1] – [3] (and, for EHL cases, the surface deformation equation) is solved using a non-segregated iteration. Lagging is used at each time step to compute the unknown values.

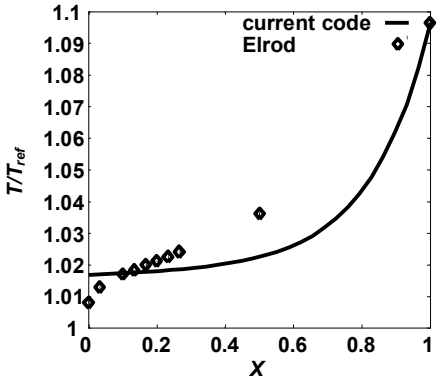

Fig.1. Temperatures at ζ=0.4472 for a flat slider with h_1/h_2=4

Figure 1 present comparisons between current results and the data obtained by Elrod *(2)* for a flat slider with reversed flow at the inlet. Elrod utilized 30x8 grid points, while the current results were obtained on a 30x4 mesh. Figure 2 presents a three dimensional temperature distribution within lubricant for an EHL line contact with W = 1.3 x 10^{-4}, U = 7.3 x 10^{-11}, G=3500 and 20% sliding.

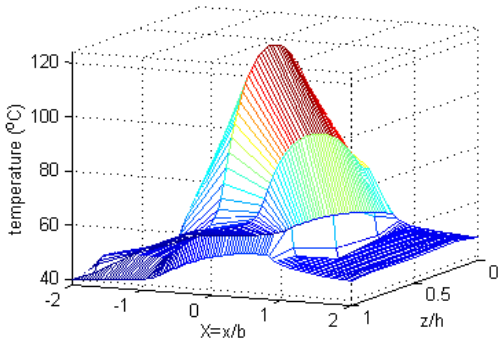

Fig.2. EHL temperature distributions within lubricant for W = 1.3 x 10^{-4}, U = 7.3 x 10^{-11}, G=3500, 20% sliding

CONCLUSIONS

Accurate solutions of thermal lubrication problems generally require intensive numerical integration over a very fine mesh. Integrations across the film can be accelerated by the use of Gaussian quadrature. Elrod and Brewe (1986) proposed an efficient algorithm based on the use of Lobatto point version of the Gauss quadrature. Within this approach, the unknown variables across the film are written in a series of Legendre polynomials. This paper presents a Lobatto quadrature algorithm which is applicable for thermal lubrication problems with compressible lubricants. In this case both density and viscosity of the lubricant are taken to be temperature and pressure dependent. The transverse velocity is obtained from the continuity equation. Use of the Labatto point calculation method has resulted in greater accuracy without the use of a larger number of grid points.

ACKNOWLEDGMENTS

This work was supported in part by the Rotorcraft Industry Technology Alliance (RITA). The authors would like to express their appreciation to our industrial and governmental supporters for their suggestions and support.

REFERENCES

(1) Elrod, H.G. and Brewe, D.E., (1986), "Thermohydrodynamic Analysis for Laminar Lubricating Films", NASA Technical Memorandum 88845.

(2) Elrod, H.G., (1991), "Efficient Numerical Method for Computation of the Thermohydrodynamics of Laminar Lubricating Films", *ASME Journal of Tribology*, 113, No.3, pp.506-511

(3) Hamrock, B.J., (1994) Fundamentals of Fluid Film Lubrication, McGraw Hill

(4) Moraru, L., (2005) "Numerical Predictions and Measurements in the Lubrication of Aeronautical Engine and Transmission Components", PhD. Dissertation, The University of Toledo.

Copyright © 2005 by ASME

Downloaded From: https://proceedings.asmedigitalcollection.asme.org on 06/30/2019 Terms of Use: http://www.asme.org/about-asme/terms-of-use