

INTRODUCTION

DIFFERENT THEORETICAL PERSPECTIVES AND APPROACHES IN MATHEMATICS EDUCATION RESEARCH - STRATEGIES AND DIFFICULTIES WHEN CONNECTING THEORIES

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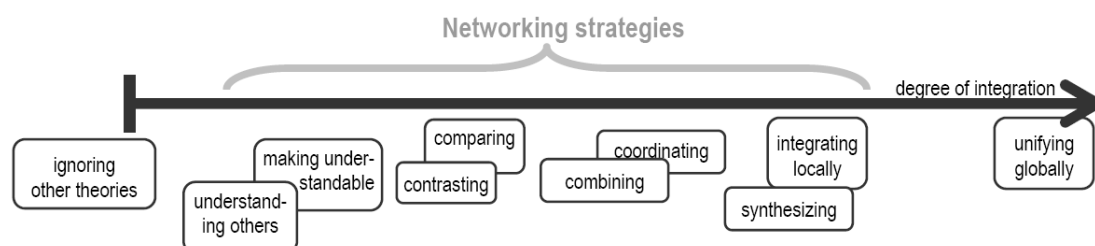
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A large diversity of different theoretical perspectives and research paradigms characterize the European mathematics education research community. Since CERME 4, the ‘Theory Working Group’ has explored differences between these theories, their expression in different research practices and possible ways to deal with this diversity (see Artigue et al. 2006, Bosch et al. 2008 and Prediger et al. 2008).

Exploiting diversity as a rich resource for grasping complex realities (Bikner-Ahsbabs & Prediger 2006) requires developing strategies for connecting theories or research results obtained using different theoretical approaches. In 2007, the Theory Working Group continued its efforts in this direction and reflected on opportunities and difficulties of what we call ‘networking theories’. We noted different intentions behind researchers efforts to network theories. In some cases, the goal is to investigate the complementary insights that are offered when we analyze given data with different theories (Kidron, 2008). In other cases, the intention is to explore the insights offered by each theory to the other theories and, at the same time, to highlight the limits of such an endeavour (Kidron et al., 2008; Radford, 2008).

The call for papers for the Theory Working Group at CERME 6 was guided by the idea of avoiding an overly abstract discussion without a concrete basis. That is why we called for papers with *concrete case studies* in which two or more theoretical approaches were connected. After an intensive peer review process, 15 substantial papers were chosen for discussion in the working group and for publication in these proceedings. The most important issues arising in the discussion of these case studies can be sketched under some key words structured according to the landscape of networking strategies as proposed by Prediger, Bikner-Ahsbabs & Arzarello, (2008).



Main issues arising in comparing and contrasting: Dimension of comparison

Comparing theories requires categories for comparison. A variety of categories have been suggested by Prediger, Arzarello & Bikner-Ahsbals (2008). The discussion this year was influenced by the following:

- the delimitation of empirical data and the kind of questions that arise, as well as the concrete formulation of results (see Ligozat & Schubauer-Leoni in this volume);
- the distinction between theoretical approaches and perspectives (discussed by Wedege in this volume);
- an ontological characterization of theories such as that proposed by Winsløw (in this volume) called the *GOA*-Model, which distinguishes theories according to nature of their objects of research, namely groups (*G*) structured by certain relationships, the organisation (*O*) of knowledge and practice, and artefacts (*A*) used to access and communicate in and about *O*.
- an epistemological characterization of theories such as that proposed by Radford (2008), distinguishing between their basic principles, their methodology, and the paradigmatic questions that are approached.

Main issues arising in combining and coordinating: Compatibility

In order to combine or coordinate different theories, it appears to us that the theories must, in some sense, be compatible; but what exactly does this mean? In working group discussions of the case studies presented in the papers, different levels were posited as possible locations for potential incompatibilities:

- the level of general principles, e.g. epistemological principles about how to interpret mathematical knowledge;
- the level of basic ‘paradigms’, the potential danger of hastily combining stability-oriented with transformation-oriented perspectives;
- the level of central constructs: although the sense or denotation of constructs may not be identical over different theories, they should not be contradictory (Gellert in this volume shows an interesting example of networking around the construct “rules”);
- the level of practical consequences: if coordinating theories in empirical work leads to contradictory practical consequences with regard to learning, then there is a need to continue reflection (see Bergsten & Jablonka in this volume);
- the level of ontology: this does not seem to present as many difficulties as some of the above since different grain sizes of analyses and focuses might help in combining theories (see, for example, Jungwirth in this volume).

In the working group discussion it was suggested that when paradigmatic research questions and/or objects diverge in different perspectives, the combination of these

perspectives in the course of analysing an empirical phenomenon might produce incommensurable, but not contradictory, results, as shown by the paper of Bergsten & Jablonka (in this volume). This raises the question of whether it is acceptable that different results can, without contradiction, lead to radically opposed interpretations.

On the other hand, we found some aspects that facilitate the connection of theories. Theories might be linked more easily when they are not too strong with respect to their grammar or their methodologies (i.e. when they are at an early level of elaboration) or when they are complementary with respect to their hypothetical scope or empirical load (see Jungwirth in this volume).

Main issue arising in integrating and synthesizing: Substrategies

The working group discussion regarding strategies for integrating and synthesizing theories led to the tentative proposal to identify substrategies which included: ‘bricolaging’ (that is adapting non-conflicting principles, notions or local analysis methods of different grand theories); ‘subordinating’ (see Gellert); ‘zooming in and out’ (see Jungwirth); and ‘metaphorical structuring’, the use of single concepts based on metaphors from one theory that converge into another (see Gellert with regard to rules).

As Radford (2008) stated, although connections between theories are possible, there is a *limit* to what can be connected and this limit is determined by the *goal* of the connection and the *specificities* of the theories that are being connected. In the following, we differentiate between different goals in the networking process.

Networking with different aims

In order to link theories beyond comparing and contrasting, we discussed the aims of the papers.:

- Some of the papers propose networking strategies with the aim of understanding an empirical phenomenon that seems difficult to entirely grasp within one single theory. These can be described as having an initial combining strategy that ends up with the construction of local coherence between the notions or principles used. In this sense, Arzarello, Bikner and Sabena (in this volume) combine theories for analysing data about a failed teaching strategy and integrating them (very) locally for the purpose of making sense of the situation described. The paper of Schäfer (in this volume) combines theories for constructing a local theory that improved his potential to approach a ‘practical’ question about low achieving students. Wedege (in this volume) presents a study in which some aspects of two theoretical perspectives are coordinated. Stadler (in this volume) coordinates different perspectives within one empirical study, describing how a research interest in the transition between mathematics studies at secondary and tertiary levels generates the need for different theoretical approaches.
- A different goal presented by some papers is to network with the aim of dealing with new problems. For example Ligozat & Schubauer-Leoni’s and Sensevy’s papers are hybrids which borrow constructs from distinct theories for local integra-

tion with conversions in order to address specific research problem, the issue of joint action of the teacher and the students.

- Networking is also an important tool to elaborate existing theories with the aim of increasing their scope by questioning them from the outside. Artigue, Bosch, Gascón & Lenfant (in this volume) show how a theory can evolve locally when an effort is made to approach a question formulated by another theory. The strategy here is to work within one theoretical framework and develop it in interaction with others, for instance by enlarging the set of paradigmatic research questions or its empirical unit of analysis. The work of Jungwirth (in this volume) presents a method of synthesizing local theories for ‘zooming in and out’ of the data.
- Other papers consider networking with the aim of satisfying the need for an enlarged framework in relation to some new domain of research, assuming the existing frames are insufficient. For instance, Lagrange & Monaghan (in this volume) incorporated Saxe’s four parameters model in order to understand the situation of teachers using technology. To these authors, the existing frameworks they considered for viewing teachers’ activities in technology-based lessons are insufficient because they focus on teachers’ established routines but technology interferes with these routines.

Different kinds of dialogues

Within these aims we may distinguish different kinds of *dialogues* between theories. We use the word ‘dialogue’ not only to describe that which enables mutual understanding in the way we communicate our theories but also to emphasize differences in the use of language. Different kinds of dialogues were offered in the papers by Ligozat & Schubauer-Leoni, by Sensevy and by Artigue et al. One important characterization is that the dialogues in these papers are between neighbouring approaches - theoretical approaches which were born in the same educational and didactic culture, which may be considered as belonging to the same ‘paradigm’. Even so, when we explore the dialogues in depth important differences between the theories can be seen and some interesting questions arise:

- Do these “neighbouring approaches” use the same words with the same meanings? For instance, is the word *milieu* in the Anthropological Theory of the Didactic (ATD) equivalent to the *a didactic milieu* in the Theory of Didactic Situations (TDS)? The same question could be asked in relation to other terms, e.g. *institution* or *contract*. The question could arise also for theories which are not necessarily *neighbouring approaches*.
- Do the different theories deal with different ways of addressing similar issues? For instance, comparing the Joint Action Theory in Didactics (JATD), as described in both Ligozat & Schubauer-Leoni and Sensevy’s papers, with ATD and TDS, we may ask what is the difference between *ATD media milieu dialectic*, *TDS a didactic and didactic situations*, and *JATD dialectic between contract and milieu*. Sensevy states that in order to situate JATD in relation to TDS and ATD it can be ar-

gued that whereas these two theories initially focus, from a logical point of view, on the nature of *knowledge* (what is the knowledge which is taught?), JATD initially focuses on the *diffusion process* (what is going on when a specific piece of knowledge is taught?). The aim of the networking is to construct a new theory JATD which makes use of existing theories, ATD and TDS. Therefore we may ask what supplementary insights and/or what new questions/problems are offered to ATD and TDS by JATD's analysis of the *diffusion process*? For example, the JATD may raise the following question: within the contract-milieu dialectic how may the teacher link the topogenesis and the chronogenesis processes with respect to the piece of knowledge at stake, and how might these processes lead the teacher, in specific cases, to enact a new learning game? In this question there are some notions from ATD and TDS which are reconceptualized in that they are used in a new way, and there is a new notion (learning game). From an abstract viewpoint, this kind of question is not impossible in ATD and TDS, and it is clearly understandable in these two theories. But the probability that this question is raised in these two theories is not high because their fundamental concerns are not focused on the problems of didactic joint action even though they are interested in didactic action.

In Artigue et al. (in this volume) the notion of 'minimal unit of analysis' appears as a basic aspect of the modelling of educational phenomena proposed by each theory. Starting from the way each perspective reformulates a given research question, we could specify what units of analysis are considered in each case and how they can be connected. The authors add that this could be a good way to improve our capacity for describing and comparing not only the concrete research or practical problem formulated by each theory but also the types of problems that can be proposed, the kind of empirical data needed and the set of 'acceptable answers' that can be provided. When we choose a *specific unit of analysis*, we make decisions not only about the empirical data we consider but also about our different priorities with regard to the focus of the analysis (Bosch & Gascón, 2005).

Final remarks

The discussions that took place in our working group about affordances and constraints of different networking strategies made us aware that the theoretical frameworks used in our research are 'living entities' that evolve through our studies. Some have been around and have developed for many decades, others are less mature. They are our working tools, providing us with new ways of looking at reality, new descriptions of empirical phenomena, new methods of analysis and new possible answers to the difficulties of teaching and learning mathematics. They are imbedded in researchers' social, cultural and institutional inheritances and their development is also impregnated with the personal interactions between researchers and the cooperative work done in our community. When we embody 'theories' into research practices that, at the same time, use theories and produce them, it becomes clear that our reflec-

tions about ‘networking theories’ are methodological reflections, referring to the kind of tools we can or cannot use, the basis and the aim of our research, as well as the kind of rules we follow.

Considering the networking of theories as the *networking of research practices* may lead us further not only in our capacity to collaborate between different groups of researchers (and thus accumulate efforts and results) but also to gain insight about the very nature – and the rationale – of our own research in mathematics education.

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