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Application of the ant colony search algorithm to reactive power pricing in an open electricity market

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ABSTRACT

Reactive power management is essential to transfer real energy and support power system security. Developing an accurate and feasible method for reactive power pricing is important in the electricity market. In conventional optimal power flow models the production cost of reactive power was ignored. In this paper, the production cost of reactive power and investment cost of capacitor banks were included into the objective function of the OPF problem. Then, using ant colony search algorithm, the optimal problem was solved. Marginal price theory was used for calculation of the cost of active and reactive power at each bus in competitive electric markets. Application of the proposed method on IEEE 14-bus system confirms its validity and effectiveness. Results from several case studies show clearly the effects of various factors on reactive power price.

1. Introduction

The traditional regulated and monopoly structure of power industry throughout the world is eroding into an open-access and competitive environment. Thus, planning and operation of the utilities are based on the economic principles of open-access markets. In this new environment electric markets are essentially competitive. Until now, effort has been directed primarily toward developing methodologies to determine remuneration for the active power of the generators. Although the investment in electric power generation and the fuel cost, represent the most important costs of power system operation, reactive power is becoming more and more important, especially from the security point of view and the economic effect caused by it [1].

Reactive power compensation and optimization sustains the exchange of electric power greatly as a part of ancillary services. The consumption of the reactive power follows a similar demand against time curve as the active power, especially for motor loads and furnaces. Therefore, the operation and cost allocation of reactive power is very important to the running and management of generation and/or transmission companies [1].

A fixed tariff on the remuneration for reactive power is insufficient to provide a proper signal of reactive power cost [2]. Berg et al. [3] pointed out the limitations of a reactive power price policy based on power factor penalties, and suggested the use of economic principles based on marginal theory [4]. However, these prices represent a small portion of the actual reactive power price [5–7]. Hao and Papalexopoulos [8] note that the reactive power marginal price is typically less than 1% of the active power marginal price and depends strongly on the network constraints. Assessing the cost of reactive power production is difficult, because of differences in reactive power generation equipment and local characteristics of reactive power [9]. Several models for cost of reactive power production have been developed [10–18]. However, despite the complexity, these models lack a precise definition for the cost of reactive power production. Also, the methodology to obtain the cost curves is not described adequately.

In a competitive electric market the generators may provide the necessary reactive power compensation if they are remunerated by the service, provided the loss of opportunity in the commercialization of active power is taken into account [12]. Static compensators (capacitive and inductive) may be remunerated according to their investment costs and depreciation of their useful lives [13].

To address the above mentioned needs, in present paper, both active and reactive power production costs of generators and capital cost of capacitors are considered in the objective function of OPF problem.

Then a new method based on the Ant Colony Optimization (ACO) and advanced sequential quadratic programming, is employed to solve the OPF problem.

Currently, most works are carried out in the direction of applying ACO to the combinatorial optimization problems [19,20]. For most of these applications, the results show that the ACO can outperform other heuristic methods. In power systems, the ACO has

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been applied to solve the optimum generation scheduling problems [21], unit commitment [22], economic dispatch of power systems [23] and the constrained load flow [24]. It is rather difficult to find a single search space, configuration and a parameter set of an ACO that can satisfy every optimization problem. Therefore, there is a need for the development of an improved version of the ACO tailored to solve the reactive power pricing. The ACO proposed in this paper formulates the reactive power pricing problem as a combinatorial optimization problem.

In several case studies, the IEEE 14-bus system was used to verify the validity of the proposed method. Different objective functions are applied in the simulation tests to observe their impacts on reactive power prices.

The paper is organized as follows: in Section 2 the objective function and constraints of reactive power pricing are presented. Section 3 describes the proposed ant colony search algorithm. In Section 4 the simulation results for IEEE 14-bus test system is illustrated.

2. Objective function and constraints of reactive power pricing

Active and reactive marginal prices are normally obtained through solving the optimal power flow in which an objective function subject to a set of equality and inequality constraints is minimized. The objective function is proposed as the summation of active and reactive power production costs, produced by generators and capacitor banks:

$$C = \sum_{i=1}^{N_g}[C_{gpi}(P_{Gi}) + C_{gqi}(Q_{Gi})] + \sum_{j=1}^{N_c} C_{Cj}(Q_{Cj}) \tag{1}$$

where N_g is the number of generators, $N_c$the number of buses which capacitor banks are installed, $C_{gpi}(P_{Gi})$ the active power cost function in ith bus, $C_{gqi}(Q_{Gi})$ the reactive power cost function in ith bus and $C_{Cj}(Q_{Cj})$ is the capital cost function of capacitor bank in jth bus.

Cost function of active power used in (1) is considered as follows:

$$C_{gpi}(P_{Gi}) = a + bP_{Gi} + cP_{Gi}^2 \tag{2}$$

The capacity of generators is limited by the synchronous generator armature current limit, the field current limit, and the under-excitation limits. Because of these limits, the production of reactive power may require a reduction of real power output. Opportunity cost is the lost benefit of this reduction of real power output of the generator.

Opportunity cost depends on demand and supply in market, so it is hard to determine its exact value. In simplest form opportunity cost can be considered as follows:

$$C_{gpi}(Q_{Gi}) = \left[C_{gpi}(S_{Gi,\max}) - C_{gpi}\left(\sqrt{S_{Gi,\max}^2 - Q_{Gi}^2}\right)\right] \cdot k \tag{3}$$

where $S_{Gi,\max}$ is the maximum apparent power in ith bus, Q_{Gi} the reactive power of generator in ith bus and k is the reactive power efficiency rate (usually between 5% and 10%).

Modified triangle method is an alternative strategy for reactive power cost allocation (see Fig. 1).

According to Fig. 1 we can write:

$$P' = P\cos(\theta) = S\cos^2(\theta) \tag{4}$$

$$Q' = Q\sin(\theta) = S\sin^2(\theta) \tag{5}$$

Using (4) and (5) we have:

$$\begin{aligned} &P' + Q' = S \\ &Cost(P') + Cost(Q') = Cost(S) \end{aligned} \tag{6}$$

For expressing active power cost, we replace (4) in (2) as follows:

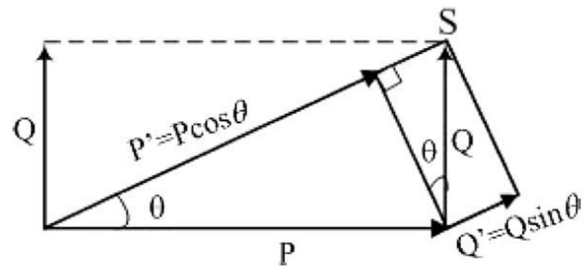

Fig. 1. An illustration of modified triangle method for reactive power cost allocation.

$$Cost(P') = Cost(P\cos(\theta)) = a + b\cos(\theta)P + c\cos^2(\theta)P^2 = a + b'P + c'P^2 \tag{7}$$

Using (2) and (5) the new frame of reactive power pricing can be written as given below:

$$\begin{aligned} Cost(Q') &= Cost(S\sin^2(\theta)) = Cost\left(\frac{P}{\cos(\theta)}\sin^2(\theta)\right) \\ &= a + b\sin(\theta)Q + c\sin^2(\theta)Q^2 = a + b''Q + c''Q^2 \end{aligned} \tag{8}$$

It is assumed that the reactive compensators are owned by private investors and installed at some selected buses. The charge for using capacitors is assumed proportional to the amount of the reactive power output purchased and can be expressed as:

$$C_{Cj}(Q_{Cj}) = r_j Q_{Cj} \tag{9}$$

where r_j and Q_{Cj} are the reactive cost and amount purchased, respectively, at location j. The production cost of the capacitor is assumed as its capital investment return, which can be expressed as its depreciation rate. For example, if the investment cost of a capacitor is \$11600/MVA, and their average working rate and life span are 2/3 and 15 years, respectively, the cost or depreciation rate of the capacitor can be calculated by:

$$r_j = \frac{\text{investment cost}}{\text{operating hours}} = \frac{\$11600}{15 \times 365 \times 24 \times 2/3} = \frac{\$0.1324}{\text{MVA h}} \tag{10}$$

In the reactive power cost optimization, the active power output of generators is specified. The bus voltage, the reactive power output of generators and capacitors are the control variables. The equality and inequality constraints include the load flow equations, active and reactive power output of generators, reactive power output of capacitors, and the bus voltage limits at the normal operating condition, as shown below:

Load flow equations:

$$\begin{aligned} &P_{Gi} - P_{Di} - \sum |\dot{V}_i||\dot{V}_j||Y_{ij}|\cos(\theta_{ij} + \delta_j - \delta_i) = 0 \\ &Q_{Gi} - Q_{Di} + \sum |\dot{V}_i||\dot{V}_j||Y_{ij}|\sin(\theta_{ij} + \delta_j - \delta_i) = 0 \end{aligned} \tag{11}$$

Active and reactive power generation limits:

$$\begin{aligned} &P_{Gi,\min} \leqslant P_{Gi} \leqslant P_{Gi,\max} \\ &Q_{Gi,\min} \leqslant Q_{Gi} \leqslant Q_{Gi,\max} \end{aligned} \tag{12}$$

Capacitor reactive power generation limits:

$$0 \leqslant Q_{Cj} \leqslant Q_{Cj,\max} \tag{13}$$

Transmission line limit:

$$|P_{ij}| \leqslant P_{ij,\max}, P_{ij} = |\dot{V}_i||\dot{V}_j||Y_{ij}|\cos(\theta_{ij} + \delta_j - \delta_i) - |\dot{V}_i|^2|Y_{ij}|\cos\theta_{ij} \tag{14}$$

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Bus voltage limits:

$$V_{i,\min} \leqslant |V_i| \leqslant V_{i,\max} \tag{15}$$

where P_{Di} and Q_{Di} are the specified active and reactive demand at ith load bus, respectively; $Y_{ij} \angle \theta_{ij}$ the element of the admittance matrix; $\dot{V}_i = V_i \angle \delta_i$ the bus voltage at ith bus; $P_{Gi,\min}$ and $P_{Gi,\max}$ are the lower and upper limits of active power generation at ith generator, respectively; $Q_{Gi,\min}$ and $Q_{Gi,\max}$ are the lower and upper limits of reactive power generation at ith generator, respectively; $Q_{Cj,\max}$ the upper limits of reactive power output of the capacitor and $V_{i,\min}$ and $V_{i,\max}$ are the lower and upper limits of voltage at ith bus, respectively.

The general-purpose optimization problem can be expressed as:

$$\begin{aligned} &\min_x f(X) \\ &g_i(X) = 0, \quad i = 1, 2, 3, \ldots, p \\ &h_j(X) \leqslant 0, \quad j = 1, 2, 3, \ldots, m \end{aligned} \tag{16}$$

The corresponding Lagrange function of the problem is formed as:

$$L(X, \lambda) = f(X) + \sum_{i=1}^{p} \lambda_i g_i(X) + \sum_{j=1}^{m} \lambda_{p+j} h_j(X) \tag{17}$$

where λ_i $(i = 1, 2, \ldots, p + m)$ is the Lagrange multiplier for the ith constraint.

Based on the above mathematical model the corresponding Lagrangian function of this optimization problem takes the form of (18).

$$\begin{aligned} L = &\sum_{i\in G}[C_{gpi}(P_{Gi}) + C_{gqi}(Q_{Gi})] + \sum_{j\in C} C_{Cj}(C_{Cj}) \\ &- \sum_{i\in N} \lambda_{pi}\left[P_{Gi} - P_{Di} - \sum |\dot{V}_i||\dot{V}_j||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i)\right] \\ &- \sum_{i\in N} \lambda_{qi}\left[Q_{Gi} - Q_{Di} + \sum |\dot{V}_i||\dot{V}_j||Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i)\right] \\ &+ \sum_{i\in G} \mu_{pi,\max}(P_{Gi,\min} - P_{Gi}) + \sum_{i\in G} \mu_{pi,\max}(P_{Gi} - P_{Gi,\max}) \\ &+ \sum_{j\in C} \mu_{Cj,\min}(Q_{Cj,\min} - Q_{Cj}) + \sum_{j\in c} \mu_{Cj,\max}(Q_{Cj} - Q_{Cj,\max}) \\ &+ \sum_{i\in G} \mu_{si}(P_{Gi}^2 + Q_{Gi}^2 - S_{Gi,\max}^2) + \sum_{i\in N} \sum_{\substack{j\in N \\ j \neq i}} \eta_{ij}(|P_{ij}| - P_{ij,\max}) \\ &+ \sum_{i\in N} \nu_{i,\min}(V_{i,\min} - |V_i|) + \sum_{i\in N} \nu_{i,\max}(|V_i| - V_{i,\max}) \end{aligned} \tag{18}$$

Definition of active and reactive power marginal cost prices:

The marginal price of electricity at a location (bus) is defined as the least cost to service the next increment of demand at that location consistent with all power system operating constraints. Marginal pricing plays an important role in many recently restructured wholesale power markets. In this scheme, a generating unit injecting energy at a given node is paid the marginal price corresponding to that node. Conversely, a demand receiving energy from a given node pays the locational marginal price corresponding to that node.

According to microeconomics, the marginal prices for active power and reactive power at ith bus are λ_{pi} and λ_{qi}, respectively and are defined as [15]:

$$\begin{aligned} \lambda_{pi} &= \frac{\partial L}{\partial P_{Di}} \\ \lambda_{qi} &= \frac{\partial L}{\partial Q_{Di}} \end{aligned} \tag{19}$$

3. Ant colony algorithm

Ant Colony Optimization method handles successfully various combinatorial complex problems. Dorigo has proposed the first ACO method in his PhD thesis [19]. ACO algorithms are developed based on the observation of foraging behavior of real ants. Although they are almost blind animals with very simple individual capacities, they can find the shortest route between their nest(s) and a source of food without using visual clues. They are also capable of adapting to changes in the environment; for example, finding a new shortest path once the old one is no longer feasible due to a new obstacle. The studies by ethnologists reveal that such capabilities are essentially due to what is called "pheromone trails", which ants use to communicate information among individuals regarding path and to decide where to go. During their trips a chemical trail (pheromone) is left on the ground. The pheromone guides other ants towards the target point. Furthermore, the pheromone evaporates over time (i.e. it loses quantity if other ants lay down no more pheromone). If many ants choose a certain path and lay down pheromones, the quantity of the trail increases and thus this trail attracts more and more ants [20]. Each ant probabilistically prefers to follow a direction rich in pheromone rather than a poorer one.

The basic ACO method was inspired by the behavior of real ant colonies in which a set of artificial ants cooperate in solving a problem by exchanging information via pheromone deposited on a graph. The basic ACO is often to deal with the combinatorial optimization problems. The ACO can be used to solve the continuous or discontinuous, nonconvex, nonlinear constrained optimization problems. The characteristics ACO are positive feedback, distributed computation, and the use of constructive greedy heuristic. The ACO method will find an optimal solution if it is run long enough, but it should be noted that optimality is traded for efficiency. Their main advantage is that in practice they often find reasonably good solutions in a short time [20].

The proposed ACO algorithm has the following features:

1. The points in feasible region are regard as "ants". After some iteration, the ants will centralize at the optimum points which could be one or more points. There are two choices for an ant in each iteration: moving to other ants' point or searching in neighborhood.
2. The iteration would be guided by changing the distribution of intensity of pheromone in feasible region.
3. Sequential quadratic programming (SQP) is used as neighborhood-searching algorithm to improve the precision of convergence.
4. The roulette wheel selection and disturbance are used to prevent the sub-optimization in ACO.

The convergence property of ACO is studied based on the fixed-point theorem on a complete metric space, presents several sufficient conditions for convergence.

3.1. The ACO procedure

The ACO procedure can be described as follows:

- *Step 1*: Initialization
- *Initial population*: An initial population of ant colony individuals X_i $(i = 1, 2, \ldots, N)$ is selected randomly from the feasible region S. Typically, the distribution of initial trials is uniform. The initial ant colony can be written as: $C^0 = (X_1, X_2, \ldots, X_N)^T$ for $X_i \in S$
- *Intensity matrix*: At initialization phase, the elements of trail intensity matrix $(\tau_{N\times N})$ are set to a constant level: $\tau_{ij} = \tau_0$, $\tau_0 > 0$

- *Number of ants*: Let $b(i)$ ($i = 1, 2, \ldots, N$) be the number of ants in point i and at the beginning $b(i) = l$.
- *Ant's visibility*: Ant's visibility can be defined as:

$$D(k) = 2\left(1 - \frac{1}{1 + e^{-\frac{\alpha k}{T}}}\right) D_0 \tag{20}$$

where k is the cycles counter, T is the upper limit of iteration number and D_0 is the upper limit of ant's visibility. With the running of ACO, the visibility $D(k)$ decreases and the exactitude of search increases gradually. If $\|X_i - X_j\| \leqslant D(k)$ then the ants can transfer from point i to point j, where $\|.\|$ is a kind of norm, which is defined as:

$$\|X\| = Max|x_i|_{1<i<n} \quad X = [x_1, x_2, \ldots, x_n]$$

- *Step 2*: For the ants on the point i ($i = 1, 2, \ldots, N$), $b(i) > 1$, the neighborhood search for transition is defined as:

$$A_i = \{X_j | \|X_i - X_j\| \leqslant D(k)\}$$

If $A_i \neq \Phi$ go to step 3, else go to step 4. Here Φ is empty set.

- *Step 3*: Let m be the quantity of elements in the set A_i, we set:

$$\eta_{ij} = F(X_i) - F(X_j), \quad \forall X_j \in A_i$$
$$\eta_{ii} = \frac{1}{m}\left(\frac{2}{1 + e^{-\frac{\alpha k}{T}}} - 1\right) \sum_{X_j \in A_i} \eta_{ij} \tag{21}$$

where $F(X)$ is objective function.

Transition probability is defined as:

$$P_0 = \frac{(\eta_{ii})^{\gamma_1}\left(\frac{1}{m}\sum_{X_j \in A_i}(\tau_{ij})\right)^{\gamma_2}}{(\eta_{ii})^{\gamma_1}\left(\frac{1}{m}\sum_{X_j \in A_i}\tau_{ij}\right)^{\gamma_2} + \sum_{X_j \in A_i}(\eta_{ij})^{\gamma_1}(\tau_{ij})^{\gamma_2}} \tag{22}$$

$$P_{ij} = \frac{(\eta_{ij})^{\gamma_1}(\tau_{ij})^{\gamma_2}}{(\eta_{ii})^{\gamma_1}\left(\frac{1}{m}\sum_{X_j \in A_i}\tau_{ij}\right)^{\gamma_2} + \sum_{X_j \in A_i}(\eta_{ij})^{\gamma_1}(\tau_{ij})^{\gamma_2}} \tag{23}$$

where γ_1 and γ_2 are parameters that control the relative importance of trail vs. visibility and P_0 is the probability of neighborhood search. If $F(X_j)$ decrease then the τ_{ij} and P_{ij} increase. Using (22) and (23) the following relation can be obtained:

$$\sum_{X_j \in A} P_{ij} + P_0 = 1 \tag{24}$$

The roulette wheel is used for stochastic selection. If the selection result is a P_{ij} carry out the update rule l.

Update rule 1: Moving an ant from point i to point j.

$b(i) = b(i) - l$, $b(j) = b(j) + l$, $\Delta\tau_{ij} = P_{ij}$, $Xi \leftarrow Xj$ and go to step 5. If the selection result is P_0, carry out the update rule 2.

Update rule 2: Carrying out search by sequential quadratic programming (SQP) algorithm in the neighborhood of X_i. The neighborhood defined by:

$$S_{X_i} = \{Y | \|X_i - Y\| < \alpha \cdot D(k)\}$$

where α is a positive parameter and $\alpha \in (0, 1)$. Let the result of neighborhood search be Y, then $X_i \leftarrow Y$ and:

$$\Delta\tau_{ij} = \frac{(F(X_i) - F(Y))^{\gamma_1}\left(\frac{1}{m}\sum_{X_j \in A_i}\tau_{ij}\right)^{\gamma_2}}{\sum_{X_j \in A_i}(\eta_{ij})^{\gamma_1}(\tau_{ij})^{\gamma_2}} \tag{25}$$

Go to step 5.

- *Step 4*: Searching in neighborhood with sequential quadratic programming (SQP) algorithm. Let the result be Y, carry out the update rule 3.

Update rule 3: $X_i \leftarrow Y$, $\Delta\tau_{ij} = r$, where r is a positive constant.

- *Step 5*: Updating the trail intensity matrix according to the following formula:

$$\tau_{ij}(k+1) = \rho\tau_{ij}(k) + \Delta\tau_{ij}, \quad \forall\ i \neq j,\ X_j \in A_i \tag{26}$$

where ρ is a coefficient such that $(1 - \rho)$ represents the evaporation of trail between time k and $k + 1$.

- *Step 6*: After iteration all ants have complete one move, calculate the results for every. Here C^k is the ant colony in k iterations.
  (1) If dissatisfying the convergence condition, cancel the result from step 2 to step 4 and go to step 2.
  (2) If the results are not changed after a definite iteration, disturb the ant colony by increasing the visibility and neighborhood of search.
  (3) If $k < T$ then $k = k + 1$ and go to step 2, else print best result and stop.

4. Simulation results and discussion

To investigate the validity of the proposed method, it has been applied to IEEE 14-bus (see Fig. 2) [25]. Generators characteristics are given in Table 1. The base of apparent power is 100 MVA. A 50 MVar capacitor bank is installed in bus 5 which can be adjusted continuously. The other system operation limits are:

1. Transmission limit: $|P_{ij}| \leqslant 1.8^{\text{p.u}}$.
2. Voltage limit: $0.95^{\text{p.u}} \leqslant |V_i| \leqslant 1.05^{\text{p.u}}$.
3. Swing bus settings: $V_1 = 1.05^{\text{p.u}}$ and $\delta_1 = 0$.

The ACSA parameters are: $N = 20$, $\tau_0 = 1$, $\gamma_1 = \gamma_2 = 1$, $T = 100$, $\alpha = 20$, $\rho = 0.9$ and $D_0 = 100$.

In order to study the impacts of various factors on the marginal price of reactive power, two scenarios are studied. In the first scenario, the impact of the various terms of cost function is analyzed. The effect of changes in load power factors is considered in the second scenario.

4.1. Scenario no. 1

In this scenario, four different objective functions have been used for OPF as below:

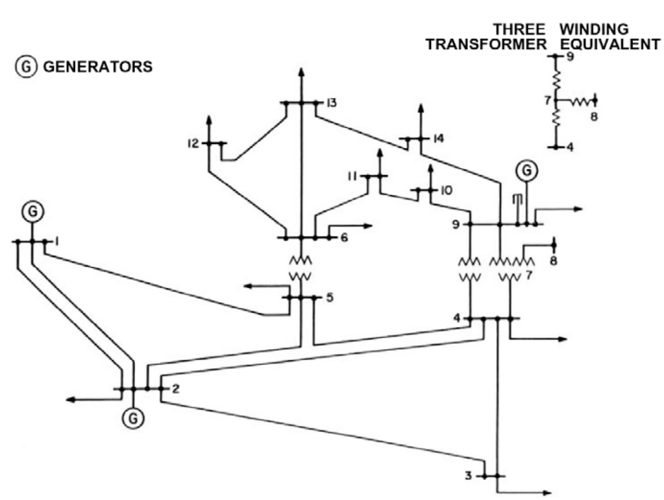

Fig. 2. IEEE 14-bus test system.

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Table 1
Generators characteristics.

Gen. no.	Bus no.	S_{max} (MVA)	$P_{g,min}$ (MW)	$P_{g,max}$ (MW)	$Q_{g,min}$ (MVar)	$Q_{g,max}$ (MVar)
1	1	125	20	125	−100	100
2	2	125	20	125	−100	100
3	9	80	20	80	−80	80
$C_{gpi}(P_{Gi}) = 75 + 750P_{gi} + 420P_{gi}^2 (\$/h)$						

1. In case 1, the objective function has only the first term of (l), i.e. only the cost of active power produced by generators is considered.
2. In case 2, the costs for both active and reactive power are considered in the objective function and the capacitor cost is ignored. In this case, the opportunity cost method is used for reactive power cost allocation.
3. In case 3, while the costs for both active and reactive power are included in the objective function, the cost function has been modeled according to the modified triangle method.
4. In case 4, the objective function has all three items as described in (1), where reactive power cost allocation is calculated based on the opportunity cost method.
5. In case 5, all three items as described in (1) are included in the objective function where reactive power cost is modeled using the modified triangle method.

The five cases are used to study the impacts of OPF objective functions and reactive power cost allocation method on Reactive Power Marginal Price (RPMP). The computer test results for cases 1–5 are listed in Table 2.

According to Table 2 the following remarks can be made:

- The active power marginal prices at various buses varies slightly when the objective function changes (The maximum change is 0.7% in bus 3).
- For each test case, active power marginal prices at various buses changes fairly slightly where the RPMP fluctuates significantly from bus to bus (see Figs. 3 and 4). Generally the active power marginal price is much higher than the RPMP at a certain bus. In our case, it is about 100 times as much as RPMP under normal conditions.

Table 2
Test results of cases 1–5.

Objective function	Case 1	Case 2	Case 3	Case 4	Case 5
Reactive power pricing method	–	Opportunity cost method	Triangle cost method	Opportunity cost method	Triangle cost method
$S_{Gi} = P_{Gi} + jQ_{Gi}$ $(i = 1, 2, 9)$	$\begin{bmatrix} 0.9096 - j0.2363 \\ 0.9443 + j0.3966 \\ 0.7974 + j0.065 \end{bmatrix}$	$\begin{bmatrix} 0.9103 - j0.0003 \\ 0.9453 + j0.1551 \\ 0.7964 + j0.0756 \end{bmatrix}$	$\begin{bmatrix} 0.9006 - j0.025 \\ 0.954 + j0.11869 \\ 0.7971 + j0.0683 \end{bmatrix}$	$\begin{bmatrix} 0.9104 + j0.0246 \\ 0.9469 + j0.1842 \\ 0.7948 + j0.0913 \end{bmatrix}$	$\begin{bmatrix} 0.9002 + j0.0154 \\ 0.9566 + j0.1965 \\ 0.795 + j0.089 \end{bmatrix}$
Reactive power output of capacitor on bus 5 (p.u)	0.5	0.5	0.5	0.4304	0.4289
System losses (p.u)	0.0613	0.0620	0.0617	0.0621	0.0618
Total active power cost of generators ($/h)	3202.489	3203.784	3171.453	3204.021	3166.595
Total reactive power cost of generators ($/h)	0	1.122	257.392	1.617	262.967
Total capital cost of capacitors ($/h)	0	0	0	5.699	5.679
Total cost ($/h)	3202.489	3204.906	3428.845	3211.337	3435.241
Marginal price of active power ($/MW h)	$\begin{bmatrix} 15.141 \\ 15.432 \\ 16.533 \\ 15.919 \\ 15.819 \\ 16.109 \\ 15.767 \\ 15.767 \\ 15.684 \\ 15.839 \\ 16.01 \\ 16.397 \\ 16.401 \\ 16.372 \end{bmatrix}$	$\begin{bmatrix} 15.145 \\ 15.442 \\ 16.595 \\ 15.951 \\ 15.845 \\ 16.148 \\ 15.794 \\ 15.794 \\ 15.708 \\ 15.868 \\ 16.046 \\ 16.445 \\ 16.45 \\ 16.417 \end{bmatrix}$	$\begin{bmatrix} 15.062 \\ 15.352 \\ 16.494 \\ 15.857 \\ 15.753 \\ 16.054 \\ 15.701 \\ 15.701 \\ 15.615 \\ 15.775 \\ 15.952 \\ 16.349 \\ 16.353 \\ 16.321 \end{bmatrix}$	$\begin{bmatrix} 15.151 \\ 15.450 \\ 16.616 \\ 15.968 \\ 15.860 \\ 16.171 \\ 15.809 \\ 15.809 \\ 15.722 \\ 15.885 \\ 16.066 \\ 16.472 \\ 16.477 \\ 16.444 \end{bmatrix}$	$\begin{bmatrix} 15.061 \\ 15.355 \\ 16.513 \\ 15.873 \\ 15.765 \\ 16.076 \\ 15.714 \\ 15.714 \\ 15.628 \\ 15.790 \\ 15.972 \\ 16.378 \\ 16.387 \\ 16.370 \end{bmatrix}$
Marginal price of reactive power ($/Mvar h)	$\begin{bmatrix} 0 \\ 0 \\ 0.286 \\ 0.098 \\ 0.004 \\ 0.062 \\ 0.114 \\ 0.114 \\ 0.121 \\ 0.177 \\ 0.153 \\ 0.164 \\ 0.222 \\ 0.315 \end{bmatrix}$	$\begin{bmatrix} 0.001 \\ 0.112 \\ 0.401 \\ 0.184 \\ 0.079 \\ 0.154 \\ 0.203 \\ 0.203 \\ 0.211 \\ 0.27 \\ 0.246 \\ 0.259 \\ 0.318 \\ 0.413 \end{bmatrix}$	$\begin{bmatrix} 0.001 \\ 0.100 \\ 0.389 \\ 0.178 \\ 0.074 \\ 0.145 \\ 0.198 \\ 0.198 \\ 0.207 \\ 0.265 \\ 0.239 \\ 0.249 \\ 0.308 \\ 0.406 \end{bmatrix}$	$\begin{bmatrix} 0.017 \\ 0.134 \\ 0.436 \\ 0.228 \\ 0.133 \\ 0.222 \\ 0.249 \\ 0.249 \\ 0.259 \\ 0.321 \\ 0.306 \\ 0.329 \\ 0.386 \\ 0.477 \end{bmatrix}$	$\begin{bmatrix} 0.001 \\ 0.120 \\ 0.427 \\ 0.228 \\ 0.132 \\ 0.249 \\ 0.270 \\ 0.270 \\ 0.291 \\ 0.352 \\ 0.335 \\ 0.361 \\ 0.423 \\ 0.549 \end{bmatrix}$

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- The total reactive power production cost changes apparently in conjunction with the objective function (from 0 to 262.967$/h). Although the cost is small, it can accumulate into a large amount.
- When the capacitor cost and the reactive power generation cost are neglected, the corresponding reactive power source bus(es) will have zero or very little RPMP(s) for the free reactive power available locally. The nearby buses also get benefits and have small RPMP(s). For example bus 6 of case 2, which is close to bus 5 where the capacitor is installed, has much smaller RPMP as compared with bus 14 which is far from reactive power sources (see Fig. 3).
- When the reactive power generation cost is taken into consideration in cases 2 and 3, the corresponding RPMP increases in comparison with case 1 (see Fig. 4). The results of RPMP are approximately equal in cases 2 and 3, but the reactive generation cost and the total production cost of the system in case 2 are noticeably greater than that in case 3. The system losses in case 3 are therefore reduced with respect to case 2. Reactive power output of capacitor on bus 5 has the maximum value in these cases, since the capacitor cost is neglected.
- The corresponding RPMP increases in some buses when the reactive power generation cost and the capacitor cost is considered in cases 4 and 5. This gives the load an incentive to reduce its reactive power demand. As it can be seen from Table 2, the total cost assigned to reactive power in triangle method (i.e. 262.967 $/h) is much greater than that of opportunity method (i.e. 1.617 $/h), which in turn, may encourage the reactive power producers to invest and provide enough reactive power. This will result in a more secure operation of the system in the future specially in restructured power systems. On the other hand, in power markets where the reactive power is priced based on opportunity method, there will not be any motivation for expansion of reactive power suppliers. It should be emphasized that in spite of the fact that reactive power is very important for enhancement of secure operation of the system, its cost is not comparable with that of active power.

Simulation was coded with MATLAB ver. 7 using a personal computer with Pentium 4 CPU (2.5 GHz) and 3 GB RAM. Table 3 shows performance of the proposed ant colony algorithm during 75 runs for each case. The best, worst and average value for total costs and average computational time are listed in Table 3. Thus the proposed method based on the ant colony algorithm and advanced sequential quadratic programming is capable of finding global optimum solution for the OPF problem.

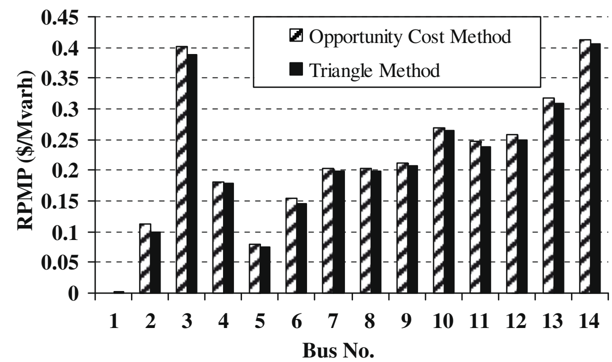

Fig. 4. Reactive power marginal price for cases 2 and 3.

Table 3
Performance of the proposed ACO for cases 4 and 5.

Case	Total cost ($)			Simulation time (s)
	Best	Mean	Worst	
Case 4 (opportunity method)	3211	3239	3306	36
Case 5 (triangle method)	3435	3451	3484	41

4.2. Scenario no. 2

In this scenario, the effect of changes in load power factors on reactive power marginal price and total reactive power cost is analyzed.

The objective function in this scenario has all three terms as described in (1).

Fig. 5 shows the impact of load power factor changes on average of RPMP that allocated by opportunity cost and triangle method. In low power factors, average of RPMP in triangle method is very much greater than that of opportunity cost method. The effect of the load power factor changes on RPMP(s) in triangle method is therefore more than the other method and may imply an economic incentive for consumers to reduce their reactive power.

Fig. 6 illustrates the effect of load power factor changes on reactive power generation cost. When load power factor increases the total reactive power production cost of generators decreases. The total reactive power cost resulted from triangle method obviously increases when load pf decreases.

Based on this study the following conclusions can be made:

- The reactive power production cost and the capital investment of capacitors should be considered in reactive power spot pricing for their noticeable impacts on reactive power marginal price.

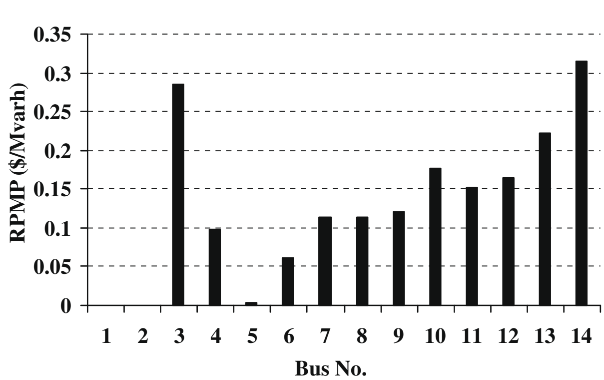

Fig. 3. Reactive power marginal price for case 1.

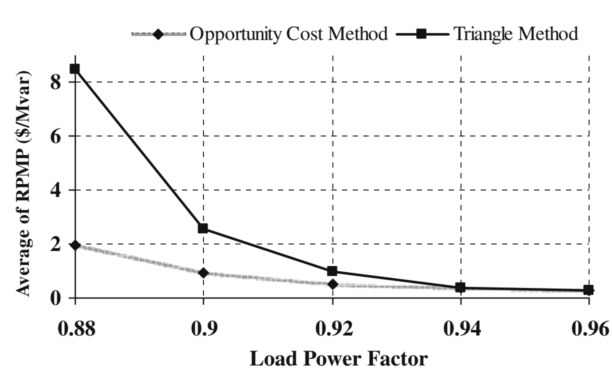

Fig. 5. Average of reactive power marginal price vs. load power factor.

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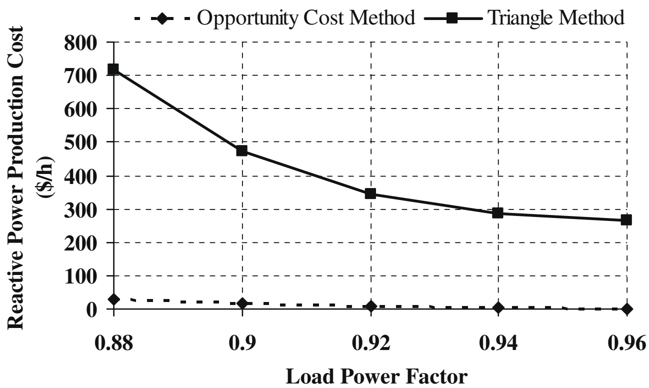

Fig. 6. Total reactive power cost vs. load power factor.

- Reactive power marginal cost can serve as a system index which is associated with the urgency of the reactive power supply, system voltage support, and as such is an incentive to improve load power factor and reduce reactive power demand.

5. Conclusions

In the study of reactive power marginal price in this paper, both active and reactive power production costs of generators and capital cost of capacitors are considered in the objective function of OPF problem. A new method based on ant colony algorithms and advanced sequential quadratic programming is employed to solve the OPF problem. The IEEE 14-bus system is used to verify the validity of the methodology, considering three objective functions. Test results may show that the reactive power production cost and the capital investment of capacitors should be considered in reactive power spot pricing for their noticeable impacts on reactive power marginal price.

Results confirm that reactive power cost allocation based on opportunity cost method may lead to wrong signals for market participants. However, triangle reactive pricing method seems to be accurate and fair when compared with opportunity cost method, and hence, is more compatible with non-discriminatory philosophy of open-access deregulated systems.

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