A Combined 3D Linear and Circular Interpolation Technique for Multi-Axis CNC Machining

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In multi-axis CNC machining of sculptured surfaces, a linear interpolation technique has been used to generate the command signals for positions along the straight line segments that connect each consecutive data point. Due to the rotational movements superimposed on the translational movements in multi-axis CNC machining, the actual cutter contact (CC) point moves along a space curve path, while the linear interpolation technique generates positions along the straight line path. The nonlinear curve segments deviate from the linearly interpolated line segments resulting in nonlinearity errors, which in turn, commonly cause difficulties to ensure high precision machining. An interpolator design technique for solving the nonlinearity errors problem in multi-axis CNC machining is presented. A combined 3D linear and circular interpolation principle is developed on the basis of the 3D linear and circular interpolation principles. The new designed interpolator is capable of driving the rotation movement pivot along a predesigned 3D curve path, so that the CC point motion trajectory is via a straight line connecting machining data points. Therefore, the proposed interpolator design technique on-line eliminates nonlinearity errors, and provides a solution to the nonlinearity errors problem for multi-axis CNC machining. [DOI: 10.1115/1.1445154]

1 Introduction

Sculptured surfaces are encountered in many objects such as turbine blades, automobile parts and airplane engine fan and compressor airfoils. For machining such complex surfaces, multi-axis CNC machining offers several advantages over the traditional three-axis machining, including better cutter access ability, higher metal removal rates, better surface finish, and more precise part surface in one setup. In multi-axes CNC machining, the cutter positions and its axis orientations, referred to as cutter location data (CLDATA), vary in space relative to the workpiece. Five-axis CNC machines or machining centers, which contain at least two rotary axes, are used to provide better access ability to realize the cutter location spatial changes. Five-axis CNC machining involves simultaneous translational and rotational movements since every new cutter axis orientation requires the motion of at least one other (and usually more) axis. And, there are coupling effects of the rotary movements on the translational movements, for changing the orientation of the cutter axis will affect the position of the tool. The simultaneous and coupled movements result in nonlinear machining motion trajectories. Consequently, a nonlinearity machining error arises in each motion step and causes the machining precision problem.

Many factors contribute to the machining error. One of the factors is due to the machining method. The most common method in multi-axes CNC machining is the "position contouring" method. Basically, the method connects each two consecutive NC-codes by using a straight line to approximate the desired cutting curve, of either a concave surface or a convex surface as shown in Fig. 1(*a*) and Fig. 1(*b*), and linear interpolation technique is used to coordinate the intermediate points of the line segment. The desired cutting curve is the design curve (either concave or convex) on the machined surface. The deviation of the straight line segment from the design curve segment is one portion of machining error. This linear approximation error is called the linearity error, δ_t, which depends upon the step-forward-size and the local geometry of the machined surface [1]. Apart from the linearity error in five-axis CNC machining, there is an additional machining error at each move, referred to as nonlinearity error, δ_n (see Fig. 1). Nonlinearity errors arise from the fact that the actual cutting point trajectory is a curve segment which deviates from the straight line segment (the cutter gage length is constant and the machine control point is interpolated along the line segment). In the case that the design surface is concave (see Fig. 1(*a*)), the total machining error is equal to the difference of the nonlinearity error from the linearity error: $\delta_{\text{total}} = \delta_t - \delta_n$ (in the case of concave design surfaces, the nonlinearity error is usually smaller than the linearity error). The nonlinearity error compensates for the total machining error. Thus, linear interpolation technique is desirable for machining of concave surfaces. On the contrary, for the machining of convex surfaces as shown in Fig. 1(*b*), the total machining error for each machining step is the sum of the linearity error and the nonlinearity error: $\delta_{\text{total}} = \delta_t + \delta_n$. That is, nonlinearity errors add onto linearity errors and increase the total machining errors, which commonly cause difficulties for ensuring ultra-precision machining requirements. On average, 5 percent to 15 percent of the total machining error is attributed to the nonlinearity errors. Nonlinearity errors depend upon the actual machining motion trajectory which is a function of the machine configuration and the rotational movements. The simultaneous and coupled rotational and translational movements generate the nonlinearity motion trajectory, and the linear interpolation machining method is not capable to curve fit the nonlinear path. Therefore, the source of the nonlinearity machining errors problem is due to the rotational movements and the use of the linear interpolation method.

One method to reduce the nonlinearity errors problem in five-axis CNC machining is to manipulate the cutter locations off-line. The off-line methods used by postprocessor producers and researchers, referred to as the "linearization process," treats nonlinearity errors by inserting additional cutter location data. Another off-line method is the "minimum error tool path generation method" which reduces nonlinearity errors by modifying the cutter orientations to attain the machining precision without inserting any data point. The ideal solution is to eliminate nonlinearity er-

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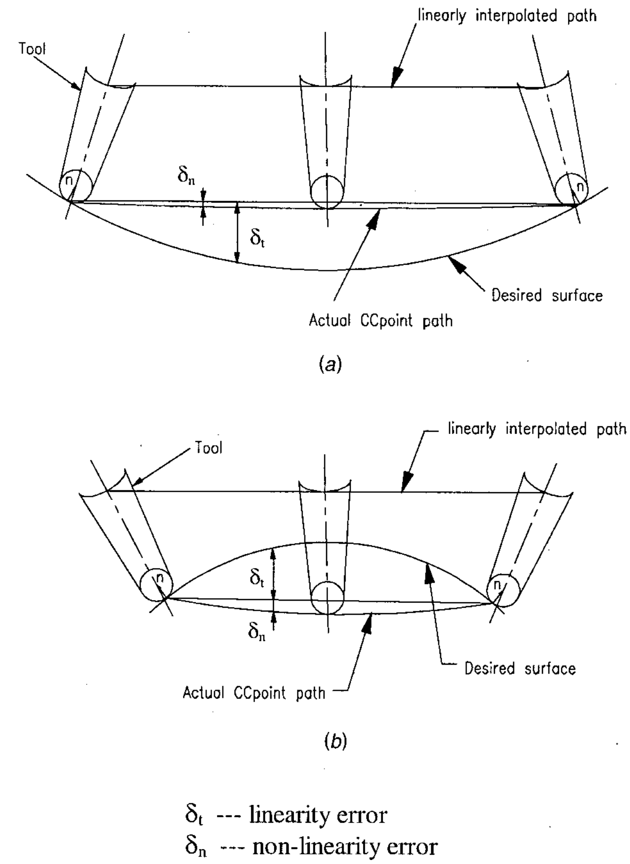

Fig. 1 The multi-axis CNC machining errors

rors, but the off-line methods can only reduce these errors. Since one of the problem sources is the utilization of the linear interpolation method, another solution route is to design new on-line interpolators. A combined 3D linear and circular (3D L&C) interpolation technique for solving the multi-axis CNC machining nonlinearity errors problem is proposed in this research.

In Section 2, methods for solving the multi-axis CNC machining errors problem and literatures on interpolator designs are reviewed. The development of the proposed combined 3D L&C interpolation principle is presented in Section 3. The proposed interpolator design technique for solving the nonlinearity errors problem in five-axis CNC machining is presented in Section 4. A computer simulation for machining the airfoil surfaces of a typical turbine impeller using the proposed interpolator is outlined in Section 5, which illustrates the elimination of nonlinearity machining errors and thus validates the proposed interpolation technique.

2 Literature Review

To solve the nonlinearity errors problem in multi-axis CNC machining, efforts have been made to treat the multi-axis CNC machining nonlinearity errors off-line. Takeuchi et al. [2] and Cho et al. [3] used the so called "linearization process" to modify NC-codes in multi-axes CNC machining processes. The function of the "linearization processes" is to insert additional data points between the adjacent NC-codes where the total machining error exceeds the specified tolerance range. Takeuchi et al. [2] calculated the insertion points by subdividing the straight line segments with equally spaced intervals and the cutter orientations were set to vary linearly in the successive insertion positions. Cho et al. [3] inserted intermediate cutter position data with linearly varying cutter orientations in nonequally spaced successive positions. In the Automation Intelligence Generalization Postprocessor (AIGP) [4] and the Vanguard Custom Postprocessor [5], "linearization processes" were designed in the NC-codes generation procedure. The method relies upon testing the deviations of the actual nonlinear tool path from the line segment connecting the NC-codes. The function is to bisectionally insert additional CLDATA points between adjacent CLDATA, which in turn, are transformed into NC-codes to ensure that the machining errors do not exceed the specified range. The "linearization processes" as described above produce NC-codes that satisfy the machining requirement. But, the NC-codes may contain dense sets of non-equally spaced data positions and incorrect cutter orientations (cutter orientations may not necessarily be linearly varying). As a consequence, application of the methods may deteriorate the feedrate fidelity and increase the machining time. In fact, the linear interpolation technique generates data points along a straight line segment, in which each path segment is subdivided into increments. With a specified feedrate and an interpolation period, the linear interpolator outputs a fixed increment in each interpolation interval. Since the different path segment length may not be an integer multiple of the fixed increment, the last increment in a segment is usually shorter than the fixed increment. But, this last increment is also machined at the same interpolation period, and this reduces the average feedrate along the path segment. This effect becomes significant when the path segment is very short. Furthermore, owing to the acceleration and deceleration effects at the beginning and the end of each segment, the tool may never reach the desired feedrate. As a result, the feedrate along the curve is not constant, which in turn, causes deterioration of the machined surface finish and increases the machining time. In addition, in the extreme case, "linearization process" could insert many data points between a pair of NC-codes, which results in the cutter position change to be near to zero. Thus, the rotational movements may cause the CC point to move randomly, which may cause a damaged workpiece and/or breaking the tool itself. In order to overcome the drawbacks of the "linearization process," one off-line method that minimizes multi-axis machining errors had been explored [6]. The basic idea of the "minimum error tool path generation method" is to reduce nonlinearity errors by manipulating cutter orientations based on machine type-specific kinematics and motion trajectories. This off-line tool path generation method reduces nonlinearity errors without inserting additional machining points, and therefore, the method does not result in the undesired consequences as encountered in the "linearization process." The off-line methods are capable of reducing, but cannot eliminate nonlinearity errors in five-axis CNC machining.

An interpolator is an essential component in a CNC machine, which is used to generate commands for tool motion between adjacent tool path data points as per accuracy requirements. Linear interpolation is commonly used in machining sculptured surfaces on five-axis machine tools because it satisfies the requirement that the interpolated curve can be easily converted from a position parameter to the time domain. However, linear interpolation has an inherent position error and has the drawback of velocity discontinuity at the tool path data points. Sata et al. [7] presented an analytical interpolation method, which used an incremental method for generating the Bézier curves to connect a series of discrete tool path data points. With this improvement, the number of interpolation segments is reduced as compared to linear interpolation. Stadelmann [8] developed a speed interpolator which can be used when the surface topology is available as a series of Bézier curves. By using parametric discretization, the method significantly reduced the computation time as compared with Sata's interpolator. Both of these interpolators ensured velocity-continuity from one segment to the next based on the assumption of constant acceleration over the entire segment. Makino [9] presented a trajectory control method using planar Clothoid curves to interpolate two lines or a series of points. By using the Clothoid curve, the direction and the curvature of the interpolated curve is kept continuous, so that a higher speed continuous path control could be achieved. Papaionnou and Kiritsis

[10] presented an extrapolated algorithm in which the interpolation points are determined by solving a constrained optimization problem. To satisfy the second order continuity requirement, the cubic spline interpolation technique has been applied by researchers. This interpolation technique entails the fitting of a composite third order parametric curve to the set of tool path points. Chou and Yang [11,12] proposed an analytical off-line interpolator for command generation, in which the cubic spline interpolation technique was used to generate the parametric cubic spline curve tool path, instead of the straight line segment of the linear interpolation. On the basis of Chou and Yang's [11] proposition, Huang and Yang [13] presented a realtime version of the interpolator which is capable of generating position commands with variable speed control and better speed accuracy for parametrically represented tool paths in three-axis CNC machining. To satisfy the requirement of easy conversion of the position from a tool path to the machining trajectory and the requirement of fast execution of interpolation, a tool path ideally should be parameterized in arc-length of the spline segment. The arc-length for each segment, however, is unknown, hence, the method of cubic spline interpolation is used to approximate the arc length. Usually, the chordal length is used as an approximation, but a relatively large error in the speed will occur. Renner and Pochop [14] and Renner [15] developed methods of fitting a composite cubic spline with closely being arc-length parameterization. Wang and Yang [16] presented a composite quintic spline interpolation method. The resulting composite quintic spline, in comparison with cubic spline, are nearly arc-length parameterization. Kiritsis [17] presented an incremental step interpolation algorithm for interpolating parametric curves. The method showed a non point-to-point interpolation scheme for accurate interpolator design.

The off-line part programming approaches decompose the design surface into line segments. These segments are then interpolated and converted into machining trajectory. The drawbacks of this off-line procedure are: (1) the acceleration and deceleration at each line segment is required, which produces less smooth curves and substantially increases machining time, (2) cutter orientations in five-axis machining are interpolated inaccurately, which causes position errors and unsmooth surfaces, and (3) the size of the tool path file could be very large for complicated parts and could cause memory shortage problems and data transmission errors [18]. Shpitalni et al. [19] presented a real-time interpolator which calculates new commands for the control loop during the execution time of the current commands. By using the real-time interpolator, the cutting curves are broken into segments by the machine control program and executed by the interpolator. Lin and Koren [20] presented a real-time five-axis interpolator for machining ruled surfaces. Using this real-time interpolator, the geometric information of cutting curves and the machining parameters were directly input into the interpolator, the cutter positions and orientations were calculated on-line by the interpolator, then, inverse kinematically transformed into the command signals to the control loops. Koren [21] developed a real-time interpolator for five-axis CNC machining. The interpolator calculates the cutter position and orientation during the same time period needed for sampling the control-loop feedback devices. Lo [22] presented a real-time surface interpolator in which the surface parameters in both of the tool path (step-forward) direction and the tool interval (step-over) direction, and the cutting conditions (such as feedrate, scallop height limit, etc.) are fed into the machine control unit. Since the CC point path is generated in real-time according to the present feedrate, the desired feedrate can be maintained. Lo and Hsiao [23] presented an interpolator with a contour error compensation procedure that is based on previous machining results. Applying the interpolator in the repeated machining process, the contour error is interpolated based on the previous extracted data and is added to the reference position commands. Thus, the previous contour machining result can be used to compensate the machining contour errors. The literature survey reveals those existing interpolation techniques are not considering nonlinearity machining errors in the current five-axis CNC machining process, therefore, designs of new interpolators are desired.

3 A Combined 3D Linear and Circular Interpolation Principle

3.1 The Multi-axis Machining Motion Trajectory. The motion trajectory in five-axis CNC machining depends on different machine configurations and the rotational movements. The common configuration of five-axis CNC machine tools includes the swivel head type and the rotary table on rotary table type (hereafter called the rotary table type). For the swivel head type five-axis CNC machine, the spindle chuck acts as the swivel (the rotational) movements pivot. The 3D circular swivel movements about the pivot superimposed on the 3D linear motion of the pivot, is the make up of the combined 3D linear and circular motion trajectory. For the rotary table type five-axis CNC machine, the two rotary axis intersection point acts as the rotational movements pivot. The machine table's 3D rotational movements about the pivot superimposed on the translational movements of the pivot, is the construct of the combined 3D linear and circular motion trajectory. To analyze 3D motion trajectories in five-axis CNC machining, without lost of generality, the OMINIMILL SERIES-1 (OM-1) milling center, which is the rotary table type, is considered in this research work.

The configuration of the OM-1 milling center is shown in the schematics of Fig. 2. The machine z_m axis is horizontal since the machine spindle is horizontal. The machine x_m axis is horizontal and the y_m axis is vertical. The machine B_m axis is vertical and coincides with the y_m axis, and the machine C_m axis is perpendicular to the y_m axis and parallel to the machine table top surface. The machine coordinate system origin is set at the top center of the machine table. The five-axis motions of the OM-1 are such that the machine table rotates about the B_m axis and the C_m axis, translates along the z_m axis and the x_m axis, and the spindle translates vertically relative to the y_m axis.

From the OM-1 configuration (see Fig. 2), both the B_m axis and the C_m axis move as the machine table translates and rotates. The two axes are perpendicular and intersect at a point, P, at all times during the machining processes. Since the intersection point P is on both the B_m axis and the C_m axis, its motion is only translational. Thus, the intersection point P acts as a pivot for the machining rotational movements, such that the part together with the machine table rotates about the moving pivot P tracing out a combined 3D nonlinear path. In other words, the actual CC point trajectory is the 3D circular movements (about the pivot P) which are superimposed on the linear interpolated motions of the pivot P. From this geometrical perspective, the CC point trajectory model for the OM-1 was determined as given by the following equations [6]:

$$\begin{aligned} x_{cc} &= x_p + l*\sin(B_m)*\sin(C_m) \\ y_{cc} &= y_p + l*\cos(C_m) \\ z_{cc} &= z_p + l*\cos(B_m) \end{aligned} \tag{1}$$

with:

$$\begin{aligned} x_p &= x_m \\ y_p &= C_{\text{pivot}} \\ z_p &= z_m \\ l &= \sqrt{(x_m - x_{\text{home}})^2 + (y_m - C_{\text{pivot}})^2 + (z_m - GL)^2} \end{aligned} \tag{2}$$

where, (x_{cc}, y_{cc}, z_{cc}) are the coordinates of the CC point, (x_p, y_p, z_p) are the coordinates of the rotation pivot P in reference to the machine coordinate system, l is the distance between the CC point and the pivot P, B_m and C_m are the rotational move-

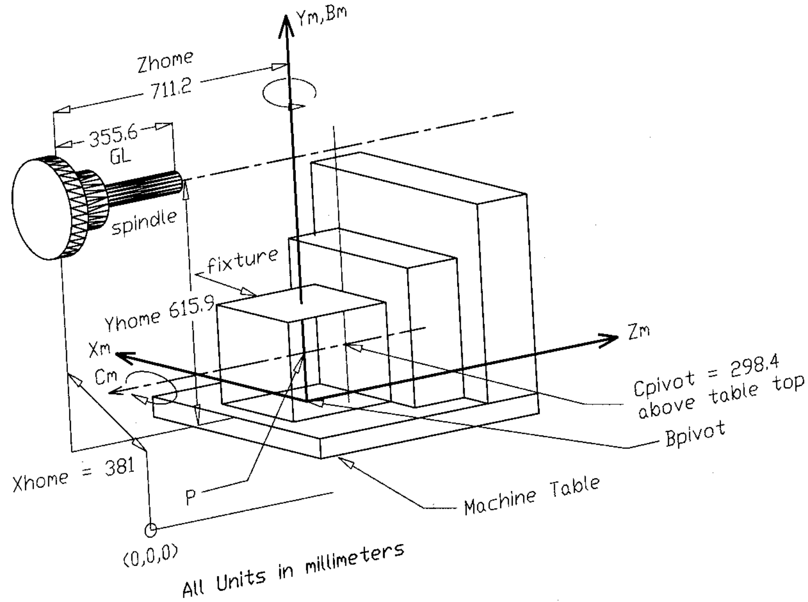

Fig. 2 The schematic configuration of the OM-1 CNC milling center

ments of the machine table, x_{home} is the spindle home position x coordinate, C_{pivot} is the C_m axis pivot coordinate which is a constant, and GL is the tool gage length.

3.2 The Rotational Movements Interpolation Principle. In multi-axis CNC machining, the rotational movement increments are the angles measured in degrees. The common interpolation method for the rotational movements is using the direct function algorithm [24]. This algorithm is designed for parametric space curves and is particularly suitable to rotational movements angles. The 3D machine movements are parametric functions of time, because each axis coordinate (linear position in length or circular position in degree units) can be a parametric function of time. The direct function algorithm calculates the interpolated rotational coordinate as follows [24]:

$$\begin{aligned} B_i &= B_0 + \tau(B_1 - B_0) \\ C_i &= C_0 + \tau(C_1 - C_0) \end{aligned} \tag{3}$$

where, (B_i, C_i) represent the interpolated coordinates of the rotational movements in degree units. (B_0, C_0) and (B_1, C_1) represent the start and the end coordinates of the rotational angles for the move respectively, τ represents a parameter which is proportional to time and varies in the range of (0,1), i.e., $\tau=0$ at (B_0, C_0) and $\tau=1$ at (B_1, C_1).

3.3 The 3D DDA Interpolation Principles. The DDA interpolation is based on the solution of differential equations, therefore, the methods have the advantage that uniform feedrate can be attained for nonlinear path interpolation. The 2D DDA linear interpolation principles are well known [25,26]. Based on the 2D DDA linear interpolation principle, the 3D linear interpolation formula can be derived as [27]:

$$\begin{aligned} x_{i+1} &= x_i + \lambda_l (x_e - x_s) \\ y_{i+1} &= y_i + \lambda_l (y_e - y_s) \\ z_{i+1} &= z_i + \lambda_l (z_e - z_s) \end{aligned} \tag{4}$$

where, (x_i, y_i, z_i) and $(x_{i+1}, y_{i+1}, z_{i+1})$ represent the coordinates of the present interpolated and the next interpolated points respectively, (x_s, y_s, z_s) and (x_e, y_e, z_e) represent the start point and end point coordinates of each segment. $\lambda_l = FT/L$ represents a scale factor determined from the specified machining feedrate, F, the chosen interpolation period, T, and the segment length, L. Using Eq. (4), the motion is coordinated with a constant feedrate on each segment.

On the basis of the 2D DDA circular interpolation principle [26], the 3D DDA circular interpolation principle can be derived [27] as shown in the following equation:

$$\begin{aligned} X_{i+1} &= X_i + \lambda_c [\mathbf{n}_z (Y_i - y) - \mathbf{n}_y (Z_i - z)] \\ Y_{i+1} &= Y_i + \lambda_c [\mathbf{n}_x (Z_i - z) - \mathbf{n}_z (X_{i+1} - x)] \\ Z_{i+1} &= Z_i + \lambda_c [\mathbf{n}_y (X_{i+1} - x) - \mathbf{n}_x (Y_{i+1} - y)] \end{aligned} \tag{5}$$

where, (x, y, z) represent the interpolation circle center coordinates, (X_i, Y_i, Z_i) and $(X_{i+1}, Y_{i+1}, Z_{i+1})$ represent the coordinates of the present 3D circular interpolated and the next 3D circular interpolated points on the interpolation circle, $\mathbf{n}_x, \mathbf{n}_y, \mathbf{n}_z$ are the unit normal vector of the circular interpolation planes and λ_c represents the circular interpolation scale factor which depends on the specified machining feedrate, the interpolation period and the radius of the interpolation circle.

3.4 A Combined 3D Linear and Circular Interpolation Principle. Conventional five-axis CNC machining uses the linear interpolation method to coordinate the translational movements along each 3D straight line segment and the rotational movements are interpolated based on the direct function algorithm of Eq. (3). As a result, the CC point trajectories are the curved segments as shown by the machining motion trajectory model of Eq. (1) and as shown in Fig. 1(*b*). These curved motion trajectories are the concave curve segments opposite to the convex curve of the designed surface and deviate from the linear interpolated line segment. Thus, the multi-axis machining nonlinearity errors result. To eliminate the nonlinearity errors, it is desired to conduct the CC point to move along the 3D straight line segment. From the geometrical perspective, this can be achieved if the linear interpolated points of the rotational pivot is shifted to opposite the nonlinear motion trajectory with the same amount of nonlinearity error at each interpolated points. Since the interpolated points on the nonlinear motion trajectory are following the 3D curve segment, the interpolated rotation pivot may be shifted to a predesigned curve which is a 3D curve the same as the nonlinear motion trajectory curve. In other words, the CC point will move on the linear line segment (the tool gage length is constant) if the interpolated rotation pivot is shifted to a convex curve designed by combining the 3D linear and circular curve. Upon this geometrical analysis and on the basis of the 3D linear interpolation principle of Eq. (4) and the 3D circular interpolation principle of Eq. (5), a combined 3D L&C interpolation principle was derived as follows [27].

By substituting Eq. (4) into Eq. (5) as the interpolation circle center, one may obtain the interpolated x coordinate:

$$\begin{aligned} X_{i+1} &= X_i + \lambda_c[\mathbf{n}_z(Y_i - (y_i + \lambda_l(y_e - y_s))) \\ &\quad - \mathbf{n}_y(Z_i - (z_i + \lambda_l(z_e - z_s)))] \\ &= X_i + \lambda_c[\mathbf{n}_z(Y_i - y_i) - n_y(Z_i - z_i) \\ &\quad + \lambda_l(\mathbf{n}_y(z_e - z_s) - n_z(y_e - y_s))] \end{aligned} \tag{6}$$

From the 3D DDA circular interpolation, replacing (X_i, Y_i, Z_i) by the initial point (x_s, y_s, z_s), and replacing $(X_{i+1}, Y_{i+1}, Z_{i+1})$ by the interpolated point (x_i, y_i, z_i), one may obtain the following equation:

$$x_i - x_s = \lambda_c[\mathbf{n}_y(z_i - z_s) - \mathbf{n}_z(y_i - y_s)] \tag{7}$$

Thus, by replacing the interpolated point with the final end point (x_e, y_e, z_e) of the segment, and using the linear interpolation scale factor λ_l, one may obtain:

$$x_e - x_s = \lambda_l[\mathbf{n}_y(z_e - z_s) - \mathbf{n}_z(y_e - y_s)] \tag{8}$$

By substituting Eq. (8) into Eq. (6), the $(i+1)$-th interpolated x coordinate is obtained as:

$$X_{i+1} = X_i + \lambda_c[(x_e - x_s) + \mathbf{n}_z(Y_i - y_i) - \mathbf{n}_y(Z_i - z_i)] \tag{9}$$

Similarly, the $(i+1)$-th interpolated y coordinate and the z coordinate functions can be determined. The developed equations are shown as the following:

$$\begin{aligned} X_{i+1} &= X_i + \lambda_c[(x_e - x_s) + \mathbf{n}_z(Y_i - y_i) - \mathbf{n}_y(Z_i - z_i)] \\ Y_{i+1} &= Y_i + \lambda_c[(y_e - y_s) + \mathbf{n}_x(Z_i - z_i) - \mathbf{n}_z(X_{i+1} - x_{i+1})] \\ Z_{i+1} &= Z_i + \lambda_c[(z_e - z_s) + \mathbf{n}_y(X_{i+1} - x_{i+1}) - \mathbf{n}_x(Y_{i+1} - y_{i+1})] \end{aligned} \tag{10}$$

where, all variables and parameters have previously been defined. By applying the developed interpolation principle, a combined 3D L&C interpolation technique is proposed as follows.

4 A Proposed Interpolator Design Technique

A methodology by applying the combined 3D L&C interpolation principle is proposed, which coordinates the translational axes and so does the rotation pivot along the predesigned 3D curve segment, such that the CC point moves along the 3D line segment connecting each pair of consecutive machining data points. Therefore, nonlinearity errors of five-axis CNC machining can be eliminated. The method interpolates iteratively the CC point coordinates which are determined from the machine motion trajectory model. An algorithm for performing the proposed interpolation principle is as follows:

A Combined 3D Linear and Circular Interpolation Algorithm

(1) Compute the start and the ending CC point coordinates for each segment by using the machine motion trajectory model (for OM-1, using Eq. (1) and Eq. (2));

(2) Off-line preparatory calculation of the interpolation scale factors for each segment:

(2.1) Calculate each segment length:

$$L = \sqrt{(x_{cc}(e) - x_{cc}(s))^2 + (y_{cc}(e) - y_{cc}(s))^2 + (z_{cc}(e) - z_{cc}(s))^2} \tag{11}$$

where, the index e represents the end point and the index s represents the start point.

(2.2) Calculate the linear interpolation scale factor: $\lambda_l = FT/L$;

(2.3) Calculate the rotational (3D circular) movements radius, R, which equals to the distance between the CC point and the rotation pivot. Hence, one may have:

$$R = \sqrt{(x_m - x_{\text{home}})^2 + (y_m - C_{\text{pivot}})^2 + (z_m - GL)^2} \tag{12}$$

where (x_m, y_m, z_m) are the NC-codes coordinate, x_{home} is the spindle position x-coordinate, C_{pivot} is the C_m axis pivot constant and GL is the cutter gage length.

(2.4) Calculate the circular interpolation scale factor: $\lambda_c = FT/R$;

(3) Interpolation routine:

(3.1) Interpolate the CC point's coordinates based on the start point $(x_{cc}(s), y_{cc}(s), z_{cc}(s))$ and the end point $(x_{cc}(e), y_{cc}(e), z_{cc}(e))$ of the segment:

$$\begin{aligned} x_{cc}(i+1) &= x_{cc}(i) + \lambda_l(x_{cc}(e) - x_{cc}(s)) \\ y_{cc}(i+1) &= y_{cc}(i) + \lambda_l(y_{cc}(e) - y_{cc}(s)) \\ z_{cc}(i+1) &= z_{cc}(i) + \lambda_l(z_{cc}(e) - z_{cc}(s)) \end{aligned} \tag{13}$$

(3.2) Interpolate the translational axes coordinates in the order of: x-axis$\rightarrow$$y$-axis$\rightarrow$$z$-axis, by using the combined 3D L&C interpolation principle of Eq. (10) and by substituting the linearly interpolated CC point's coordinates in step (3.1) as the circle center coordinates of Eq. (10);

(3.3) Interpolate the rotational axes circular movements coordinates according to the direct function algorithm of Eq. (3);

(4) Calculate the length between the start point and the present interpolated CC point:

$$l(i) = \sqrt{(x_{cc}(i) - x_{cc}(s))^2 + (y_{cc}(i) - y_{cc}(s))^2 + (z_{cc}(i) - z_{cc}(s))^2} \tag{14}$$

(5) Compare the coordinates of the interpolated point with the end point of the segment by performing the test:

$$(L - l(i)) \geqslant FT \tag{15}$$

If the test is true, perform step (6), otherwise go to step (7).

(6) Repeat the interpolation routine on the present segment by performing steps (3) to (5).

(7) If the test is false, determine the modified feedrate:

$$F' = \frac{(L - l(i))}{T} \tag{16}$$

and modify the interpolation scale factors: $\lambda_l = F' \cdot T/L$ and $\lambda_c = F' \cdot T/R$;

and repeat step (3) to reach the end point of the segment.

(8) Repeat steps (1) to (7) on the next segment until the machining arrives at the end of the NC-codes.

To implement the proposed interpolator design technique in five-axis machining, a software interpolation routine has been developed. The new software interpolation routine computes the coordinates based on the proposed algorithm. It includes the following functions:

(1) A function for accepting the machining NC-codes and the machining parameters such as the specified feedrate and the chosen interpolation period;
(2) A function to determine the CC point coordinates that correspond to the end points of each movement by employing the machine motion trajectory model;
(3) Three functions to compute the interpolated translational axes coordinates based on the combined 3D L&C interpolation principle of Eq. (10);
(4) Two functions to calculate the interpolated rotational movements coordinates based on the direct function algorithm of Eq. (3).

The program starts by inputting the machining parameters: the specified feedrate F, the chosen interpolation period T, and the machining NC-codes. Then, the CC point coordinate of the end points of each segment is computed. Using the machining parameters and the NC-codes, the linear and circular interpolation scale factors can be determined. These are constants for each machining step move and are calculated off-line prior to the on-line execution of the interpolation. Then, the real-time interpolation starts by initializing each axis coordinate for each segment. The translational axes interpolated positions and the rotational axes interpolated points are then computed in succession. The iterative computation of the interpolated positions for each axis is performed under the condition that the total interpolation incremented length is less than the segment length and the remaining length of $(L - l(i))$ is greater than the interpolation increment as given in Eq. (15). In the case that the remaining length is smaller than the interpolation increment, it is used to modify the feedrate which, in turn, is used to command the axes to reach the end point of the segment. This interpolation routine repeats until the machining completes all of the NC-codes. For the final interpolation point, it should be noted that each axis may not arrive at the segment end at the same time since each axis segment length is different. The iterative interpolation is performed under the condition of Eq. (15) and a feedrate adaptation procedure, so that the interpolation process on the next segment is commenced at the same time for each axis.

5 An Application Using the Proposed 3D L&C Interpolation Technique

Machining the airfoil surfaces of a turbine impeller, five-axis point milling is commonly used. In the following, the conventional five-axis CNC machining process for machining the airfoil surfaces of an impeller is simulated. This simulation result is then compared with that from the simulation of machining the same airfoil surface using the proposed 3D L&C interpolation technique.

5.1 Simulation of Machining Blade Surface Using Linear Interpolation Technique. To determine the interpolation increments, the machining feedrate and the interpolation period must be determined for considering the interpolation accuracy. The feedrate in this study was specified as $F = 25$ mm/sec (60 ipm) and the interpolation period $T = 10$ ms was determined for the machining tolerance of $t_r = 0.005$ mm $(0.0002'')$.

A set of NC-codes for machining an impeller airfoil were obtained from a sample of a real machining process in industry. The 3D cutting curve (or the tool path) on the airfoil of the impeller in reference to the part coordinate system is shown in Fig. 3. The cutter locations for machining the airfoil of the impeller in reference to the part coordinate system is shown in Fig. 4. The translational axes, i.e., the coordinates of the machine rotation pivot P, were interpolated based on the 3D linear interpolation principle of Eq. (4). The rotational axes were interpolated circularly based on the direct function algorithm of Eq. (3). The linear interpolation and the direct function algorithms interpolated the translational and the rotational movements coordinates in each interpolation period as in the real machining process. The corresponding cutting point (CC point) moved along the machining motion trajectory. The CC point coordinates, for the OM-1 milling center, was then calculated based on the OM-1 machining motion trajectory model of Eqs. (1) and (2).

The resulting CC point path in reference to the machine coordinate system is plotted as shown in Fig. 5. It shows that the CC point path is formed by a series of space curve segments, which is a fact that is obvious from the OM-1 nonlinear machining motion trajectory model (the space curve segments looking not smooth are due to the effect of the reduced 2D plot of the 3D curve). The enlarged closer view of the CC point path between a pair of adjacent machining data points shows that the CC point moves along the nonlinear curve segment, which deviates from the straight line segment. The machining error resulting from this simulation consists of both linearity and nonlinearity errors in each step move. For instance, in step one, the linearity error is 0.101 $\times 10^{-3}$ [inch] and the nonlinearity error is 0.346×10^{-3} [inch]. The total machining error is 0.447×10^{-3} [inch]. The simulation results confirm the machining error analysis and show the inadequacy of the linear interpolation method when applied to current multi-axis CNC machining of convex surfaces.

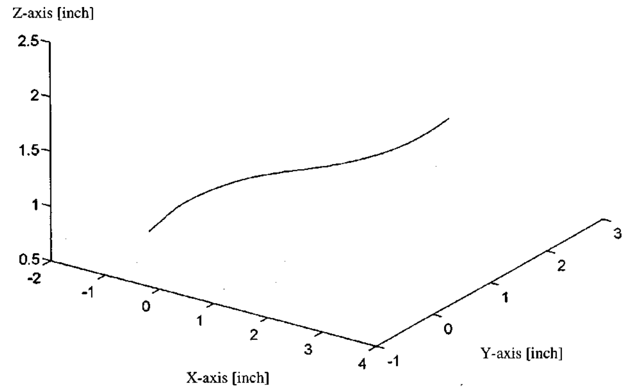

Fig. 3 The cutting curve on the airfoil surface

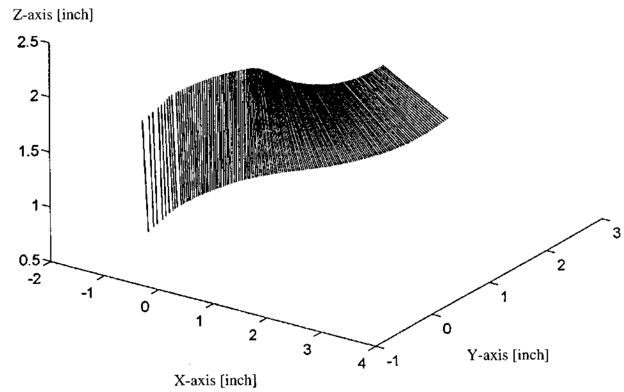

Fig. 4 The cutter locations for machining the airfoil surface

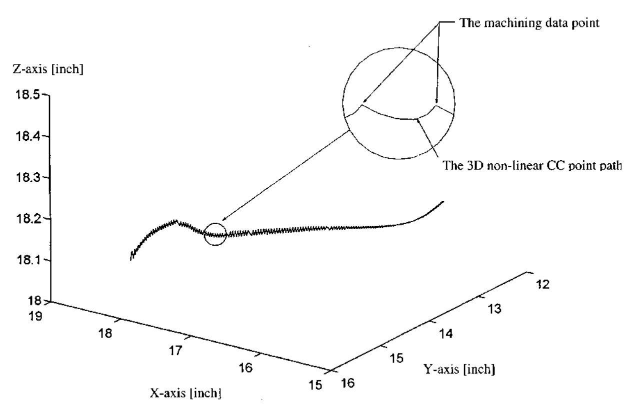

Fig. 5 The simulated CC point path by linear interpolation

5.2 Simulation of Machining Airfoil Surfaces Using the 3D L&C Interpolator. Using the proposed combined 3D L&C interpolation technique, a simulation for machining the same airfoil of the impeller as in Section 5.1 was performed. The same feedrate and interpolation period as in Section 5.1 were used to calculate the interpolation scale factors. The initial interpolation coordinates of the pivot P was assigned as the start point coordinates of each segment. In the first interpolation period, the first point of translational movements was interpolated by using the combined 3D L&C interpolation formula of Eq. (10). The rotational movements were interpolated in the same way as in the linear interpolation simulation of Section 5.1. The interpolated position and rotation coordinates were input into each machine axis control loop as the reference-words. It should be indicated that the control loops were not simulated in this interpolator function simulation study. After inputting the interpolated coordinates to each of the control loops, the interpolated CC point increment was compared with the present segment length. Upon the comparison test, the next point of the translational axes and the rotation incremented coordinates were interpolated on the basis of the previous interpolated coordinates. This iterative interpolation procedure repeated until the interpolated point reached the end point of the segment (the number of interpolations performed depends on the segment length of the machining step move). When the test showed the difference between the length of the total increments and the segment was shorter than the interpolation increment F^*T, the feedrate was modified as in the algorithm above and the interpolation scale factors were also re-calculated. Then, the 3D L&C interpolation was continued to command the machine axes to move to the end point of the segment. The same procedure was performed on each consecutive segment of the complete set of NC-codes.

The interpolated pivot point path in reference to the machine coordinate system is plotted as shown in Fig. 6. It shows that the pivot point path is constructed by a series of 3D curve segments (the spatial curve segments looking not smooth are due to the 2D plot of 3D curves). The enlarged closer view of the pivot point path between each pair of adjacent machining data points shows that the pivot point moves along a 3D nonlinear curve segment which deviates from the straight line segment. The corresponding interpolated CC point path in reference to the machine coordinate system is plotted as shown in Fig. 7. It clearly shows that the resulting CC point path is formed by a series of smooth segments. In fact, the CC point path is connected by a series of line segments and the nonlinearity error for each step move is eliminated. The machining errors resulting from this simulation were also computed. In comparison with the linear interpolation simulation, the total machining error in the first step move is 0.101×10^{-3} [inch] which equals to the linearity error for the step move. The same results consisting of only linearity errors were obtained for all steps simulated.

Compared with the results obtained from the simulations, the linear interpolation method is simpler, but it results in both linearity errors and nonlinearity errors. Alternatively, the combined 3D L&C interpolation technique conducts the CC point to move along straight line segments, so that the only machining error for each segment is the linearity error. For the simulation, the average

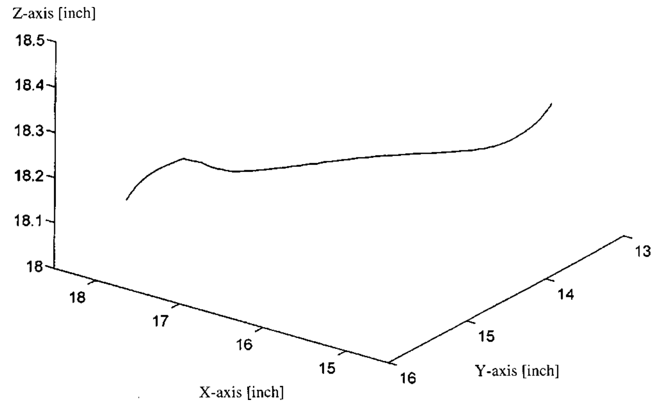

Fig. 7 The simulated CC point path by the combined 3D L&C interpolation

percentage of the reduction of the total machining error is 10 percent. Although the proposed interpolator needs more calculations to interpolate the position points as compared to the linear interpolator, the nonlinearity error for each segment move is eliminated.

6 Conclusion

A combined 3D L&C interpolation technique for five-axis CNC machining has been presented in this paper. The proposed interpolation principle was developed based on the machine motion trajectories which are constructed by superimposing the 3D rotational movements on the translational movements. The proposed 3D L&C interpolator is capable of driving the rotational movements pivot to move along a predesigned curve, rather than a straight line segment, such that the cutting point moves along the space line segment. Therefore, the proposed interpolation technique eliminates nonlinearity errors, and provides a solution to the nonlinearity errors problem in ultra-precision multi-axis CNC machining. A computer simulation procedure for machining a sculptured surface demonstrated the elimination of nonlinearity errors.

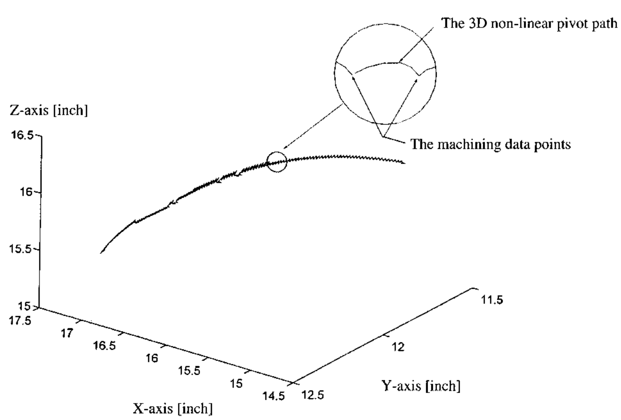

Fig. 6 The simulated pivot point path by the combined 3D L&C interpolation

Nomenclature

(B_m, C_m) = machine rotational movement variables
B_0 = the start coordinate of the B_m axis rotation movement for each move
C_0 = the start coordinate of the C_m axis rotation movement for each move
B_1 = the end coordinate of the B_m axis rotation movement for each move
C_1 = the end coordinate of the C_m axis rotation movement for each move
P = the rotation pivot of the B_m axis and the C_m axis
F = feedrate
GL = tool gage length
l = distance between the cutter contact point and the rotation pivot P
$l(i)$ = distance from the i-th interpolated point to the segment start point
L = segment length
$\mathbf{n}_x$ = the unit normal vector to the y-z coordinate plane in Cartesian coordinate system
$\mathbf{n}_y$ = the unit normal vector to the x-z coordinate plane in Cartesian coordinate system
$\mathbf{n}_z$ = the unit normal vector to the x-y coordinate plane in Cartesian coordinate system
R = rotational movement radius
t_r = specified machining tolerance
T = interpolation period

(x, y, z) = circular interpolation center coordinates
(x_m, y_m, z_m) = machine translational movement variables
(x_p, y_p, z_p) = coordinates of the rotation pivot P w.r.t. the machine coordinate system
(x_{cc}, y_{cc}, z_{cc}) = coordinates of the cutter contact point
x_s = x-coordinate of the start point for each segment
y_s = y-coordinate of the start point for each segment
z_s = z-coordinate of the start point for each segment
x_e = x-coordinate of the end point for each segment
y_e = y-coordinate of the end point for each segment
z_e = z-coordinate of the end point for each segment
x_i = the i-th linearly interpolated x-coordinate
y_i = the i-th linearly interpolated y-coordinate
z_i = the i-th linearly interpolated z-coordinate
X_i = the i-th 3D L&C interpolated x-coordinate
Y_i = the i-th 3D L&C interpolated y-coordinate
Z_i = the i-th 3D L&C interpolated z-coordinate
δ_n = nonlinearity error
δ_t = linearity error
δ_{total} = total machining error
λ_l = the linear interpolation scale factor
λ_c = the circular interpolation scale factor
τ = the rotational movements interpolation parameter

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