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Compromise: An Effective Approach for Designing Composite Conical Shell Structures

The layout of fiber composite structures compared to that of structures made from conventional homogeneous isotropic materials is far more difficult, because a fiber composite (laminate) is built up of several unidirectional layers (UD-layers) with fibers set at different angles. A contribution to the structural analysis and preliminary design of a fiber-reinforced conical shell is made in this paper. The equations of the membrane theory are used for analyzing the shell behavior. The design, with the objective of obtaining minimal deformation at minimal weight, subject to a set of failure constraints, is achieved by formulating and solving a compromise Decision Support Problem. Some designs of a fiber reinforced conical shell subjected to pressure load and temperature are presented.

1 Design of Composite Material Shells

In modern lightweight structures, shells of revolution fabricated of fiber composite materials, e.g., fuel tanks, are becoming increasingly important. These shell structures can mostly be built up from different well known shell types, e.g., spherical, cylindrical and conical shells, simplifying the stress and deformation calculations. A more difficult problem is calculating the stress concentrations which appear at the connections of the different shell types. The reason being that the deformations of the different shell types under similar loading are not the same and so bending and shear effects appear. The high stresses at the connection decrease rapidly away from it, so that for the most part of the shell the membrane stresses are important. Thus, to design a shell structure one must use both the membrane theory and bending theory.

The bending theory for composite material structures is much more complicated than for structures made of homogeneous and isotropic materials. Therefore, we have to find a way to design shell structures using only the membrane theory. This is possible if the stiffnesses of the different shell types can be changed in a way that the strains (or the deformations) at the connections are the same. By using composite materials, instead of isotropic materials, the stiffness can be changed by using different layer orientations in a laminate and increasing or decreasing the layer thickness. To find the appropriate layer orientation and thickness optimization methods can be applied.

Before one can build up a whole shell structure out of spherical, cylindrical and/or conical shells the deformation behavior of the different shell types has to be studied in detail. In references [1] and [2] several parametric studies and the "optimum" design of spherical and cylindrical shells under pressure and temperature loads are given. Battermann and Pavicic [3] published a paper about weight minimization of laminated shells of revolution where the laminate is built up as a symmetrical angle-ply laminate. They found the optimal results by doing a lot of calculations with different laminate parameters, e.g., fiber angles and laminate thicknesses. Most of the publications in the field of optimum design of composite shells have dealt with weight minimization including stability and/or vibration constraints, see [4, 5].

Our paper deals with the structural analysis and preliminary design of a thin conical shell subjected to a pressure load and a temperature distribution along the meridian direction, as shown in Fig. 1, with respect to minimal deformation at minimal weight. The deformations and stresses of the shell are calculated with the basic equations of the membrane theory for composite material shells [2, 6]. This approach is appropriate for preliminary design. Further refinement of the design would have to be accompanied by a more accurate analysis, using modern numerical methods such as finite element analysis, to determine the response of the shell to applied loads. It should be noted that this would also increase the computational time. The choice to build up the laminate from 0 deg, 90 deg and ± 45 deg, layers is based on manufacturing considerations and is not a requirement of our design approach. Woven mats with these fiber orientations are readily available commercially and so the laminate can be constructed with ease. The general practice in the industry is to select a certain number of fiber directions and to change the number of laminae in these fiber directions.

A comprehensive approach called the Decision Support Problem (DSP) Technique [7] is being developed and implemented at the University of Houston to provide support for human judgment in design synthesis. The DSP Technique consists of three principal components: a design philosophy expressed at present in terms of paradigms, an approach for identifying and formulating Decision Support Problems (DSPs), and the software necessary for solution. The design of

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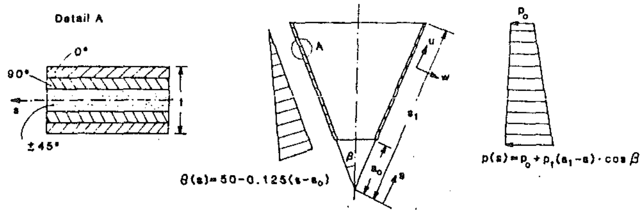

Fig. 1 Laminated conical shell subjected to pressure load and temperature

the conical shell is carried out within the framework of the DSP Technique. This is done by formulating a compromise DSP and solving it using the DSIDES (Decision Support In the Design of Engineering Systems) package. DSIDES has been used extensively to solve a variety of multiobjective, nonlinear programming problems in engineering. These applications include: preliminary ship and ship structural design [9, 10] aircraft [11], design of damage tolerant structures [12], design of thermal energy system [13], and the design of various mechanical engineering components like pressure vessels [14], and helical compression springs [15].

2 Structural Analysis of a Composite Conical Shell

The following equations, applicable to the membrane theory of a composite conical shell, are derived from the basic equations of inhomogeneous anisotropic shells [2, 6]. It is assumed that the thickness of the shell, t, is small compared to the other geometrical dimensions and the shell can be described by its middle surface, which halves the thickness at each point. Furthermore, the deformations of the shell are assumed to be small.

For the conical shell shown in Fig. 1, the pressure load and the temperature distribution are given by

$$p_s = p_\upsilon = 0,\ p = p(s) = p_0 + p_1 \cos\beta\ (s_1 - s) \tag{1a}$$

and

$$\Theta = \Theta(s) = 50.0 - 0.125(s - s_0) \tag{1b}$$

where

p_s = pressure load in meridian direction,
p_υ = pressure load in circumferential direction,
p = pressure load normal to the middle surface,
p_0 = $1.4715*10^{-2}$ N/mm^2
p_1 = $9.81*10^{-6}$ N/mm^3
Θ = temperature difference from the stress-free temperature.

The temperature difference from the stress-free temperature Θ is constant across the thickness t and both the pressure load and the temperature distribution are independent of the circumferential angle υ. Thus, the equilibrium equations are given as follows [6]:

$$d(sN_{ss})/ds - N_{\upsilon\upsilon} = 0 \tag{2a}$$

$$d(sN_{s\upsilon})/ds + N_{s\upsilon} = 0 \tag{2b}$$

$$N_{\upsilon\upsilon} - p\ s\ \tan\beta = 0 \tag{2c}$$

where

N_{ss} = normal force in meridian direction,
$N_{\upsilon\upsilon}$ = normal force in circumferential direction,
$N_{s\upsilon}$ = shear force in the s,υ-plane.

To calculate the deformations of the shell the strain-displacement relations and the material law are needed. The strain-displacement equations, which relate the deformations and their derivatives to the strains in the shell, are the following:

$$\epsilon_{ss} = du/ds \tag{3a}$$

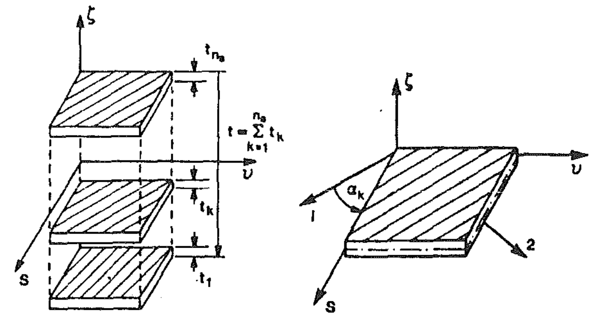

Fig. 2 Laminate configuration of a fiber composite

$$\epsilon_{\upsilon\upsilon} = (u + w\ \cot\beta)/s \tag{3b}$$

$$\gamma_{s\upsilon} = dv/ds - v/s \tag{3c}$$

where

ϵ_{ss}, $\epsilon_{\upsilon\upsilon}$ = normal strains in meridian and circumferential direction,
$\gamma_{s\upsilon}$ = shear strain in the s,υ-plane,
u,v = displacements in meridian and circumferential direction,
w = displacement normal to the middle surface.

The second set of equations, representing the material law, are the most difficult ones for composite materials. The reason is that in general a structural member made from composite materials consists of n_s single orthotropic layers turned at an angle α_k as shown in Fig. 2. Thus the material characteristics are discontinuous across the laminate thickness. In contrast to homogeneous materials, with constant material properties across the thickness, composite materials are inhomogeneous and anisotropic. In [2] the material law equations of general shell structures made of composite material are described.

Based on these equations and with the following assumptions:

- linear elastic material behavior,
- pre-stresses are not taken into consideration,
- the existence of the membrane state, i.e., bending moments and shear forces normal to the middle surface are zero,
- the displacements between the single layers of the laminate are constant, i.e., $(\epsilon_{ss})_k = \epsilon_{ss}$, $(\epsilon_{\upsilon\upsilon})_k = \epsilon_{\upsilon\upsilon}$, $(\gamma_{s\upsilon})_k = \gamma_{s\upsilon}$,
- the laminate is symmetrical to the middle surface and built up from 0 deg, 90 deg and ±45 deg layers only, see Fig. 1, where the 0 deg lamina is aligned with the u-direction,

the material law can be written as,

$$\mathbf{N} = t\mathbf{Q}^*\epsilon - \mathbf{N}_\Theta \tag{4a}$$

or

$$\epsilon = \mathbf{D}(\mathbf{N} + \mathbf{N}_\Theta) \tag{4b}$$

where

$\mathbf{Q}^* = \varphi_0 \cdot \mathbf{Q}_0 + \varphi_{90} \cdot \mathbf{Q}_{90} + \varphi_{\pm 45} \cdot \mathbf{Q}_{\pm 45}$ = laminate stiffness matrix,
$\mathbf{D} = (1/t)\mathbf{Q}^{*-1}$ = laminate compliance matrix,
$\mathbf{N} = [N_{ss}, N_{\upsilon\upsilon}, N_{s\upsilon}]^T$
$\epsilon = [\epsilon_{ss}, \epsilon_{\upsilon\upsilon}, \gamma_{s\upsilon}]^T$
$\mathbf{N}_\Theta = t(\varphi_0\mathbf{Q}_0\alpha_{T0} + \varphi_{90}\mathbf{Q}_{90}\alpha_{T90} + \varphi_{\pm 45}\mathbf{Q}_{\pm 45}\alpha_{T\pm 45})\Theta(s)$
$\alpha_{Tk} = [\alpha_{ss}, \alpha_{\upsilon\upsilon}, 0]^T$
$\mathbf{Q}_k$ = stiffness matrix of the kth layer in the s,υ-coordinate system,
α_{Tk} = vector of the thermal expansion coefficients of the kth layer in the s,υ-coordinate system,
t = laminate thickness,
φ_k = thickness ratio t_k/t of the kth layer,
t_k = sum of all kth layer thicknesses.

To derive the laminate stiffness matrix $\mathbf{Q}^*$ one must transform the stiffness matrix $\mathbf{Q}_{UD}$ and the vector of the thermal coefficients α_{TUD} of a single unidirectional layer for all layers with different fiber angles α_k into the global s,υ-coordinate system of the conical shell

$$\mathbf{Q}_{UD}=\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \tag{5a}$$

and

$$\alpha_{TUD}=[\alpha_{T11},\ \alpha_{T22},\ 0]^T \tag{5b}$$

where

$Q_{11}=E_{11}/(1-\nu_{12}\nu_{21}),\ Q_{12}=\nu_{12}E_{22}/(1-\nu_{12}\nu_{21}),$

$Q_{22}=E_{22}/(1-\nu_{12}\nu_{21}),\ Q_{66}=G_{12}.$

Therefore, one has to use the following transformation matrix $\mathbf{T}_k$:

$$\mathbf{T}_k=\begin{bmatrix} \cos^2\alpha_k & \sin^2\alpha_k & \sin 2\alpha_k \\ \sin^2\alpha_k & \cos^2\alpha_k & -\sin 2\alpha_k \\ -(1/2)\sin 2\alpha_k & (1/2)\sin 2\alpha_k & \cos 2\alpha_k \end{bmatrix} \tag{6}$$

Using equations (5) and (6) the stiffness matrix of each layer in the s,υ-coordinate system is given by:

$$\mathbf{Q}_k=\mathbf{T}_k\mathbf{Q}_{UD}\mathbf{T}_k{}^T \tag{7}$$

and specially for

$\alpha_k=0$ deg

$$\mathbf{Q}_0=\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \tag{8a}$$

$\alpha_k=90$ deg

$$\mathbf{Q}_{90}=\begin{bmatrix} Q_{22} & Q_{12} & 0 \\ Q_{12} & Q_{11} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \tag{8b}$$

$\alpha_k=\pm 45$ deg

$\mathbf{Q}_{\pm 45}=$

$$(1/4)\begin{bmatrix} Q_{11}+Q_{22}+2Q_{12}+4Q_{66} & Q_{11}+Q_{22}+2Q_{12}-4Q_{66} & 0 \\ Q_{11}+Q_{22}+2Q_{12}-4Q_{66} & Q_{11}+Q_{22}+2Q_{12}+4Q_{66} & 0 \\ 0 & 0 & Q_{11}+Q_{22}-2Q_{12} \end{bmatrix} \tag{8c}$$

Also, the thermal coefficients of each layer have to be transformed:

$$\alpha_{Tk}=(\mathbf{T}_k{}^T)^{-1}\alpha_{TUD} \tag{9}$$

and in particular we have

$$\alpha_k=0 \text{ deg } \alpha_{T0} \quad =[\alpha_{T11},\ \alpha_{T22},\ 0]^T \tag{10a}$$

$$\alpha_k=90 \text{ deg } \alpha_{T90} \quad =[\alpha_{T22},\ \alpha_{T11},\ 0]^T \tag{10b}$$

$\alpha_k=\pm 45$ deg $=$

$$\alpha_{T\pm 45} \tag{10c}$$

$$\begin{bmatrix} E_{11}\alpha_{T11}+E_{22}\alpha_{T22}+\nu_{12}E_{22}(\alpha_{T11}+\alpha_{T22})]/[E_{11}+E_{22}(1+2\nu_{12})] \\ E_{11}\alpha_{T11}+E_{22}\alpha_{T22}+\nu_{12}E_{22}(\alpha_{T11}+\alpha_{T22})]/[E_{11}+E_{22}(1+2\nu_{12})] \\ 0 \end{bmatrix}$$

To calculate the forces and the deformations using equations (1) to (10) one has to take the following boundary conditions into account:

$$N_{ss}(s=s_1)=0,\ N_{s\upsilon}(s=s_1)=0,\ u(s=s_0)=0. \tag{11}$$

Finally, to make the solution of the problem more transparent, both the forces and deformations due to pressure load and temperature distribution are shown separately:

(a) constant pressure load $p=p_0$

$$N_{ss}=p_0\tan\beta(s^2-s_1{}^2)/2s$$

$$N_{\upsilon\upsilon}=p_0 s\tan\beta \tag{12a}$$

$$N_{s\upsilon}=0$$

$$u(s)=p_0\tan\beta[(D_{11}+2D_{12})(s^2-s_0{}^2)-2D_{11}s_1{}^2\ln(s/s_0)]/4$$

$$v(s)=0 \tag{12b}$$

$$w(s)=p_0\tan^2\beta[(2D_{22}+D_{12})s^2-D_{12}s_1{}^2-2u(s)/(p_0\tan\beta)]/2$$

(b) linearly distributed pressure load, $p=p_1\cos\beta(s_1-s)$.

$$N_{ss}=p_1 s_1{}^2\sin\beta[(s/s_1)/2-(s/s_1)^2/3-(s_1/s)/6]$$

$$N_{\upsilon\upsilon}=p_1 s_1{}^2\sin\beta[(s/s_1)-(s/s_1)^2] \tag{13a}$$

$$N_{s\upsilon}=0$$

$$u(s)=p_1\sin\beta[s_1(s^2-s_0{}^2)(D_{11}+2D_{12})/4 - (s^3-s_0{}^3)(D_{11}+3D_{12})/9-s_1{}^3\ln(s/s_0)D_{11}/6]$$

$$v(s)=0 \tag{13b}$$

$$w(s)=p_1\sin^2\beta/\cos\beta\{s_1{}^3(D_{22}[(s/s_1)^2-(s/s_1)^3]+D_{12}[(s/s_1)^2/2-(s/s_1)^3/3-1/6])-u(s)/(p_1\sin\beta)\}$$

(c) linear temperature distribution, $\Theta=50.-0.125(s-s_0)$.

$$N_{ss}=N_{\upsilon\upsilon}=N_{s\upsilon}=0 \tag{14a}$$

$$u(s)=\alpha_{Tss}\{50(s-s_0)-0.0625(s^2+s_0{}^2)+0.125\ s\ s_0\}$$

$$v(s)=0 \tag{14b}$$

$$w(s)=\tan\beta\{\alpha_{T\upsilon\upsilon}s[50-0.125(s-s_0)]-u(s)\}$$

3 Compromise Decision Support Problems: An Overview

The conical shell problem is formulated as a compromise DSP and solved using the DSIDES software [8]. The solution scheme, based on the Adaptive Linear Programming algorithm, utilizes a modified sequential linear programming appraoch. For details of the algorithm see [8].

Compromise DSPs belong to a class of constrained, multiobjective optimization problems. They are defined using the following system descriptors: system and deviation variables, system constraints and bounds, system goals and a deviation function. These descriptors have been explained in [7, 9, 14, 15]. The compromise DSP formulation is tailored to handle common engineering design situations where physical limitations manifest themselves as system constraints and bounds and where the design has to meet multiple objectives. In the compromise DSP formulation the multiple objectives are formulated as goals with appropriate target values and the deviation from the target value is then minimized. The compromise DSP represents a hybrid formulation. The traditional mathematical programming formulations, as presented in [16], are a subset of compromise DSPs and the compromise

DSPs in turn are a subset (with a twist) of generalized goal programming [17, 18].

The formulation and solution of compromise DSPs enables a variety of analyses. The formulation helps create a better understanding of the original problem. The solution yields, besides the values of the design variables, the set of active constraints which would give an idea of the criticality of the design. Further, the investigation of the stability of the current solution and the effects of changes in design goal priorities can be carried out. Also, by varying the target values for the weight goal the functional efficient boundary in the criterion space can be plotted. It is also easy to isolate and study the effects of the pressure and temperature loading separately. The compromise DSP presented in this paper can be further generalized by adding fiber orientation in the laminate layers as variables.

3.1 The Conical Shell Design Problem as a Compromise DSP

3.1.1 Problem Statement. It is desired to minimize the maximum deformation $|w|_{max}$ and the weight W of a laminated conical shell (see Fig. 1). The shell is loaded by forces due to a constant pressure and hydrostatic pressure and a linear temperature distribution. Either the thickness fractions of the single layers and the total thickness of the laminate or the thicknesses of each layer are to be determined so that the two conflicting objectives, that of minimizing maximum deflection and that of minimizing weight, are achieved as best as possible.

3.1.2 Mathematical Formulation. The objective function in this case represents the deviation from the goals of the design. The goals along with the target values represent the demands on the design. In the present problem the two goals are those of minimizing weight and minimizing the maximum deflection are obvious. The expression for the weight W is as follows,

$$W=\rho_{UD} g\ \pi\ t(s_1^2-s_0^2)\sin\beta \tag{15}$$

where $t=t_0+t_{90}+t_{\pm 45}$.

The deflection w as a function of the s-coordinate is calculated based on the membrane theory solutions, see equations (12), (13), and (14). For example, in case of both the pressure and temperature loads, the deflection w would be given by the sum of expressions for $w(s)$ from equations (12b), (13b), and (14b). The maximum deflection $|w|_{max}$ is then determined by a search procedure along the meridian length. In Fig. 3, the interaction of the optimization and analysis schemes is illustrated.

Two sets of constraints are used in this problem, namely, the bonding break failure constraint

$$(\sigma_{11}/\sigma_{11BB})^2+(\sigma_{22}/\sigma_{22BB})^2+(\tau_{12}/\tau_{12BB})^2\le 1 \tag{16a}$$

and the break failure constraint for the fiber

$$(\sigma_{11}/\sigma_{11B})^2\le 1, \tag{16b}$$

where

$$\sigma_{11BB}=\ 1990\ \text{N/mm}^2\ \text{tension/compression}$$

$$\sigma_{11B}=\begin{cases}1330\ \text{N/mm}^2 & \text{tension}\\ -1220\ \text{N/mm}^2 & \text{compression}\end{cases}$$

$$\sigma_{22BB}=\begin{cases}70\ \text{N/mm}^2 & \text{tension}\\ -150\ \text{N/mm}^2 & \text{compression}\end{cases}$$

$$\tau_{12BB}=\ 85\ \text{N/mm}^2.$$

The failure criteria (16a,b) have to be satisfied in each layer. It should be noted that the maximum stress in each layer occurs at different locations and it is this maximum value which is to be used in the constraint expressions. Thus, a search procedure has to be used to locate the maximum values of the constraint functions too (see Fig. 3).

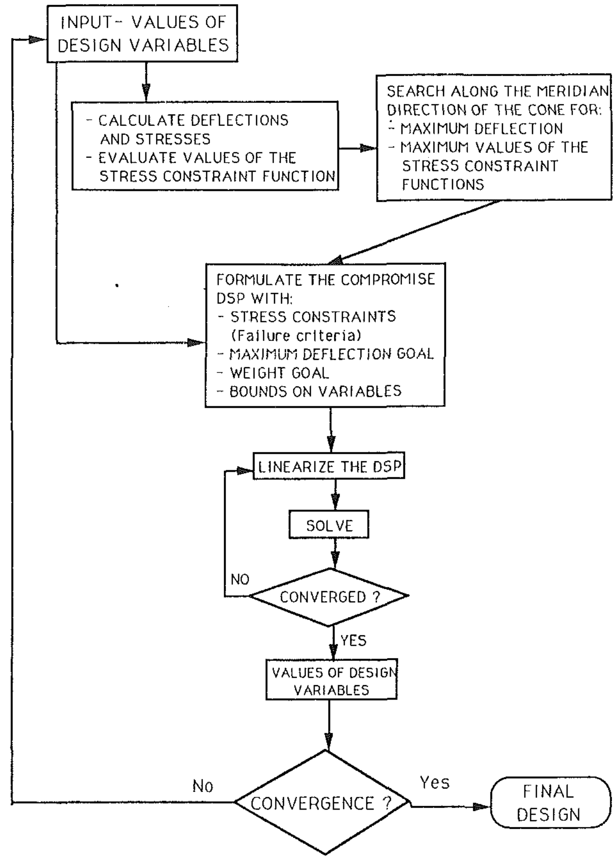

Fig. 3 The design procedure using DSIDES

Now, the mathematical formulation of the compromise DSP for this problem can be described by the following template:

Given

A composite material conical shell of laminate structure (0 deg/90 deg/ $\pm$45 deg)$_s$. Membrane theory solution of the conical shell; deflections and stresses under pressure and temperature loading.

Pressure load, see (1a),

$p(s)=p_0+\gamma\cos\beta(s_1-s)$, where $\gamma(=p_1)$ is the fluid density.

Temperature distribution, see (1b),

$\Theta(s)=50.0-0.125(s-s_0)$.

The unidirectional layers are made of

Fiber-T300 (Volume fraction = 60 percent)
Matrix-914C (Epoxy resin)
Material properties: density
ρ_{UD}, E_{11}, E_{22}, G_{12}, ν_{12}, α_{T11}, α_{T22}.

Find

Thicknesses of each of the lamina in the laminate, namely, t_0, t_{90}, and $t_{\pm 45}$

Satisfy

System Constraints
Failure criteria for each layer.
Bonding break failure, see (16a)
$[(\sigma_{11}/\sigma_{11BB})^2+(\sigma_{22}/\sigma_{22BB})^2+(\tau_{12}/\tau_{12BB})^2]_{max}\le 1$

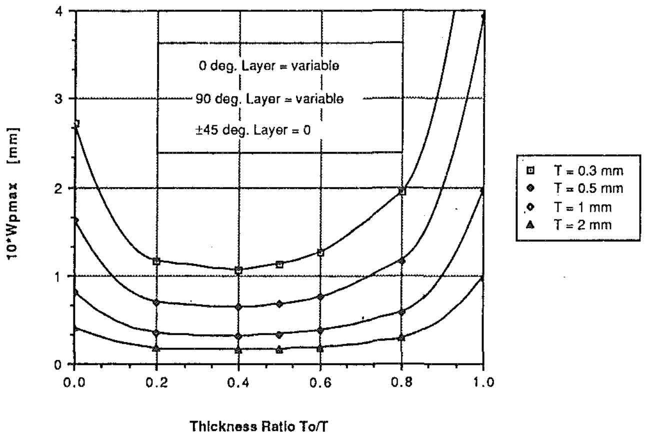

Fig. 4(*a*) Parametric studies of a laminated shell under pressure load

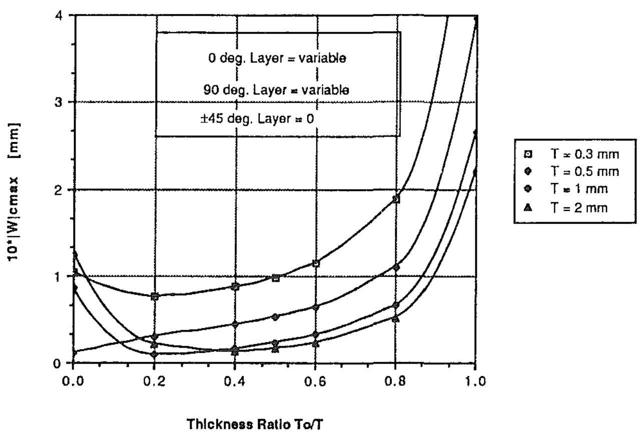

Fig. 4(*b*) Parametric studies of a laminated shell under pressure load and temperature

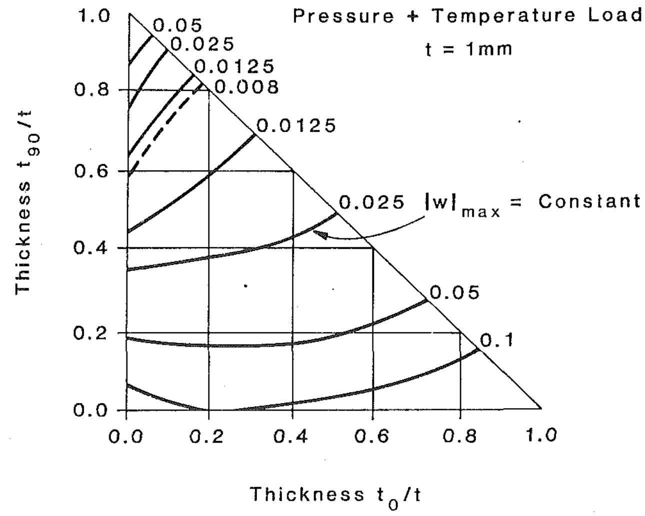

Fig. 5 Deformation behavior of a conical shell built up from a (0 deg/90 deg/ ± 45 deg)$_S$ laminate with $t = 1$ mm

Table 1 Material Properties of a UD-layer (Fiber T300/Matrix 914C)

Fiber Volume Fraction	$V_f = 0.6$
Young's Modulus in fiber direction	$E_{11} = 132700$ N/mm^2
Young's Modulus in transverse direction	$E_{22} = 9300$ N/mm^2
Shear Modulus	$G_{12} = 4600$ N/mm^2
Poisson's ratio	$\nu_{12} = 0.26$
Thermal Coefficient in fiber direction	$\alpha_{T11} = 0.23 \times 10^{-5}$ 1/°K
Thermal Coefficient in transverse direction	$\alpha_{T22} = 29 \times 10^{-5}$ 1/°K
Density	$\rho_{UD} = 1.55 \times 10^{-6}$ kg/mm^3

Break failure of the fiber, see (16b)

$$[(\sigma_{11}/\sigma_{11B})^2]_{max} \leq 1$$

System Goals

Weight W of the conical shell, see (15), should not exceed WEIMAX.

$$\rho_{UD} g \; \pi \; t(s_1{}^2 - s_0{}^2)\sin\beta + d_1{}^- - d_1{}^+ = \text{WEIMAX}$$

Maximum deflection of the conical shell should be zero, see (12), (13), and (14).

$$|w(s, t, \varphi_0, \varphi_{90}, \varphi_{\pm 45})|_{max} + d_2{}^- - d_2{}^+ = 0$$

Bounds on the design variables

$$0 \leq t_0, \; t_{90}, \; t_{\pm 45} \leq 3.0$$

Minimize

The deviation from target values (right-hand sides) of the system goals. This is expressed as a function of the deviation variables:

$$Z = \{(d_1{}^+), (d_1{}^- + d_2{}^+), (d_2{}^-)\}.$$

4 Discussion of Results

A parametric study was first performed to gain better understanding about the deformation behavior of a composite conical shell. The results are discussed in Section 4.1. After that, since it is not possible to handle multiple objectives with a parametric study, we solved the compromise DSP of Section 3.1.2, to find designs with respect to minimum deformation at minimum weight. These designs are presented in Section 4.2, as plots of the functional-efficient boundaries. The results in the two cases were found to be in agreement with each other. In obtaining designs for plotting the functional-efficient boundary both the weight and deflection goals are accorded the same priority. If necessary the priorities can be changed by the designer in the post-solution phase to pick designs on the functional-efficient boundary. In all our calculations the following geometrical data are used:

$$s_0 = 200 \text{ mm}, \; s_1 = 600 \text{ mm}, \; \beta = 30 \text{ deg}.$$

The material property values are as shown in Table 1.

4.1 Parametric Studies. In the parametric study the thickness fractions of each layer and the total thickness of the laminate are varied. In Fig. 4 the deflection behavior of the composite conical shell with only the 0 deg and 90 deg layers is shown. Under pressure load the least $|w|_{max}$ is seen only for approximately 40 percent of the 0 deg layer and 60 percent of the 90 deg layer, see Fig. 4(a). Also one can see that with increasing total thickness of the shell the minimum value of $|w|_{max}$ decreases and becomes insensitive to t_0/t in the range of $0.2 \leq t_0/t \leq 0.6$. Using only the 90 deg layer shows less deflection than using only the 0 deg layer. Under pure temperature effect the deflection is found to be independent of the thickness, using the membrane theory. Therefore, the minimum value of $|w|_{max}$ is given for nearly 50 percent of each type of fiber orientation. But it must be mentioned that in the range of $0 \leq t_0/t \leq 0.5$ the real deformation w of the shell under temperature load is negative. Thus, under combined temperature and pressure effect it is observed that there is a particular thickness beyond which the minimum $|w|_{max}$ does not change irrespective of thickness, Fig. 4(b). This value is slightly more than 0.5 mm and the required design is seen to

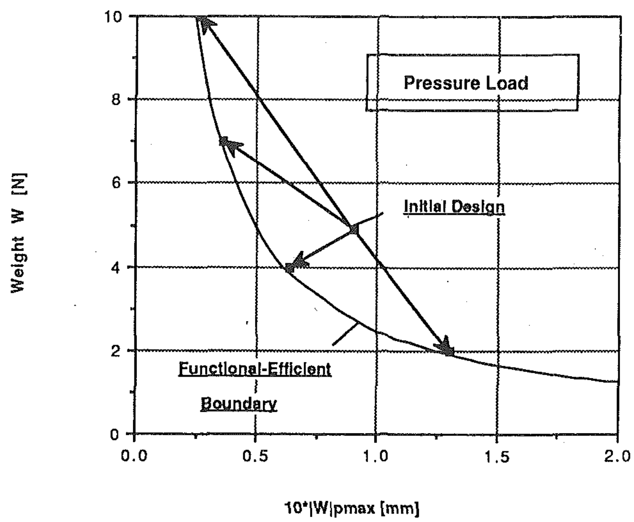

Fig. 6 Functional-efficient boundary of a laminated conical shell under pressure load

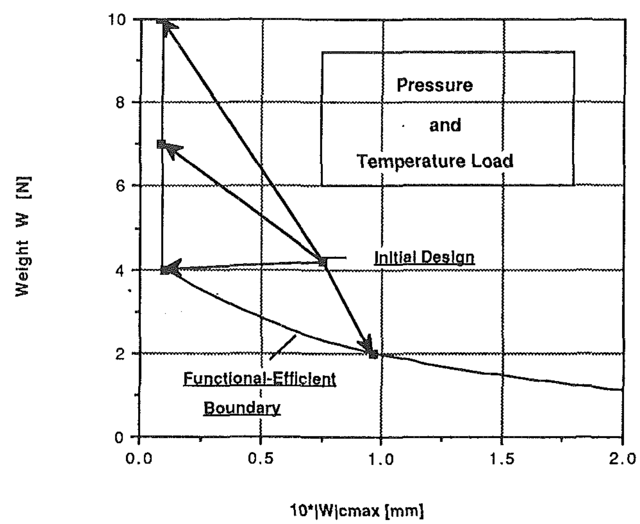

Fig. 7 Functional-efficient boundary of a laminated conical shell under pressure load and temperature

converge towards the 90 deg fiber direction only. The reason for this is that the deformation under pressure load decreases on increasing the thickness of the laminate and above this particular laminate configuration the smaller positive deformation due to pressure can be compensated by the negative deformation under temperature.

Similar studies were carried out for combinations of 0 deg and ±45 deg, and 90 deg and ±45 deg layers. It is seen that no combination with ±45 deg fiber direction gives lower deflections than those obtained for the 0 deg and 90 deg case. This fact is also apparent from Fig. 5. In this carpet plot the thick lines are contours of equal deformation. Equivalent designs, i.e., with the same minimum $|w|_{max}$, can be identified on these contours. Any point in the interior of the plot indicates the thickness fractions of the layers (the fractions for the 0 deg and 90 deg layers can be read off from the axes. If the sum of the two is less than one the remainder is accounted for by the ±45 deg layer). The hypotenuse represents data obtained in the 0 deg/90 deg case mentioned earlier. It is seen that to minimize deformation one would have to necessarily use the 90 deg layer. With a total laminate thickness of 1 mm the minimum deformation of 0.008 mm is shown with a broken line. Since all the designs on this contour are equivalent manufacturing considerations would have to be employed, to choose a particular design.

4.2 Compromise DSP Solution. The solutions of the compromise DSP are shown as functional-efficient boundaries, which represent the "optimum" solution set of objective functions in the criterion space. In our case it means that a reduction of the functional value of the deformation can only be achieved by increasing the weight. In Fig. 6 the functional-efficient boundary is plotted for pressure load only. As mentioned, the deflection is seen to decrease if the weight increases. This was also seen in the parametric studies. As presumed in the parametric studies only the 0 deg and 90 deg layers are present along the functional-efficient boundary. Both the layers are in about the same proportion.

The functional-efficient boundary for the conical shell under both pressure load and temperature is shown in Fig. 7. It is seen that the functional-efficient boundary exists only for laminate weights less than 4 N, this is equivalent to a laminate thickness of nearly 0.5 mm. For laminate weights greater than 4 N the minimum value of $|w|_{max}$ is the same as that for a laminate with 4 N weight. So increasing the weight, and hence the thickness, will only increase the cost without any advantage of lower deflection. As mentioned before, the reason being that the deflection under pressure load can be best compensated by the deflection under temperature up to a certain thickness only. Beyond that the effect of pressure load on the deflection of the shell decreases. With both the pressure load and temperature acting the 90 deg layers are dominant around the total thickness of 0.5 mm for the laminate. Above and below that thickness we see the presence of some 0 deg and ±45 deg layers also. These inferences match those drawn from Fig. 4(*b*).

In determining good designs using DSIDES some problems with convergence were experienced. One reason could be the method of computation of the derivatives of the constraint functions. DSIDES uses the central difference formula. This aspect needs further investigation. Another reason could be that, at least for pressure load, the sensitivity of deflection to thickness is very small near the solution point, i.e., a flat curve is seen in Fig. 4, thus making it difficult to determine the "optimum" design.

As seen from Fig. 3 the design process involves search procedures within the main program. Search is required to locate maximum deflection and maximum values of the bonding break failure and fiber break failure constraints in each layer. This slows the design process considerably.

5 Future Work

Coupled selection compromise DSPs can also be formulated and solved using DSI DES. This means that the selection of the type of composite material can be combined, very easily, with the synthesis of the dimensions and the configuration of the structural member. Additionally, in principle, the DSPs can be used to model hierarchical decisions. Typically these arise in the simultaneous design and integration of various parts of a larger system. Consider the design of a fuel tank, mentioned in Section 1, cylindrical in shape with conical and/or spherical ends. A compromise DSP could be used to design the cylindrical, conical and/or spherical portions separately, using composite laminates. The designs could then be integrated by a higher level compromise DSP. Work on both these aspects is now in progress. Additionally, the possibility of including more manufacturing related information in the compromise DSP template, in preliminary design, is being actively looked into [19]. This is especially important for composite material structures because the interaction between design and manufacturing is extremely strong.

6 Conclusion

This paper contributes to the design of a conical shell made of carbon fiber composites. In order to minimize the weight and the maximum deformation of the conical shell the compromise DSP includes two goals. From this formulation the results of a multi-objective optimization problem, for the minimum weight and minimum deflection objectives, are derived. In our study the laminate is built up by a (0 deg/90 deg/$\pm$45 deg)$_s$ configuration. As design variables, the thickness of each layer was chosen. Among the essential constraints we have used the elastic ultimate loads, such as bonding break failure and break failure of the fiber. For the determination of the maximum deformation, as well as the various ultimate stresses, the basic equations of the membrane theory for a conical shell have been solved. The compromise DSP is solved using the DSIDES package.

From the presented results it is seen that the solution of the DSP leads to the same answers as revealed by the parametric study. So it can be concluded that the compromise DSP formulation is able to handle structural design problems using composite materials. This obviates the need to carry out a detailed parametric study, in the early stages of design, thus saving time and effort. The appropriate DSP would directly give the required design which can be further validated and refined. Also, valuable additional sensitivity information can be obtained. In particular, the effect of changing the priorities on the weight and deflection goals can be studied.

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References

1 Eschenauer, H., and Fuchs, W., 1987, "Optimale Auslegung einer Zylinderschale aus Faserverbundwerkstoff unter Drunck- und Temperaturbelastung," *ZAMM, Zeitschrift für Angewandte Mathematik und Mechanik*, Spring.

2 Fuchs, W., 1986, "Strukturanalyse und Optimierung anisotroper Schalen aus Faserverbundwerkstoff, (Structural Analysis and Optimum Design of Anisotropic Fiber-Reinforced Shells), "Doctoral Thesis, University of Siegen, (In German).

3 Battermann, S. C., and Pavicic, N. J., 1973, "Optimum Design of Fiber Reinforced Shells of Revolution," Optimization in Structural Design, IUTAM Symposium Warsaw/Poland 1973, Berlin, Göttingen, Heidelberg, Springer Verlag, pp. 372–394.

4 Hirano, Y., 1982, "Optimization of Laminated Composite Plates and Shells," Z. Hashin and C. T. Herakovich, eds., *Mechanics of Composite Material, Proceedings of the IUTAM Symposium on Mechanics of Composite Materials 1982*, New York, Oxford, Toronto, Sydney, Paris, Frankfurt, Pergamon Press, pp. 355–365.

5 Nshanian, Y. S., and Pappas, M., 1983, "Optimal Laminated Composite Shells for Buckling and Vibration," *AIAA Journal*, Vol. 21, No. 3, pp. 430–437.

6 Schnell, W., and Eschenauer, H., 1984, "Elastizitätstheorie II; Schalen," Mannheim, Wien, Zürich, Bibliographisches Institut.

7 Mistree, F., Muster, D., Shupe, J. A., and Allen, J. K., 1989, "A Decision Based Perspective for the Design of Methods for Systems Design," NASA CP 3031.

8 Mistree, F., and Hughes, O. F., 1987, *Adaptive Linear Programming: An Approach for Solving Multiobjective Decision Support Problems*, Systems Design Laboratory Report, Dept. of Mechanical Engineering, Univ. of Houston, Houston, Texas, November.

9 Lyon, T. D., and Mistree, F., 1985, "A Computer-based Method for the Preliminary Design of Ships," *Journal of Ship Research*, Vol. 29, No. 4, 1985, pp. 251–289.

10 Hughes, O. F., Mistree, F., and Zanic, V., 1980, "A Practical Method for the Rational Design of Ship Structures," *Journal of Ship Research*, Vol. 24, No. 2, June, pp. 101–113.

11 Marinopoulos, S., Jackson, D., Shupe, J., and Mistree, F., 1987, "Compromise: An Effective Approach for Conceptual Aircraft Design," AIAA Paper No. AIAA-87-2965.

12 Shupe, J. A., and Mistree, F., 1987, "Compromise: An Effective Approach for the Design of Damage Tolerant Structural Systems," *Computers and Structures*, Vol. 27, No. 3, pp. 407–415.

13 Bascaran, E., Mistres, F., and Bannerot, R. B., 1987, "Compromise: An Effective Approach for Solving Multiobjective Thermal Design Problems," *Engineering Optimization*, Vol. 12, No. 3, pp. 175–189.

14 Nguyen, N., and Mistree, F., 1986, "A Computer-based Method for the Rational Design of Horizontal Pressure Vessels," ASME Journal of Mechanisms, Transmissions, and Automation in Design, Vol. 108, June, pp. 203–210.

15 Jivan, A., and Mistree, F., 1985, "A Computer-based Method for Designing Statically Loaded Helical Compression Springs," ASME 11th Design Automation Conference, Cincinnati, September, ASME Pamphlet No. 85-DET-75.

16 Vanderplaats, G. N., 1984, *Numerical Optimization Techniques for Engineering Design: With Applications*, McGraw-Hill Book Co.

17 Ignizio, J. P., 1976, *Goal Programming and Extensions*, Lexington Books, D. C. Heath and Co.

18 Ignizio, J. P., 1985, *Introduction to Linear Goal Programming*, Sage Publications Inc.

19 Karandikar, H. M., "Hierarchical Decision Making for the Integration of Information in the Processes of Design and Manufacture in Concurrent Engineering," 1989, Ph.D. Dissertation, Department of Mechanical Engineering, University of Houston, Houston, Texas.