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DEXTEROUS TRAJECTORY TRACKING CONTROL OF A MOBILE ROBOT

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ABSTRACT

A departure from the traditional trajectory tracking control technique of a mobile robot is presented here in order to accommodate sudden changes in the reference trajectory. It is expected that in a dynamic, uncertain environment the robot may need to make sudden changes in its navigation strategy that may necessitate such an approach. In this work, a hybrid control framework is developed that first determines a suitable control strategy for a particular subtask and then implements it by means of choosing the specific controller. A supervisor is used to determine the suitable control strategy. The switching stability among a set of trajectory tracking controllers is analyzed. Extensive simulation results demonstrate the efficacy of the proposed control technique.

1. INTRODUCTION

The wide spread applications of mobile robots in recent years pose demanding requirements on the capabilities of the controller in terms of both speed and precision. It is expected that in an uncertain dynamic environment the mobile robot will be able to effect sudden changes in its navigation strategy in order to perform its task. Examples of such tasks include battlefield target tracking and surveillance, mine counter measure, planetary exploration, search and rescue and others where the robot may be expected to negotiate with sudden and abrupt changes in task requirements.

These changes are likely to impact the performance of the robot controller. One continuous controller may not be suitable to address several needs that can arise in such environments. As a result, the overall control architecture can be designed to employ several controllers at different times based on the situation. Since hybrid controllers can achieve faster response with less overshoot (e.g., to a step input command) [18], we cast the dexterous trajectory tracking problem within a hybrid control framework.

In this work, we propose a hybrid control framework that first analyzes the specific requirement of the task at a given time and then tries to match the capabilities of a specific controller. It then chooses that particular controller from a set of controllers to perform that specific task component. Discrete events are used to indicate the need for the choice of a new controller. The supervisor continuously monitors the task performance and effects controller changes as and when necessary.

The objective of this paper is to investigate a control strategy that requires sudden changes in the reference trajectory. Consider a robot that is following a target (e.g., an enemy). If the target makes sudden changes in its path while fleeing, the robot must also be capable of effecting those sudden changes in order to successfully pursue its target. We develop a framework that allows the robot to work under such a situation. We develop two trajectory tracking controllers and switch between them to track sharp trajectories. A supervisor that employs hybrid automata performs the switching. The stability of switching is proved by multiple Lyapunov function (MLF) analysis. Simulation results are presented to verify the proposed control technique.

The paper is organized as follows. In Section 2, we briefly review recent work on the control of nonholonomic mobile robots. We present the control architecture and the stability analysis in Section 3. We present the results from detailed computer simulations that verify the proposed theoretical development in Section 4. Finally, Section 5 summarizes the contributions of our present work and identifies future research directions.

2. LITERATURE SURVEY

There has been considerable research effort in the literature on the motion planning of a wheeled mobile robot (WMR) using kinematic models. Relatively less work can be found on the dynamic control of the mobile robot with nonholonomic constraints. However, dynamic control is important for many applications where speed, dexterity and performance are important. Here we mention only the most relevant work that is concerned with mobile robot control with dynamic model.

In [7] a stable control algorithm capable of dealing with the three basic nonholonomic navigation problems is proposed and the complete dynamics of a mobile robot has been derived using backstepping. This feedback servo control scheme is valid as long as the velocity control inputs are smooth and bounded.

Normally, perfect knowledge of mobile robot parameters is unattainable, so in the literature several adaptive control techniques have been developed. In [6], a design method of an adaptive tracking controller for a nonholonomic mobile robot with unknown parameters in its kinematic part is presented. The authors proved that an adaptive tracking controller for the dynamic model can be designed by using adaptive backstepping if an adaptive tracking controller for the kinematic model exists. In [11], Wilson et al. developed a robust adaptive control system for nonholonomic mobile robots. The algorithm is robust to unmodeled dynamics and external disturbances as long as the bound on the disturbance is known.

In [12], Zhang et al. derived a simplified dynamic model that is adequate for control design and treat the remaining terms as model uncertainty. The uncertainty is analyzed and a robust control algorithm is designed.

In [8], the authors proposed a sliding mode control law for solving trajectory tracking problems of nonholonomic mobile robots. The schemes for a mobile robot with two control inputs asymptotically stabilize to a desired trajectory consisting of three posture variables. A variable structure control law was proposed [9] with which mobile robots converge to reference trajectories with bounded errors of position and velocity. In [10] Aguilar et al. presented a path following controller that was robust with respect to localization errors. The controller guaranteed a global and exponential convergence of the distance and orientation errors with respect to the moving frame.

An overview of recent developments in control of hybrid systems is described in [14]. The supervisory control of hybrid systems is introduced and discussed in [15].

Switched systems are simple models of (the continuous portion) hybrid systems. Since stability analysis is important for hybrid systems, much work has done in this field. A survey paper [4] presented the major results in the stability of finite-dimensional hybrid systems and discussed the results of switched linear (stable or and unstable) system. In [16], Hou et al. established results for the asymptotic stability of switched systems. The recent developments in three basic problems regarding stability and design of switched systems are surveyed in [5]. In [13], authors provided a summary of recent developments in control of nonholonomic systems.

In [17], Lim et al. proposed the design of hybrid control systems for the motion control of wheeled mobile robot systems with nonholonomic constraints. The motion control tasks for desired-paths with edges and dynamic path following with various initial conditions are investigated as the applications by simulation studies.

From the literature survey presented above, it is clear that relatively less work is conducted on the hybrid control of a mobile robot using a dynamic model that is subjected to nonholonomic constraints. In this work, we present a new hybrid control framework that first determines a suitable control strategy for a particular subtask and then implements it by means of choosing the specific controller that can improve the agility of WMR.

3. MODELING AND CONTROLLER DESIGN

We model a WMR as a continuous dynamic system in this work. It is controlled by a continuous feedback controller at any given time. However the WMR is expected to be able to respond to discrete events that may arise because of changes in the dynamic environment and/or task requirements. As a result, several continuous controllers may be employed to perform the whole task. This set of controllers will be accommodated by a supervisor. Thus the combined system consisting of the supervisor and the set of continuous feedback controllers will form a hybrid system.

In what follows, we first describe the proposed hybrid control methodology and then explain each component in a detailed manner with reference to the control of a mobile robot.

3.1. HYBRID CONTROL ARCHITECTURE

A schematic diagram for the proposed control architecture is shown in Fig. 1. A set of controllers is stored as the possible candidates in a bank of controllers. A supervisor collects information of discrete events related to the task and activates one of the candidates as a response. The decision logic is realized in the form of a hybrid automaton, which makes decisions concerning the requirement of current mission and characteristics of the basic controllers.

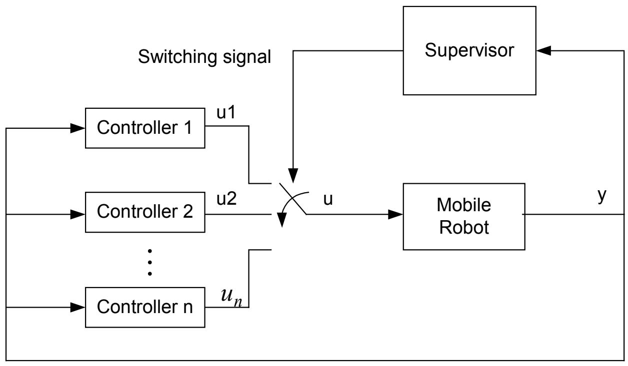


Fig. 1. Hybrid control architecture for a mobile robot

Typically, a *hybrid automaton* $H=(Q, X, f, Init, D, E, G, R)$, where [1].

Q: finite set of discrete variables;
X: finite set of continuous variables;
$init \subseteq Q \times X$ set of initial states;

D: $Q \to P(X)$	a domain or variant set;
$E \subseteq Q \times Q$	set of edges;
G: $E \to P(X)$	guard condition;
R: $E \times X \to P(X)$	reset map.

There are n controllers, *Controller_1*, *Controller_2*, …, *Controller_n*. The jump conditions are: *Controller1_2*, *Controller2_1*, …, *Controller1_n, ..., Controllern_1*. They are used to decide which controller is to be adopted at any given time. The control strategy can be summarized in Fig. 2.

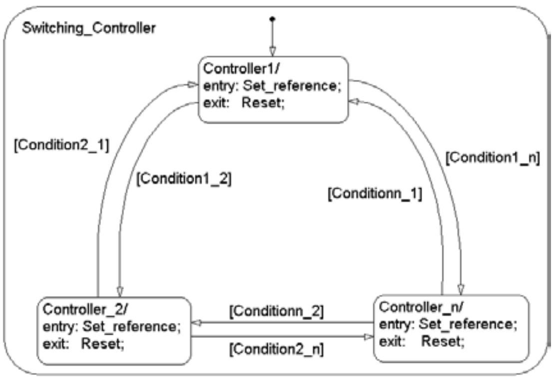


Fig. 2. Hybrid automaton for switching logic

Thus the hybrid automaton has n discrete modes: V={*Controller_1, Controller_2, ..., Controller_n*}, continuous variable X, which is the state vector to be discussed in Sec. 2.2, and jump conditions *Controller1_2*, *Controller2_1*, …, *Controller1_n, ..., Controllern_1*. The continuous dynamics of each discrete mode is given by the vector fields defined by the n controllers.

3.2. DYNAMIC MODEL OF THE WMR

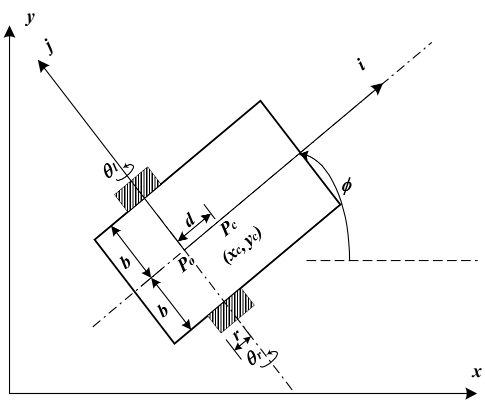


Fig. 3. Mobile robot geometry

Consider a WMR with differentially driven wheels as shown in Fig. 3 (The front passive caster is omitted.) and the relevant parameters shown in Table 1.

Table 1. Parameters of the WMR

P_o:	*the intersection of the axis of symmetry with the driving wheel axis*
P_c:	*the center of mass of the platform with coordinates* (x_c, y_c)
x, y:	*the world coordinate system*
i, j:	*the local coordinate system fixed with the WMR with (0 ,0) at Po*
d:	*the distance between Po and Pc*
b:	*the distance between either driving wheel and the axis of symmetry*
r:	*radius of each driving wheel*
c:	*r/2b*
M_c:	*the mass of the WMR*
J_c:	*the rotation inertia of the WMR about a vertical axis through Pc*
m_w:	*mass of each wheel*
I_w:	*inertia of each wheel*
ϕ:	*the heading angle of the platform*
θ_r, θ_l:	*angular positions of the two driving wheels, respectively*

The kinematic constraints arise from the pure rolling and no-slip conditions. They can be represented in the following form:

$$A(q)\dot{q} = 0 \qquad (1)$$

where

$$A = \begin{bmatrix} -\sin\phi & \cos\phi & 0 & 0 \\ -\cos\phi & -\sin\phi & \frac{r}{2} & \frac{r}{2} \end{bmatrix}$$

and $q = [x_c, y_c, \theta_r, \theta_l]^T$, $\phi = c(\theta_r - \theta_l)$.

By employing Lagrangian method [2], we obtain the following dynamic equation for the WMR:

$$M\ddot{\Theta} + C(\Theta, \dot{\Theta}) = \tau \qquad (2)$$

where

$M \in \Re^{2\times 2}$ is the symmetric, positive definite inertia matrix, $C \in \Re^{2\times 1}$ is the centrifugal and Coriolis term.

$\Theta = \begin{bmatrix} \theta_r \\ \theta_l \end{bmatrix}$ and $\tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}$ are the torques applied on two wheels.

Now we derive the state space model that is employed in controller design. We do not use a tire model. Instead, we impose pure rolling and no-slip kinematic constraints. The friction effect is not considered when we design the controller for the WMR, but both Coulomb and viscous friction were later considered as disturbance in the simulations to make the simulations more realistic.

Equation (1) includes two nonholonomic and one holonomic constraint. We can express

$$\dot{q} = S\dot{\Theta} \qquad (3)$$

where S is a matrix consisting of the null space basis vectors of A,

$$S = \begin{bmatrix} cb\cos\phi & cb\cos\phi \\ cb\sin\phi & cb\sin\phi \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Let us define the state vector

$$X=\begin{bmatrix} q \\ \dot{\Theta} \end{bmatrix}=[x_c, y_c, \theta_r, \theta_l, \dot{\theta}_r, \dot{\theta}_l]^T \quad (4)$$

From Eq. (2), Eq. (3) and Eq. (4), we can write the following state space equation of the WMR.

$$\dot{X}=\begin{bmatrix} S\dot{\Theta} \\ F \end{bmatrix}+\begin{bmatrix} 0 \\ G \end{bmatrix}\tau \quad (5)$$

where $F=-M^{-1}C$ and $G=M^{-1}$.

3.3. FEEDBACK CONTROLLER DESIGN

We design two trajectory tracking controllers that can improve the agility of the WMR by means of switching. Both controllers are designed using input-output linearization techniques. Although it has been proven that model (5) is not input-state linearizable, we can still perform input-output linearization [3]. First we design the nonlinear feedback

$$\tau=G^{-1}(U-F) \quad (6)$$

and apply Eq. (6) to Eq (5). We get

$$\dot{X}=\begin{bmatrix} S\dot{\Theta} \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ I_{2\times 2} \end{bmatrix}U$$

where U is the new input vector and I is 2 by 2 identity matrix.

Here we have used the fact that $S^TE=I_{2\times 2}$ and $E=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$.

In what follows, we introduce two controllers that will be used for the demonstration of the proposed hybrid control. One of them controls the (x_c, y_c) trajectory. The other one controls the forward displacement η, and the orientation angle ϕ.

4.0. CASE STUDY

Controller 1: $Y=[x_c \ \ y_c]^T$

In this case, it commands the position (x_c, y_c) of motion, which can be used as the default controller to follow the desired trajectory.

We can see that the output equation is a function of position state variable q only, that is, if

$$Y=h(q)=[h_1(q) \ \ h_2(q)]^T$$

Differentiating Y and substituting Eq. (3) in $\dot{Y}$, we get

$$\dot{Y}=\frac{\partial h}{\partial q}S\dot{\Theta}$$

Letting $J_h=\frac{\partial h}{\partial q}$, which is the Jacobin matrix of Y, we can obtain the decoupling matrix

$$\Phi=J_hS$$

Differentiating $\dot{Y}$, and substituting Φ in $\dot{Y}$, we get

$$\ddot{Y}=\dot{\Phi}\dot{\Theta}+\Phi\ddot{\Theta} \quad (7)$$

Further using nonlinear feedback:

$$U=\Phi^{-1}(\upsilon-\dot{\Phi}\dot{\Theta}) \quad (8)$$

where $\upsilon=[\upsilon_1 \ \upsilon_2]^T$ is the reference input vector, we can get the input-output feedback control law.

Now we derive the decoupling matrix of *Controller 1* and name it as Φ_1:

$$\Phi_1=c\begin{bmatrix} b\cos\phi-d\sin\phi & b\cos\phi+d\sin\phi \\ b\sin\phi+d\cos\phi & b\sin\phi-d\cos\phi \end{bmatrix} \quad (9)$$

Substituting Eq. (8) and Eq. (9) into Eq. (7), we get the output equations: $\ddot{Y}=\upsilon$

Controller 2: $Y=[y_1 \ y_2]^T=[\eta \ \phi]^T$

It commands the forward displacement (η) (in driving direction) and the direction of movement (ϕ), which can be used to adjust the direction when meeting with abrupt changes in the trajectory or *non-smooth* trajectory.

Follow the same procedure as that of *controller 1*:

Differentiating y_1 twice, we obtain $\ddot{y}_1=\ddot{\eta}$

We notice $\dot{\eta}=\dot{x}_c\cos\phi+\dot{y}_c\sin\phi=\frac{r}{2}(\dot{\theta}_r+\dot{\theta}_l)$

Differentiating $\dot{\eta}$, we get $\ddot{\eta}=\frac{r}{2}(\ddot{\theta}_r+\ddot{\theta}_l)$

Differentiating y_2 twice, we get $\ddot{y}_2=\ddot{\phi}$

We notice

$$\phi=c(\theta_r-\theta_l) \quad (10)$$

Differentiating ϕ twice, we get $\ddot{\phi}=c(\ddot{\theta}_r-\ddot{\theta}_l)$.

The decoupling matrix can be derived in the same way:

$$\Phi_2=\begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ c & -c \end{bmatrix} \quad (11)$$

Thus the output equations are: $\ddot{Y}=\upsilon$

4.1. STABILITY ANALYSIS

As discussed earlier, in many cases, to increase the agility and high-speed maneuverability requires switching among several controllers. In this work, we expect to switch between the above two controllers to track trajectories that pose discontinuous changes in the reference trajectories. However, such switching necessitates a stability analysis to ensure the safety and feasibility of such operations.

Stability of internal dynamics

In order to analyze the switching stability between *controllers 1* and *2* of the WMR, we must guarantee that both the linearizable part and the internal dynamics are stable under switching. Because if the internal dynamics is not stable, the system will not be able to track the desired outputs even though the design of the feedback controller is correct. The above controller design only accounts for part of the closed-loop dynamics that are linearizable and controllable. However, the system under each controller has unobservable internal dynamics. The stability of the internal dynamics is critical for a feedback control to work properly. There are very few efforts found in the literature that studies the internal dynamics of a WMR. In [19], Yun et al. studied the internal stability of a two-wheel differentially driven mobile robot. In our work, we use the similar model to their case. It was shown in their results, that for look-ahead control (which is *Controller 1* in our case), driving forward is stable but driving backward is not. In our case, we are only interested in moving forward and as result, that stability analysis of [19] is directly applicable.

Next, we need to analyze the stability of the internal dynamics of *Controller 2*. By following a similar procedure as in [19], we obtain the internal dynamics of *Controller* (η,ϕ):

$$\begin{bmatrix}\dot{z}_5\\ \dot{z}_6\end{bmatrix}=\begin{bmatrix}z_2\cos z_3 - d\cdot z_4 \sin z_3\\ z_2\sin z_3 + d\cdot z_4\cos z_3\end{bmatrix}$$

where

$$Z=T(x)=\begin{bmatrix}z_1\\ z_2\\ z_3\\ z_4\\ z_5\\ z_6\end{bmatrix}=\begin{bmatrix}h_1(x)\\ L_f h_1(x)\\ h_2(x)\\ L_f h_2(x)\\ x_c\\ y_c\end{bmatrix}=\begin{bmatrix}\eta\\ \dot\eta\\ \phi\\ \dot\phi\\ x_c\\ y_c\end{bmatrix}$$

T(x) is a diffeomorphism in the whole state space.

Since our goal is trajectory tracking instead of state stabilization, we reformulate the dynamics in the error space of the state variables. Denoting the error coordinates by (see Kanayama et al., [20]),

$$\begin{bmatrix}x_e\\ y_e\\ \phi_e\end{bmatrix}=\begin{bmatrix}\cos\phi & \sin\phi & 0\\ -\sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}x-x_r\\ y-y_r\\ \phi-\phi_r\end{bmatrix},$$

where the current posture is $(x, y, \phi)^T$, the reference posture is $(x_r, y_r, \phi_r)^T$ and error posture of the above two is $(x_e, y_e, \phi_e)^T$.

we obtain the error dynamics:

$$\begin{bmatrix}\dot{x}_e\\ \dot{y}_e\end{bmatrix}=\begin{bmatrix}\dot\phi\, y_e + u - u_r\cos\phi_e\\ -\dot\phi\, x_e + u_r \sin\phi_e\end{bmatrix} \tag{12}$$

where u and u_r donates the actual forward velocity and the reference forward velocity, respectively.

Since we control ϕ, so eventually $\phi_e \to 0$ leading $\cos\phi_e \to 1$. In addition, u is controllable (the derivative of η), which implies $u-u_r \to 0$ as $t\to\infty$. Therefore, Eq. (12) can be simplified as following.

$$\dot\xi=\begin{bmatrix}\dot{x}_e\\ \dot{y}_e\end{bmatrix}=\begin{bmatrix}\dot\phi\, y_e\\ -\dot\phi\, x_e\end{bmatrix}=\dot\phi\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}x_e\\ y_e\end{bmatrix} \tag{13}$$

Clearly, the above oscillator equation is Lyapunov stable (i.e., the energy is bounded) but is not asymptotically stable, which has an equilibrium subspace characterized by

$$E_\xi = \{\xi \mid x_e = y_e = 0\}.$$

Consider the following energy-like function.

$$V(\xi)=\frac{1}{2}{x_e}^2+\frac{1}{2}{y_e}^2$$

In the neighborhood of E_ξ, $V(\xi)$=0 if $\xi\in E_\xi$, and $V(\xi)$>0 if $\xi\notin E_\xi$. So $V(\xi)$ is positive definite with respect to E_ξ. The derivative of $V(\xi)$ with respect to the time is

$$\dot V(\xi)=\frac{\partial V}{\partial \xi}\dot\xi = x_e\dot{x}_e + y_e\dot{y}_e = 0.$$

Since, we know the internal dynamics is Lyapunov stable (i.e., stability in the BIBO sense), the design of *Controller 2* is sufficient to achieve tracking objectives.

It is well known that some switching sequences can lead to instability, even if both the individual subsystems are stable [4]. Therefore, in order to achieve stability of the switched system, we need to restrict the class of admissible switching sequences.

Stability analysis of switched system

We will briefly review the MLF [4] analysis for its later use in the control of switched systems. Consider the following switched nonlinear system, with $u_i \equiv 0, i = 1, 2, \cdots, N$,

$$\begin{aligned}\dot x &= f_i(x) + g_i(x)u_i\\ y_i &= h_i(x)\\ i &\in I = \{1, 2, \cdots, N\}\end{aligned} \tag{14}$$

and suppose that we can find a family of Lyapunov functions $\{V_i : i\in I\}$ such that the value of V_i decreases monotonically on each interval when the i^th subsystem is active, i.e.,

$$V_{i_k}(x(t_{k+1})) \le V_{i_k}(x(t_k))$$

for all $i_k \in I$. Then system (14) is Lyapunov stable.

For our specific problem, construction of above MLF can split into the following subtasks.

1. Find candidate Lyapunov functions for each subsystem; and
2. Construct the switching law to show the switched system is stable.

The Lyapunov functions in *step 1* can be chosen by $V_i = \overline{V}_i + \widetilde{V}_i$.

where $\overline{V}_i$ is taken to be the one for each linearized subsystem, and $\widetilde{V}_i$ is another Lyapunov function for internal dynamics.

For linearized subsystems, Lyapunov functions ($\overline{V}_i$) can be obtained by solving corresponding Riccati equations. For internal dynamics, Lyapunov functions can be computed given the particular structure of the individual systems.

In our case, $\overline{V}_i = e_i^T P_i e_i$.

where P_i is corresponding symmetric, positive definite matrix associated with each linearized subsystem and e_i donates the error form of each linearized subsystem

$$e_1=\begin{bmatrix}x-x_r\\ \dot x-\dot x_r\\ y-y_r\\ \dot y-\dot y_r\end{bmatrix},\qquad e_2=\begin{bmatrix}\eta-\eta_r\\ \dot\eta-\dot\eta_r\\ \phi-\phi_r\\ \dot\phi-\dot\phi_r\end{bmatrix}$$

The subscript r stands for the reference values of the particular variables.

$\widetilde{V}_1$ is taken from [19] as $1-\cos(c\cdot(\theta_r^* - \theta_l^*))$.

where $[\theta_r^*, \theta_l^*]^T$ are the equilibrium points.

Also, $\widetilde{V}_2 = \frac{1}{2}({x_e}^2 + {y_e}^2)$. Although its time derivative is not negative (it is always zero), the general Lyapunov function can still be constructed by $V_2 = \overline{V}_2 + \widetilde{V}_2$, such that its time derivative is negative for monitoring the evolution of (η, ϕ) system.

To this end, we obtain two Lyapunov functions associated with two subsystems. Based on the above functions, we can show the switched system is stable by appropriate switching between them.

For our specific system, the parameters are shown in Table 2.

Table 2 System parameters

Parameters	Value
d *(cm)*	1
b *(cm)*	10
r *(cm)*	3
M_c *(kg)*	5
J_c *(kg-m^2)*	0.01

In the hybrid automaton (Fig. 4), before switching to another system, the switching algorithm should satisfy that the end of Lyapunov function of the current active subsystem is not greater than the previous one of which the subsystem is active. This criterion is included in the jump condition.

4.2. SIMULATION RESULTS

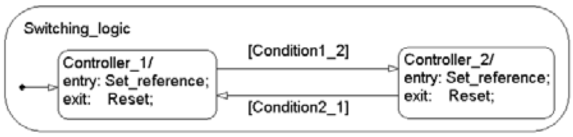
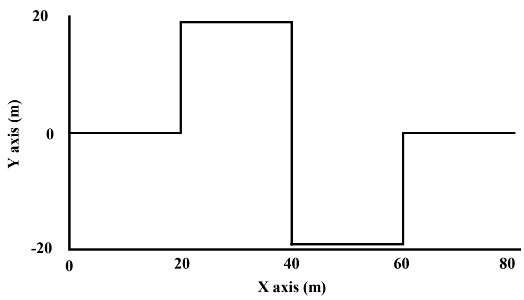
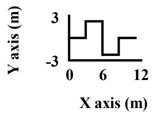


Fig.4. Hybrid automaton for switching between two controllers

We present simulation results of tracking control of the WMR that is subjected to discontinuous reference trajectories. The wheels are considered to be far lighter than the chassis, and, therefore, their masses and rotation inertias are ignored.



a)



b)

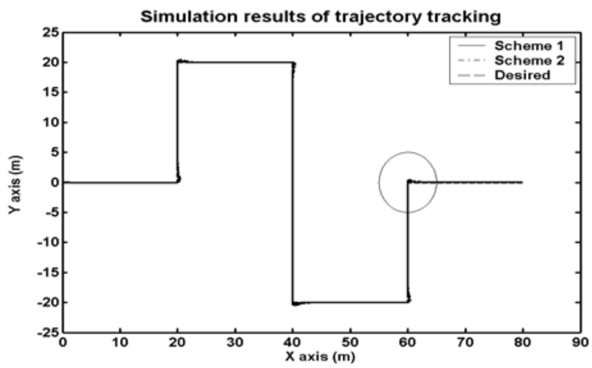
Fig. 5. Two reference trajectories shown in the Cartesian space. (a) sparsely discontinuous trajectory, and (b) densely discontinuous trajectory.

A sharply changing trajectory (Fig. 5.) is chosen to demonstrate the efficacy of the proposed methodology. We want the WMR to track the position as accurately as possible without considerable sacrifice in speed. This trajectory can be used as a benchmark trajectory for high-speed maneuverability and agility. The trajectories were generated by providing with reference velocities of *1 m/s* in both *x* and *y* directions alternatively.

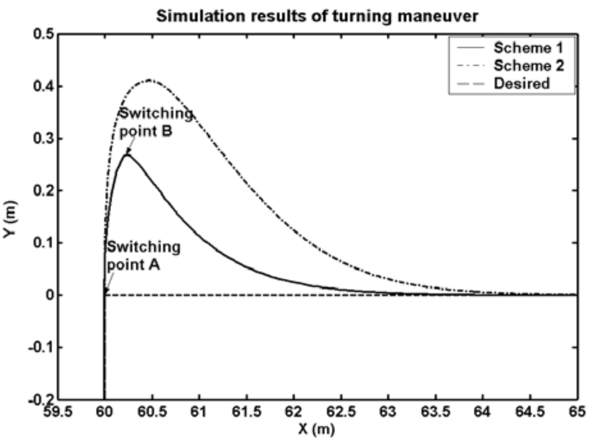
When a discontinuous trajectory occurs (such as the heading angle of the robot ϕ_r varies sharply, then $\frac{d\phi}{dt}$ becomes a large quantity), the event is reported and if the end of Lyapunov function of current active subsystem is not greater than the previous one of which the subsystem is active, the jump condition described in section 3.1 is satisfied. The transition from *controller 1* to *controller 2* is triggered consequently. *Controller 2* is then used for zero-radius turning (i.e., the forward reference velocity is zero). Once the orientation is adjusted to the desired one and the criterion for Lyapunov function is satisfied, another jump condition is met according to the corresponding event (i.e., the orientation angle is close enough to the desired one and Lyapunov function condition is satisfied). Then the converse transition happens from *controller 2* to *controller 1*.

The above scheme for dexterous trajectory tracking is called *Scheme 1*. In order to demonstrate its advantage over traditional trajectory tracking, we compare it with another schemes called Scheme 2. In *Scheme 2*, only *Controller 1* is used for the whole trajectory tracking task.

In the simulations, in order to avoid lateral slip during turning because of centrifugal force on the WMR we set a limit on the lateral acceleration, which was *3 m/s^2*. This value was obtained from the consideration of the lateral friction coefficient of tire materials. We have also set the limits on angular acceleration and linear velocity at *3 rad/s^2* and *1 m/s,* respectively. And these limits are there for both schemes for uniformity.



a)



b)

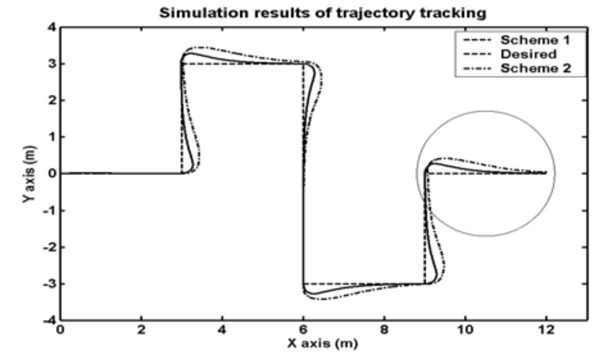
Fig. 6. Controller performance in the Cartesian space for the first reference trajectory (Fig. 5(a)):

a) the global view, b) the detailed view of the encircled point of a)

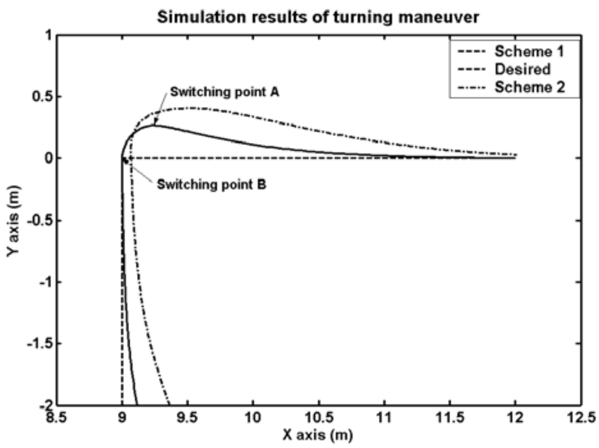
Fig. 6 (a) depicts the results of trajectory following tasks in the Cartesian space as performed by both schemes for the reference trajectory shown in Fig. 5 (a). In order to understand how well each scheme performed at the sharp edges, we present a magnified view of the encircled area of Fig. 6 (a) in Fig. 6 (b). In Fig. 6 (b), for Scheme 1, switching occurs from *Controller 1* to *Controller 2*, and *Controller 2* to *Controller 1* at point A and B respectively. So before A, (x, y) controller is active; between A and B (η, ϕ) controller is active and after B, (x, y) controller is active again etc. It is seen that the Scheme 1 has much less overshoot. In other words, the positional accuracy is significantly increased by employing a hybrid switching control methodology. This ability to follow a sharp trajectory with accuracy enables the robot with dexterous trajectory tracking capabilities. This effect is more pronounced when the trajectory has more sharp edges (Fig. 5 b)). In Fig.7 (a), we present one such simulation result where the WMR is expected to track a more densely discontinuous trajectory. In this case, the sharp changes occur every 3m in both x and y directions (as opposed to every 20m for the previous simulation). As a result, it is seen that the traditional controller (Scheme 2) performs much worse compared to the proposed hybrid controller, which is clear from the enlarged view in Fig. 7 (b).

The time taken to complete the first reference trajectory for Schemes 1 and 2 are *163.2 sec* and *160.6 sec*, respectively. The time taken to complete the second reference trajectory for Schemes 1 and 2 are *27.2 sec* and *24.6 sec*, respectively. We define the average speed by using the desired distance from the starting point to the goal point divided by the actual time consumption. By this way, the average speed taken to complete the first reference trajectory for Schemes 1 and 2 are *0.980 m/s* and *0.996* m/s, respectively. The average speed taken to complete the second reference trajectory for Schemes 1 and 2 are *0.882 m/s* and *0.976 m/s*, respectively. It can be seen that our proposed hybrid controller does not sacrifice too much speed to significantly improve the positional tracking performance.

These simulations demonstrate that it is possible to track trajectories with abrupt changes by employing multiple controllers. The advantages are more pronounced when the agility requirement is high. The results confirm that our proposed control methodology enables the robot with a higher dexterity and agility as far as trajectory tracking capabilities are concerned.



a)



b)

Fig.7. Controller performance in the Cartesian space for the second reference trajectory (Fig. 5(b)): a) the global view, b) the detailed view of the encircled point of a)

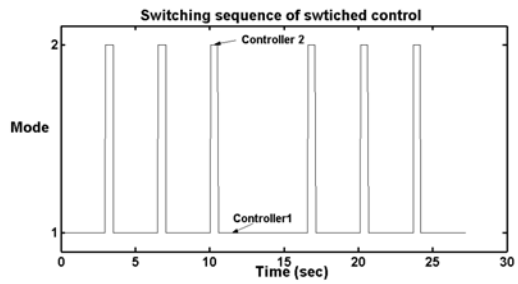


Fig.8. Switching sequence of switched control for the second reference trajectory

The switching sequence of switched control is shown in Fig.8. The supervisor controller selects the appropriate controller candidate to achieve tasks. The two different controllers are active alternately.

5. CONCLUSIONS AND FUTURE WORK

In this paper we present our work on a new hybrid control framework for a mobile robot performing a trajectory tracking task. We have developed a hierarchical framework that allows the continued operation of the mobile robot with performance guarantees in the presence of abrupt changes in the reference trajectory. We have also proved the stability of the internal dynamics of the WMR and proposed a switching sequence that guarantees stable operation. The high-precision maneuverability is increased noticeably, which is demonstrated by extensive computer simulations. The switching controller holds promise for mission accomplishment with increased agility and precision. There are a numbers of practical applications where such an agile controller can be useful for trajectory following that include battlefield target tracking and surveillance, mine counter measure, planetary exploration, search and rescue and others where the robot may be expected to negotiate with sudden and abrupt changes in task requirements. Even though we demonstrated our concepts using two controllers, the proposed framework is equally applicable to multiple controllers. We are currently developing an experimental test-bed to verify the proposed framework in actual experimentation.

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