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Minimum Drop-Loss Design of Microphotonic Microring-Resonator Channel Add-Drop Filters

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***Abstract*—Microring-resonator filters have important applications as filtering elements in microphotonic circuits. In this paper, we address the question of optimum design of resonator-based add-drop filters in the presence of finite losses, and show that symmetric coupling provides the optimum design. This conclusion contravenes previous work on this subject, and the oft-cited critically coupled resonator case. While the minimum bandwidth of a resonant filter is ultimately limited by intrinsic losses, i.e. the intrinsic Q, we show that the symmetric design can approach twice as narrow a linewidth as a critically coupled design for the same losses, in principle. We present a coupled-mode theory (CMT) model, and a complete electromagnetic device design example based on finite-difference time-domain field simulations which validates our conclusions.**

***Index Terms*—Microring resonators, channel add-drop filters, coupled mode theory, filter synthesis, power splitters.**

I. Introduction

INTEGRATED silicon based photonics has many promising applications in optical telecommunications, optoelectronics and optical signal processing [1]–[4]. The integration of silicon photonics and electronic circuits offers the prospect of low energy devices, circuits and systems for applications including on-chip and processor-to-memory interconnects [3], [4], as well as photonic analog-to-digital converters [5]. Other applications include nonlinear and quantum optical devices for applications in quantum information and computing [6].

An important photonic device, and one of the earliest concepts realized in integrated photonics, is the resonant channel add-drop filter. Microring resonators are particularly well suited for add-drop filter applications [7], [8] because of their traveling wave structure that allows for a natural separation of the four ports (in, through, drop, add in Fig. 1), without the use of circulators. Detailed techniques have been worked out for synthesizing standard Butterworth, Chebyshev [1], [9], and more advanced [10] filter responses.

These filter synthesis techniques have primarily dealt with lossless structures. However, radiation and scattering losses are not insubstantial in strong-confinement photonics, with typical losses of 2-3 dB/cm [4] and radiation Q's on the order of 250,000 in silicon, and higher in some other material systems. Regardless of the magnitude of the loss, it begins to play a major role for narrow enough bandwidth filters, when the total

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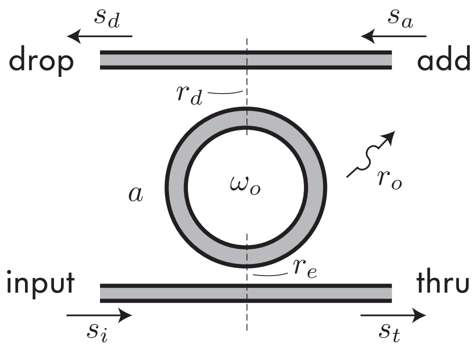

Fig. 1. Schematic of a single microring-resonator add-drop filter showing the parameters used in the CMT model.

Q approaches the loss Q, Q_o. Therefore it is of interest to investigate the optimal design of filters in the presence of loss.

Some prior work has already addressed this issue in photonics [11], and considerably more in circuit and microwave theory. Here, we show that the design of single-microring filters that provides minimum loss calls for symmetric coupling to the input and drop bus. This is in contradiction with the critically coupled design claimed to provide optimum transmission e.g. by Vörckel *et al.* [12] explicitly, and often assumed optimal in other work (e.g. [13]).

II. Coupling of modes in time (CMT) model

Coupled-mode theory in time (CMT) provides a simple model that affords all of the necessary physics of the resonant add-drop filter problem, including resonance, loss and coupling to ports. The system of equations that describes a single-resonator filter excited by a monochromatic input wave $s_i(t)$ at angular frequency ω is [1], [2]

$$\begin{aligned} \frac{d}{dt}a(t) &= j\omega a(t) = (j\omega_o - r)a(t) - j\sqrt{2r_e}s_i(t) \\ s_t(t) &= s_i(t) - j\sqrt{2r_e}a(t) \\ s_d(t) &= -j\sqrt{2r_d}a(t) \end{aligned} \quad (1)$$

where $|a|^2$ is the energy amplitude of the ring resonant mode, and s_i, s_t, and s_d are the power-normalized amplitudes of input, through and drop port waves [2] (Fig. 1). With input wave s_i incident, some excitation is picked up by the resonator, and the remaining field propagates on to the through port. It then interferes with the light leaving the resonator in the through port and is carried away by through-port wave s_t. The energy stored in the resonator is $|a|^2$ and according to Eq. (1) the energy amplitude $a(t)$ decays at the total rate r, comprising a decay rate describing external coupling to the input port, r_e, the drop port, r_d, and a loss mechanism, r_o:

$$r = r_e + r_d + r_o \tag{2}$$

The decay rates are related to decay time constants as $r_i = 1/\tau_i$, for $i \in \{e, d, o\}$. Since τ is a field time constant, the associated photon lifetime ($1/e$-intensity time) is $\tau/2$.

The drop-port response of the device is found from Eq. (1),

$$\left|\frac{s_d}{s_i}\right|^2 = \frac{4r_e r_d}{(\omega - \omega_o)^2 + r^2} \tag{3}$$

The response is Lorentzian, with a 3dB (full width at half maximum, FWHM) bandwidth of $\delta\omega_{3\text{dB}} = 2r$.

Unlike a full scattering model using transfer matrices [2], [14], the CMT model treats only given resonances (here, ω_o) of the ring and does not include geometry information that would reveal properties such as the free spectral range (FSR).

III. Optimal and Critical Coupling

Given a certain loss Q, Q_o, and corresponding loss rate $r_o = \omega_o/(2Q_o)$, our objective is to find the optimum choice of ring-waveguide couplings r_e, r_d in order to maximize on-resonance efficiency of transmission to the drop port, $|s_d/s_i|^2$. The CMT model, because of its simplicity, lends itself to closed-form analytical synthesis.

To find the optimum solution, we first note that the transmission efficiency, see Eq. (3), not surprisingly decreases with increasing loss, r_o. On the other hand, increasing r_e and r_d with a fixed r_o increases transmission, but also bandwidth, thus providing lower loss for a *different* filter. Therefore, we must ask for the best design of a fixed bandwidth. This was neglected in Ref. [12], and is the cause of its erroneous claim that critical coupling provides the minimum drop loss. Fixing bandwidth means fixing total rate r, according to $\delta\omega_{3\text{dB}} = 2r$, and together with a fixed loss, r_o, leaves only one undetermined degree of freedom, since from Eq. (2), $r_d = r - r_e - r_o$. Maximizing with respect to the remaining (input) coupling rate r_e, gives the optimal couplings for maximum drop port transmission

$$r_e = r_d = \frac{r - r_o}{2} \tag{4}$$

It is instructive to compare this solution to the critical coupling solution of the same bandwidth [1], [13] which leads to

$$r_e = r_d + r_o = \frac{r}{2} \tag{5}$$

A comparison of the transmission efficiency of the optimal (symmetric) and critical-coupling designs is given in Fig. 2, showing that the symmetric design is indeed optimal for maximizing dropped on-resonant power. We define normalized bandwidth $\alpha \equiv \Delta\omega_{3\text{dB}}/\Delta\omega_o$ as the ratio of total bandwidth $2r$ to intrinsic (loss limited) bandwidth $2r_o$. Substitution of Eqs. (4,5) into (3) yields the efficiency of the symmetric and critically coupled designs for various relative bandwidths α:

$$\left|\frac{s_d}{s_i}\right|^2 = \left(1 - \frac{1}{\alpha}\right)^2 \tag{6}$$

$$\left|\frac{s_d}{s_i}\right|^2 = 1 - \frac{2}{\alpha} \tag{7}$$

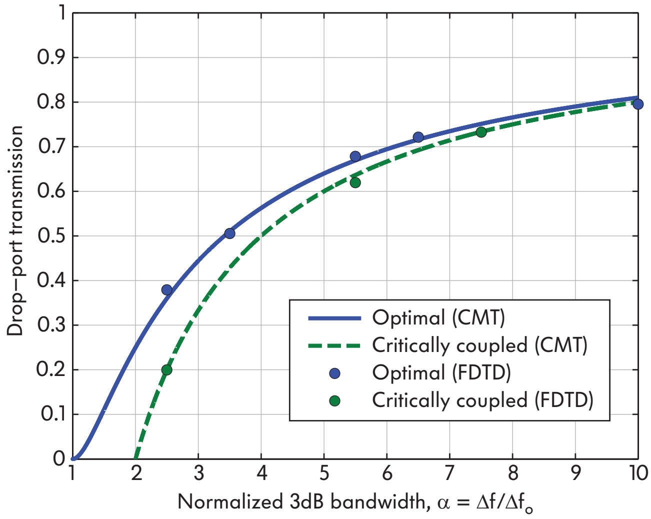

Fig. 2. Minimizing the impact of loss on a single filter stage: comparison of symmetric (optimal) and critically coupled single-ring filter designs for different normalized bandwidths, α (ratio of total bandwidth to loss-limited, intrinsic bandwidth).

This comparison is useful in the design process to determine the narrowest bandwidth that supports a desired transmission to the drop port, or the maximum transmission achievable at a certain bandwidth, given known linear losses. Fig. 2 and Eqs. (4–5) show that the optimum symmetric design has a minimum bandwidth limit of Δf_o, while the critically coupled design has a minimum bandwidth of $2\Delta f_o$. In the limit of a large relative bandwidth α, the loss plays a negligible role and the two solutions can be verified by to be equal by a first-order Taylor series expansion in α^{-1} of Eqs. (4–5). For 3dB transmission, the symmetric case can reach $\alpha = \sqrt{2}/(\sqrt{2} - 1) \approx 3.412$ times the intrinsic linewidth, while the critically coupled case is limited to $\alpha = 4$ intrinsic linewidths, a difference of $\sim$20%.

IV. Electromagnetic Design

We next verify these results on a hypothetical lossy device design, via full-wave finite-difference time-domain (FDTD) numerical simulations [15]. We first use numerical simulations to design the example filter and relate the physical geometry to CMT variables such as r_e, r_d and r_o, and then verify the total device performance against the CMT model by simulating the entire device's response using FDTD. Without loss of generality, we consider a two-dimensional (2D) model in TE polarization, because all relevant physics is in the plane. The theory applies, however, to arbitrary resonator type (microring, photonic crystal cavity, etc.), in 2D (e.g. toy models) or 3D (real devices), and for arbitrary choices of excitation mode (e.g. polarization) and loss mechanism. In our example, we consider bending loss as the source of loss, but the approach treats equally absorption, roughness-induced scattering, etc.

Fig. 1 shows the geometry of the device. First, the bus and ring waveguide widths are chosen to be the widest that still give single mode slab operation in the $1.55\,\mu$m wavelength range. For core and cladding indices $n_{\text{co}} = 3.5$ and $n_{\text{cl}} =$

1.45, the waveguide width is $w_g = 0.24\,\mu$m. The microring resonator radius is chosen next, purposefully small enough to result in substantial radiation losses, so that we can test our design approach for filters with lossy resonant elements. A circularly bent waveguide is known to produce radiation loss due to bending that exponentially increases with decreasing radius [16]. We use a two-dimensional mode solver for bent slab waveguides [17] to find a radius for which the radiation Q, $Q_o \approx 1,000$. This amounts to selecting a radius at which the real part of the propagation constant forms an integer number of wavelengths in one round trip at 1550 nm, and the imaginary part of the propagation constant yields the losses, and the target loss Q ($Q_o \approx k_o(d\beta_R/dk_o)/(2\beta_I)$). This radiation Q, Q_o, determines the minimum possible 3 dB linewidth known as the intrinsic linewidth, $\Delta f_o = f_o/Q_o$, due to decay rate r_o. The closest radius to a Q_o of 1,000 and 1550nm resonance is an outer radius of $R_o = 0.78\,\mu$m.

Next we choose the gaps that correspond to calculated coupling rates r_e, r_d in the presence of loss, r_o. The evanescent field that exists in the cladding is responsible for coupling between the bus waveguide and the ring waveguide. The fraction of power coupled to the ring normalized to the input bus power is termed the power coupling coefficient, k^2. Since the evanescent field decays exponentially far from the core, larger gaps between the bus and the ring lead to smaller coupling coefficients as [2], [14]

$$k^2(g) \approx k_o^2 e^{-\gamma(\mathrm{g}-\mathrm{g_o})}. \quad (8)$$

where $k_o \equiv k(g_o)$. In high-index-contrast, strong-confinement structures, this dependence deviates from purely exponential dependence, but rigorous FDTD simulations provide the exact relationship between the coupling coefficients and the bus-ring gap. Hence, to capture the data in a physically consistent model, we take the log of k^2 vs. gap, which is nearly linear and fit it to a low-order polynomial to account for higher-order effects captured in the full-wave simulation [8]

$$\ln k^2(g) = p_3 + p_2 g + p_1 g^2. \quad (9)$$

From this type of fit, the coupling geometry of the device can be chosen to obtain desired r_e,r_d. Finally, for a given loss Q, Q_o (i.e. r_o), we use FDTD simulations to simulate the full device design to confirm the analytic solutions obtained using the CMT model and Eqs. (4–5) in both the optimal and critically coupled case. We give one design example in this paper, for $\alpha = 2.5$. From the chosen microring cavity design, and corresponding radiation loss r_o, we obtain total bandwidth $2r = 2r_o\alpha$, and the corresponding r_e and r_d for the symmetric and critical designs. To connect the power coupling coefficient to the CMT model, we use the first-order correspondence [1]

$$k_i^2 \approx \frac{2r_i}{\Delta f_{\mathrm{FSR}}} \qquad \text{for } i \in \{e, d\} \quad (10)$$

where the FSR is $\Delta f_{\mathrm{FSR}} \equiv \frac{c}{2\pi R n_g}$, c is the speed of light in vacuum and $n_g \approx 3.62$ is the ring group index found in modesolver simulations. Using Eq. (9), the coupling gaps are

$$g_i = \frac{-p_2 - \sqrt{p_2^2 - 4p_1(p_3 - \ln k_i^2)}}{2p_1} \qquad \text{for } i \in \{e, d\} \quad (11)$$

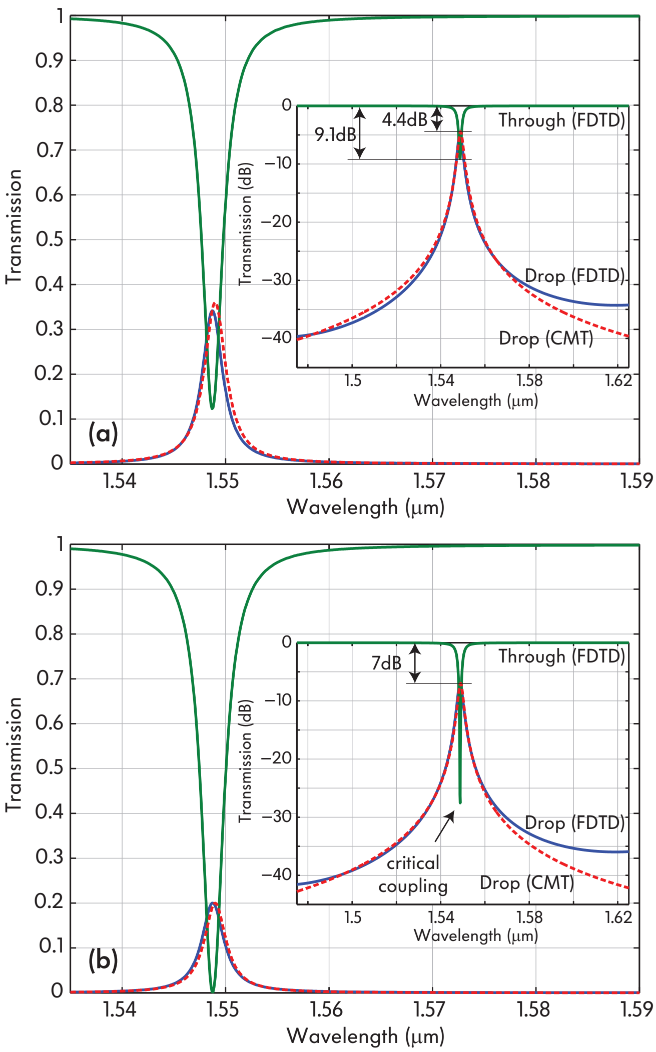

Fig. 3. Spectra of through and drop ports calculated in FDTD simulations and drop port calculated using the CMT model for a) optimally coupled and b) critically coupled microring resonator channel add-drop filters on $\alpha = 2.5$ A good matching of the analytic CMT solution with numerically simulated FDTD is observable. The insets are showing spectra on dB scale.

In Fig. 3, spectral responses resulting from FDTD simulations are shown for optimal and critically coupled filters for $\alpha = 2.5$. Overlapping the data is the CMT model of the target design, showing very good matching. CMT and FDTD spectra significantly differ only at far off-resonant detuning, where drop port transmission is less than -30 dB. The reason for this disagreement is that the CMT model here has included only one resonance, while a physical microring cavity has a finite FSR and repeating resonances. Hence, the FDTD drop response levels off on the left (shorter wavelength side) and right (longer wavelength side) because it is about to rise into another peak one FSR away. The left and right sides of the FDTD response are unequal because dispersion results in the FSR becoming smaller with increasing wavelength. We have verified that equally good agreement can be obtained for other values of the normalized bandwidth α. The on-resonant drop loss is plotted (points) alongside the analytic response (lines) in Fig. 2 and confirms the analytic model of the optimum

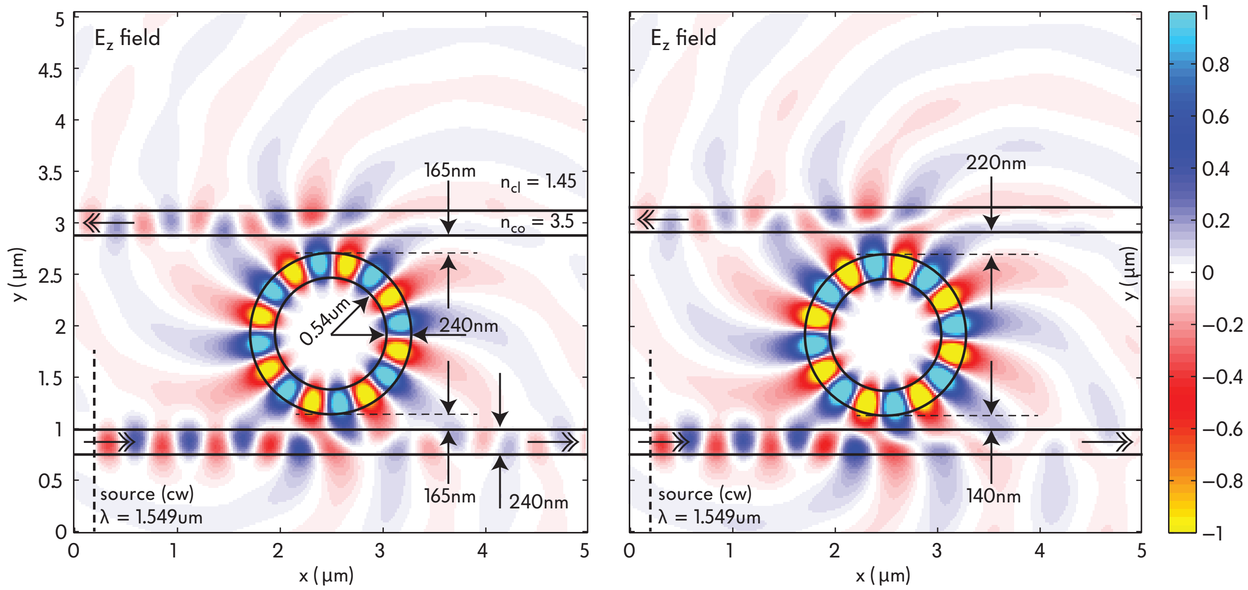

Fig. 4. Numerical FDTD simulations of symmetrically (on the left), i.e. optimally, and critically (on the right) coupled microring resonator channel add-drop filters showing the electric field propagation in the structures. Geometrical parameters are also provided. This is a snapshot under continuous-wave excitation on-resonance and it shows that more power is dropped for optimally coupled device.

symmetric and critical designs, respectively.

Figure 4 shows field snapshots of FDTD simulations of the $\alpha = 2.5$ designs whose spectra are in Fig. 3. While pulsed excitation was used to obtain full spectral response data in a single simulation, the snapshots are taken in the steady state with continuous-wave excitation at the resonant wavelength, for simpler interpretation. Fields are shown with excitation amplitudes on the same scale. More power is seen dropped by the symmetric design in Fig. 4(a) even though the critical case has less through-port transmission (it also radiates more). The guided fields look 'wobbly' because of the significant presence of radial radiation from the ring in the total field. An additional interesting observation is the larger steady-state field enhancement in the critically-coupled cavity in Fig. 4(b) even though both have the same total Q (r). This is because the input coupling r_e is stronger for the critically coupled case. Hence, the symmetric design is not only more efficient but also less sensitive to nonlinear resonance shifting and nonlinear loss due to Kerr nonlinearity and two-photon absorption.

V. Conclusion

We showed that the optimum design of a drop filter, in the context of minimizing on-resonance insertion loss, is a design with a symmetric coupling configuration. A CMT model shows that the symmetric design approaches a 2-fold narrower loss-limited bandwidth, and allows 20% narrower passbands for 3 dB insertion loss. We constructed an example geometry using mode solver and FDTD simulations of a microring cavity and directional couplers, and verified that the complete device has the CMT-predicted response from full-wave simulations of the entire device. We also noted that the symmetric design is in addition more robust to nonlinearities.

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