Proceedings of IMECE2005
2005 ASME International Mechanical Engineering Congress and Exposition
November 5-11, 2005, Orlando, Florida USA

IMECE2005-81501

MODELING OF WEB SLIP ON A ROLLER AND ITS EFFECT ON WEB TENSION DYNAMICS

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ABSTRACT
This paper considers the effect of web slip over the rollers on the span tension dynamics. In classical development of the web span tension dynamics, it is assumed that there is strict adhesion between the web and the surface of the roller and thus, there is no slippage between the web and the roller. As a result of this assumption, effect of tension disturbances in downstream spans on the upstream span tension is precluded. However, in practice, perfect adhesion between the web and roller surface is seldom achieved and tension disturbances propagate upstream also. Though web span tension dynamic models that include slippage between web and roller are proposed, these models rely to a great extent on numerical computation of slip arc angles and are prohibitively complex to be of practical use. This paper proposes an alternative, simple approach for developing web span tension dynamics to include the effect of web slip.

NOMENCLATURE
A_i Nominal cross-sectional area of web
E Young's modulus of web material
$F_{i,f}$ Frictional force on i^{th} roller
$F_{i,n}$ Normal force on i^{th} roller
h_i Thickness of web
L_i Length of i^{th} span
m_i Mass of web in i^{th} span
R_i Radius of i^{th} roller/roll
t_i Tension in i^{th} span
v_i Velocity of web on i^{th} roller/roll
$v_{r,i}$ Peripheral velocity of i^{th} roller/roll
w_i Width of web
α_i Wrap angle on i^{th} roller
ε_i Longitudinal strain in i^{th} span
ρ_i Density of web material
ω_i Angular velocity of i^{th} roller/roll

1 INTRODUCTION
A web is any material which is manufactured and processed in a continuous, flexible strip form. Examples of web include paper, plastics, textiles, strip metals, and composites. Web handling refers to the physical mechanics related to the transport and control of web materials through processing machinery. Web processing allows us to mass produce a rich variety of products from a continuous strip material. Products that include web processing somewhere in their manufacturing include aircraft, appliances, automobiles, bags, books, diapers, boxes, newspapers, and many more. Modern manufacturing processes exploit the continuous nature of the basic material in web form by transporting it through and out of the process. In such processes, it is essential to maintain continuity and avoid cracks/breakage in the web. Though tests have been conducted to determine breaking strength of webs, it is found in practice that web-breaks occur

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even when the web tension is much less than the break tension determined under test conditions. There are two main reasons for web breakage: (i) the cracks could be the result of local stress concentrations. In the event of these stress concentrations, cracks may appear and propagate even at moderate overall web tension (ii) considerable variation of tension about the mean tension. Fatigue may set in when tension fluctuations are rapid and their amplitude is considerable. In general, web breakage is probably a result of a combination of these two effects.

The local stress concentrations may be avoided by improving the manufacturing processes to reduce the severity and density of irregularities. Such efforts fall under the purview of the design of manufacturing process and are specific to the product being manufactured. On the other hand, controlling web tension within a tight tolerance band is a common feature to manufacturing processes which involve material in web form at some stage of production. Thus, there is a definite need for the study of synthesis of web tension control systems. Before attempting to devise such control systems, it is essential to find out how tension disturbances occur and how they are propagated through the system.

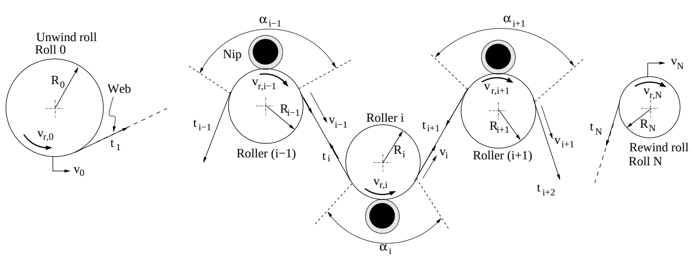


Figure 1. A SCHEMATIC SHOWING THE NOMENCLATURE

Figure 1 shows schematic of a web processing line. In this figure, the web is released from an unwind roll, transported over a series of idle/driven rollers, and is collected at a rewind roll. Various processes such as coating, perforation, and printing are performed at suitably located intermediate sections between the unwind and rewind rolls. Tension disturbances in such process lines may originate because of unevenness of unwind/rewind rolls, eccentricity of intermediate rollers, play in the bearings, etc. It can be visualized that such disturbances, originated in one part of the process line, *propagate* to the other parts along with the web transport. Since the web passes over the rollers as it is transported over them, the nature of contact between the web and the rollers plays an important role in the propagation of web tension disturbances. To ensure that motion of the driven rollers is completely imparted to the web material, nips are introduced at points of interest. Often, it may not be possible to introduce nip at a point of interest due to space constraints or the manufacturing process does not permit presence of nips. Also, it may not be possible to provide sufficient wrap angle to ensure adhesion of the web over the roller. In such cases, it is of importance to study the contact between the web and the roller and evaluate if there is slippage between the web and the roller. The phenomenon of motion transfer as a material is transported over a roller (pulley) was studied well in the context of belt drives [1–4]. The mechanics of belt drive was considered in [1] and it was concluded that the drive behavior is determined by the shear strain in the soft pliable envelop of the belt. A pair of pulleys connected by a belt was considered in [2] for analysis and it was shown, by a thermodynamic analysis of the system, that as the belt passes over the pulley, *slip* does not occur at entry point of contact. Calculation of slip arc angle and investigation of the dynamics of the belt/pulley frictional contact were reported in [3, 4].

Though the belt drives and/or band saw considered in [1–5] and the web processing lines have a semblance, namely a material passing over pulleys/rollers, they differ in some respects. The belt drives and band saw systems are closed-ended and the same belt loops around the pulleys whereas the web processing lines are open-ended with an unwind roll at one end and rewind roll at the other end. Also, most of the belt drives have side contact on the belts normal to the direction of belt travel while webs have surface contact parallel to the direction of web travel. Prediction of slip and rotational response of serpentine belt drive systems is considered in [3]. A study of band saw vibrations and the occurrence of instability is presented in [5]. This analysis assumed that the band velocity is constant. Dynamic models were developed for web process lines [6–9]. Modeling of a newspaper press is considered in [6] and it was concluded that for good printing, it is necessary to regulate the tension to within a few percent at all times, regardless of disturbances or irregularities. From then on, considerable attention is given to modeling web process lines and tension control. Dynamic models of papermaking and converting machinery and tension control schemes were reported in [7–9]. These models assumed perfect adhesion between the web and the roller. A rigorous mathematical background for including slippage between the web and the roller was reported in [10, 11]. Following this work, span tension dynamics, condition for slippage and computation schemes for slip arc angles were reported in [12, 13]. These analyses were too complex and are not amenable for use in a control scheme. This paper proposes a model of span tension dynamics to include web slippage. As an initial step, the case of web slipping over the entire contact area is considered and a static model of traction between the web and roller is used.

2 DYNAMICS OF WEB TENSION IN A FREE SPAN

Dynamics of web tension in a free span, as shown in Figure 1 are derived in [10–16] using conservation of mass principle which is summarized below. The dynamics of the web tension is derived in two stages: (i) the relation between the strain in web

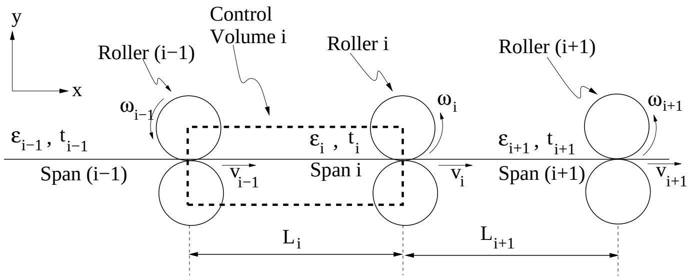


Figure 2. CONTROL VOLUME CONSIDERED FOR DERIVING WEB TENSION DYNAMICS

and velocity of web is derived considering a control volume and (ii) the relation between the strain and the web tension is written invoking the Hooke's law. To derive the relation between the strain and web velocity, consider the control volume i shown in Figure 2. This control volume is receiving material at a strain of ε_{i-1} and giving out material at a strain of ε_i. The principle of conservation of mass is applied to the control volume to say that, during any time interval $\Delta\tau$, the mass of material entering the control volume must be equal to the sum of mass accumulated in the control volume and the mass of material exiting the control volume. The mass of web in control volume i may be written as $m_{cv} = \int_0^{L_i} \rho_i(x,\tau)A_i(x,\tau)dx$. Let Δm_{cv} be the mass accumulated in the control volume during the $\Delta\tau$ time interval. During the same interval, the masses entering the control volume and leaving the control volume may be written as $m_{i-1} = \rho_{i-1}A_{i-1}v_{i-1}\Delta\tau$ and $m_i = \rho_i A_i v_i \Delta\tau$ respectively. Hence, the conservation of mass principle for any time duration $\Delta\tau$ may be written as,

$$\begin{aligned}\Delta m_{cv} &= \Delta\left[\int_0^{L_i} \rho_i(x,\tau)A_i(x,\tau)dx\right] \\ &= m_{i-1} - m_i \\ &= \rho_{i-1}A_{i-1}v_{i-1}\Delta\tau - \rho_i A_i v_i \Delta\tau.\end{aligned} \tag{1}$$

To introduce web strain into equation (1), we notice that the mass of the web in stretched state is equal to its mass in unstretched state. The mass of web in stretched state is $m_i = \rho_i A_i \Delta x_i$ and the mass of web in the unstretched state is $m_{i,u} = \rho_{i,u}A_{i,u}\Delta x_{i,u}$. Thus, we have $\rho_i A_i(\Delta x_i) = \rho_{i,u}A_{i,u}(\Delta x_{i,u})$ which implies $\dfrac{\rho_i A_i}{\rho_{i,u}A_{i,u}} = \dfrac{\Delta x_{i,u}}{\Delta x_i} = \dfrac{1}{1+\varepsilon_i}$. Using this relation for each term in (1), we obtain,

$$\Delta\left[\int_0^{L_i} \frac{\rho_{i,u}(x,\tau)A_{i,u}(x,\tau)}{1+\varepsilon_x(x,t)}dx\right] = \left[\frac{\rho_{i-1,u}A_{i-1,u}v_{i-1}}{1+\varepsilon_{i-1}} - \frac{\rho_{i,u}A_{i,u}v_i}{1+\varepsilon_i}\right]\Delta\tau \tag{2}$$

where the extra subscript, u, in the equation refers to unstretched state. To further simplify equation (2), we notice that the area of cross-section and density of the web are constant in each span. This is generally true for all web processes except the cases where the cross-sectional area is made to change intentionally as in drawing processes or hot rolling processes. Using this observation, dividing equation (2) throughout by $\Delta\tau$, and taking limit as $\Delta\tau \to 0$,

$$\frac{d}{d\tau}\left[\int_0^{L_i} \frac{1}{1+\varepsilon_x(x,t)}dx\right] = \frac{v_{i-1}}{1+\varepsilon_{i-1}} - \frac{v_i}{1+\varepsilon_i}. \tag{3}$$

Assuming the that the strain is uniform along the length of the web span ($\varepsilon_x(x,\tau) = \varepsilon_i$) we have,

$$\frac{d}{d\tau}\left[\frac{1}{1+\varepsilon_i}\int_0^{L_i} dx\right] = \frac{d}{d\tau}\left[\frac{L_i}{1+\varepsilon_i}\right] = \frac{v_{i-1}}{1+\varepsilon_{i-1}} - \frac{v_i}{1+\varepsilon_i}. \tag{4}$$

The last equation may be further simplified by noting that the strain in the strip is usually very small and consequently we may use an approximation $1/(1+\varepsilon) \approx (1-\varepsilon)$. Notice here that this approximation may be used in equation (3) either before evaluating the derivative or after evaluating the derivative, leading to two slightly varying span tension dynamics. These two variations are explained in detail in [17]. In what follows, the approximation, $1/(1+\varepsilon) \approx (1-\varepsilon)$, is used in the equation (3) first and then the derivative is evaluated to obtain,

$$L_i\frac{d\varepsilon_i}{d\tau} = v_i(1-\varepsilon_i) - v_{i-1}(1-\varepsilon_{i-1}). \tag{5}$$

At this point, we may note that the strain induced in the strip is due to tension in the strip. If we assume that the strip is purely elastic and obeys Hooke's law, we have $t_i = EA\varepsilon_i$ and thus, equation (5) is further simplified to

$$L_i\dot{t}_i = v_{i-1}t_{i-1} - v_i t_i + EA(v_i - v_{i-1}). \tag{6}$$

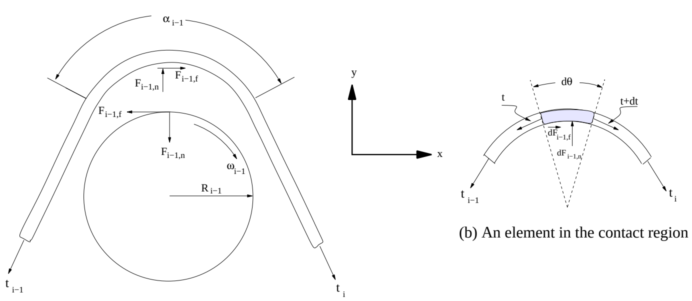
It may be noted here that v_{i-1} and v_i used in equation (6) are velocities of web material and not the roller peripheral velocities. When perfect adhesion between the web and the roller is not maintained, $v_{r,i} \neq v_i$. On the other hand, when there is perfect adhesion, the roller peripheral velocity is equal to the web velocity: $v_{r,i} = R_i\omega_i = v_i$ and the tension dynamics may be written as

$$L_i\dot{t}_i = v_{r,i-1}t_{i-1} - v_{r,i}t_i + EA(v_{r,i} - v_{r,i-1}). \tag{7}$$

Equation (7) indicates that, any disturbance in span tension t_{i-1} affects only the downstream tension. Thus, the span tension model (7) does not allow propagation of tension disturbances upstream. It can be seen, as the slip model given in subsequent sections present, that tension disturbances in one span propagate both upstream and downstream when the web slips on rollers.

3 SLIPPAGE WITHIN THE REGION OF WRAP

Consider Figure 3(a) which shows free-body diagram of web as it is slipping over the roller $(i-1)$. In Figure 3(a), $F_{i-1,f}$ is the effective frictional force and $F_{i-1,n}$ is the effective normal force. The direction of the frictional force is chosen to reflect the case $t_i < t_{i-1}$. The normal and frictional forces can be computed by considering an element in the contact region as shown in Figure 3(b). Equations (8)–(13) may be found in many standard text books on theory of machines. These equations are presented here for the sake of completeness.



(a) Friction force and the normal force

(b) An element in the contact region

Figure 3. (a) FRICTION AND NORMAL FORCES AND VELOCITIES AT ENTRY AND EXIT WHEN THE WEB SLIPS THROUGHOUT THE CONTACT REGION.(b) AN ELEMENT IN THE CONTACT REGION AND FORCES ACTING ON IT.

Force balance along the x and y directions for the element shown in Figure 3(b) gives

$$\begin{aligned} dF_{i-1,f} &= t\cos\left(\frac{d\theta}{2}\right) - (t+dt)\cos\left(\frac{d\theta}{2}\right) \\ &= -dt\cos\left(\frac{d\theta}{2}\right), \end{aligned} \tag{8}$$

$$\begin{aligned} dF_{i-1,n} &= (t+dt)\sin\left(\frac{d\theta}{2}\right) + t\sin\left(\frac{d\theta}{2}\right) \\ &= (2t+dt)\sin\left(\frac{d\theta}{2}\right). \end{aligned} \tag{9}$$

As $d\theta \to 0$, we note that $\sin(d\theta/2) \to d\theta/2$ and $\cos(d\theta/2) \to 1$. Ignoring the product $dt \cdot d\theta/2$, the force balance equations are written as

$$dF_{i-1,n} = t \cdot d\theta, \tag{10}$$

$$dF_{i-1,f} = -dt. \tag{11}$$

Integrating equation (11) over the entire slip region, we obtain,

$$F_{i-1,f} = t_{i-1} - t_i. \tag{12}$$

Using equations (10) and (11) in the equation $dF_{i-1,f} = \mu dF_{i-1,n}$, where μ is the coefficient of friction, we obtain, $dt/t = -\mu d\theta$. Integrating within the limits, we obtain,

$$\int_{t_{i-1}}^{t} \frac{dt}{t} = -\mu \int_0^{\alpha_{i-1}} d\theta \quad \Rightarrow \quad t = t_{i-1} e^{-\mu\alpha_{i-1}} \tag{13}$$

where μ is the static friction coefficient. Using equation (13) into (10) and integrating over the entire contact area, we obtain

$$\begin{aligned} F_{i-1,n} &= \int_{\text{contact}} dF_{i-1,n} = t_{i-1} \int_0^{\alpha_{i-1}} e^{-\mu\theta} d\theta \\ &= \frac{t_{i-1}}{\mu}\left[1 - e^{-\mu\alpha_{i-1}}\right] \end{aligned} \tag{14}$$

Equations (12) and (14), respectively give the effective friction force and the effective normal force when the web is slipping over the entire arc of contact. To determine the relation between the velocity of the web and the peripheral velocity of the roller ($v_{r,i} = R_i\omega_i$), a model of the traction between the web and the roller needs to be used. One such model that combines stiction, Coulomb friction and viscous friction is shown in Figure 4.

The model shown in Figure 4 can be expressed as

$$\begin{aligned} F_{i-1,f} = {} & a \cdot \text{sign}(v_{r,i-1} - v_{i-1}) + b \cdot (v_{r,i-1} - v_{i-1}) \\ & + c \cdot \delta(v_{r,i-1} - v_{i-1}) \end{aligned} \tag{15}$$

where

$$\delta(x) = \begin{cases} 1 & \text{if} \quad x = 0 \\ 0 & \text{otherwise} \end{cases}, \qquad \text{sign}(x) = \begin{cases} -1 & \text{if} \quad x < 0 \\ 0 & \text{if} \quad x = 0 \\ 1 & \text{if} \quad x > 0 \end{cases}, \tag{16}$$

$a = \mu' F_{i-1,n}$ is the Coulomb frictional force, b is the slope of the friction characteristics, and $c = \mu_0 F_{i-1,n}$ is the stiction force.

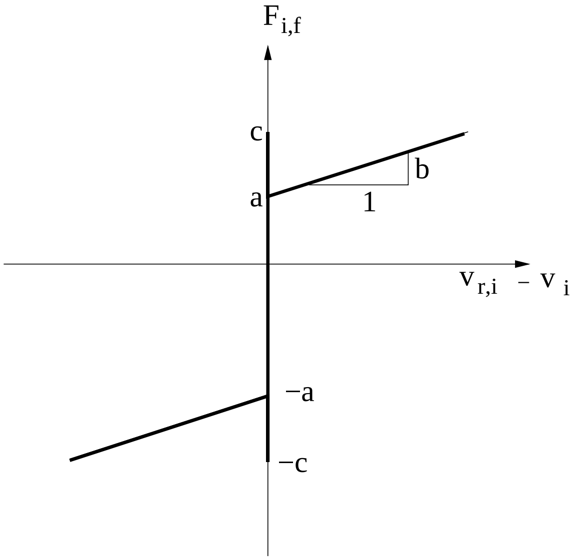


Figure 4. A MODEL OF TRACTION BETWEEN THE WEB AND THE ROLLER

Combining equations (12), (14), (15), and (16), we obtain

$$v_{i-1} = v_{r,i-1} - \frac{t_{i-1}}{b}\left[1 - p\left(1 - e^{-\mu\alpha_{i-1}}\right) - \frac{t_i}{t_{i-1}}\right] \qquad (17)$$

where $p = \mu'/\mu$.

Thus, the span tension dynamics, the roller dynamics, and the web velocity may be described by the following equations

$$\frac{d}{d\tau}\left(\frac{J_{i-1}v_{i-1}}{R_{i-1}}\right) = R_{i-1}(t_i - t_{i-1}) + u_{i-1} - b_{f,i-1}v_{r,i-1}, \qquad (18a)$$

$$v_{i-1} = v_{r,i-1} - \frac{t_{i-1}}{b}\left[1 - p\left(1 - e^{-\mu\alpha_{i-1}}\right) - \frac{t_i}{t_{i-1}}\right], \qquad (18b)$$

$$L_i\dot{t}_i = v_{i-1}t_{i-1} - v_i t_i + EA(v_i - v_{i-1}). \qquad (18c)$$

In equations (18), $u_{i-1} = 0$ corresponds to an idle roller.

4 AN EXAMPLE

Consider the free span shown in Figure 5. The tension dynamics for span 2 may be written using equation (6) as

$$L_2\dot{t}_2 = v_1 t_1 - v_2 t_2 + EA(v_2 - v_1). \qquad (19)$$

It is assumed that the web is not slipping over the roller 1 so that $v_1 = v_{r,1}$ and that the web is slipping over the roller 2 so that $v_{r,2} \neq v_2$. The velocity of the web moving over the roller 2 may be obtained by using equation (17).

$$v_2 = v_{r,2} - \frac{t_2}{b}\left[1 - p\left(1 - e^{-\mu\alpha_2}\right) - \frac{t_3}{t_2}\right]. \qquad (20)$$

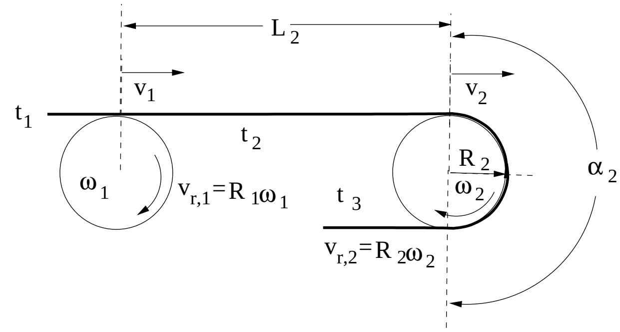


Figure 5. A FREE SPAN

EA	177920 N	L_2	3.05 m
b	26244 N-s/m	α_2	π rad
$\mu = \mu_0$	0.1	μ'	0.02
t_1	186.8 N	$t_2(0-)$	186.8 N
$t_3(0-)$	186.8 N	$v_1(0-)$	5.08 m/s
$v_1(0+)$	5.58 m/s	$v_{r,2}(0-)$	5.58 m/s
$v_{r,2}(0+)$	5.085 m/s		

Table 1. NUMERICAL VALUES USED FOR SIMULATION

The velocity of web given by the last equation above may be used in (6) to include the effect of slippage. Notice that a disturbance in tension t_3 affects the web velocity v_2 given by equation (20) which in turn affects web tension t_2 given by equation (19). Thus, tension disturbances downstream of the roller affect the dynamics of the upstream dynamics when there is slippage on the roller.

To highlight the effect of slip, a step change in the reference velocities of rollers 1 and 2 in Figure 5 is considered and the response of tension t_2 is computed and plotted as shown in Figure 6. Numerical values used in the simulation are shown in Table 1.

Figure 6 shows three plots:

1. The dashed line shows the response of t_2 ignoring the slippage, $v_{r,1} = v_1$. When there is no slippage at both the rollers, simultaneous change in the velocity of both the rollers (from 5.08 m/s to 5.58 m/s) does not change the tension. It may further be noticed that any tension disturbance in the down-

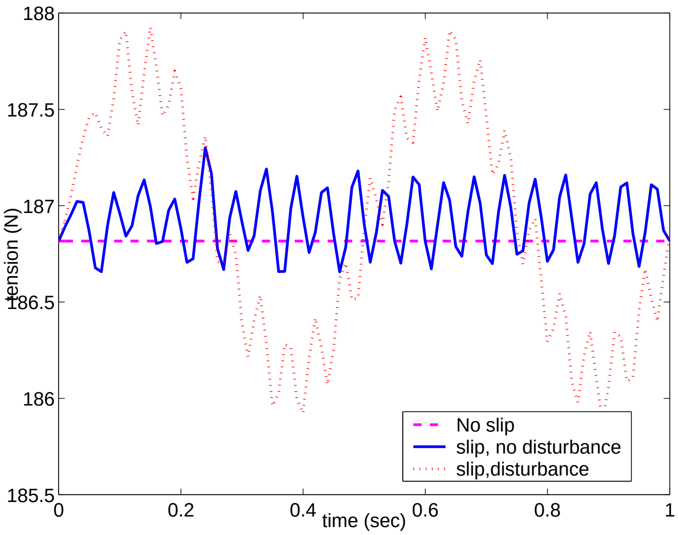


Figure 6. RESPONSE OF TENSION t_2 FOR A STEP CHANGE IN $v_{r,2}$ IN FIGURE 5

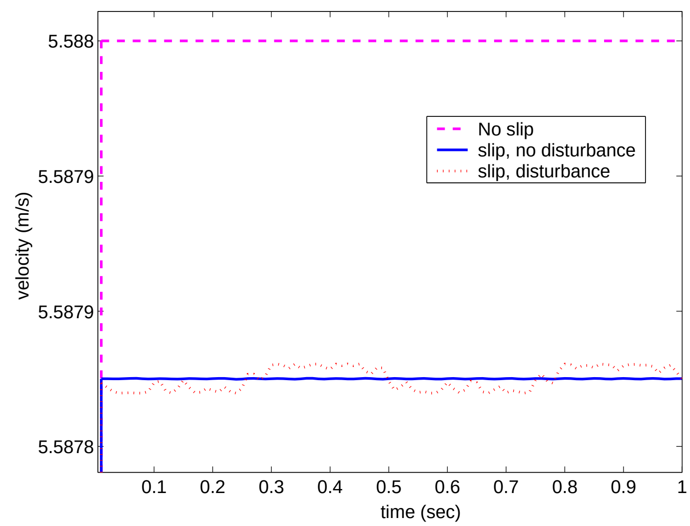


Figure 7. WEB VELOCITY FOR A STEP CHANGE IN $v_{r,2}$ IN FIGURE 5

stream tension t_3 does not affect t_2 since the span tension model without slippage (equation (19)) is independent of t_3.

2. The solid line shows the response of t_2 considering slippage at roller 2, $v_{r,1} = v_1$ and $v_{r,2} \neq v_2$. In this case, the actual web velocity computed from (20) is used in (19) to compute the response of t_2. As seen in Figure 6, variations in tension are seen even when no disturbance is assumed in tension t_3.
3. The dotted line shows the response considering slippage at roller 2 and also a tension disturbance in span 3, $t_3 = 186.8 + 44.48\sin(4\pi t)$. Equations (19) and (20) are used to obtain the response of t_2. When the web slips at all points of contact in the contact region, any tension disturbances in t_3, affect the span tension t_2, as shown in Figure 6.

Figure 7 shows the velocity of the web on roller 2 for the three cases of simulation considered above. The dashed line shows the velocity of web on roller 2 when no slippage is assumed on roller 2. In this case, the web velocity follows the roller velocity accurately. The solid line shows the case when slippage is assumed on roller 2. It is seen that there is a drop in the web velocity due to slippage. Though the web velocity appears to be almost *flat* in this case, there are very small variations as shown in Figure 8. It is interesting to see that such small variations shown by solid line in Figure 8 result in considerable variations in the tension. This is due to the fact that the small velocity variations get multiplied by EA as they appear in tension dynamics given by (19). The dotted line in Figure 7 shows the web velocity when slippage is considered on roller 2 and a disturbance in t_3 is considered. It is seen that there are perceptible web velocity variations due to the disturbance in t_3.

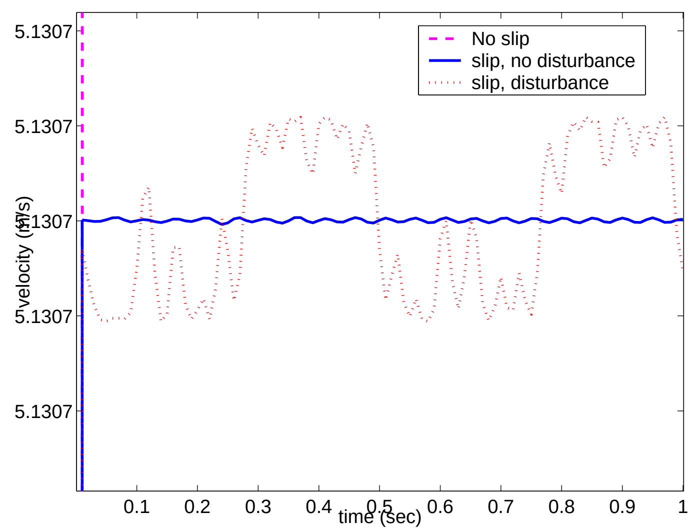


Figure 8. WEB VELOCITY VARIATIONS DUE TO SLIP

5 DISCUSSION ON TENSION IN CONTACT AREA

In general, the area of contact between the web and the roller could be divided into three regions [10] as shown in Figure 10. The boundaries between these regions are defined by the angles γ_{i-1} and β_{i-1} which are measured from the entry point in the direction of the roller. These three regions are defined as

1. $0 \leq x \leq R_{i-1}\gamma_{i-1}$ - an entry region of slip. In this region, the tension is a function of both the distance from entry point of contact and time.
2. $R_{i-1}\gamma_{i-1} \leq xR_{i-1}\beta_{i-1}$ - a central region of active adhesion where the tension is imparted into the material.

3. $R_{i-1}\beta_{i-1} \leq xR_{i-1}\alpha_{i-1}$ - an exit region of slip. Again, in this region the tension is a function of both the distance from entry point of contact and time.

The existence and spread of these three regions depends on whether the tension $t_{i-1}(t)$ changes quickly or slowly. It is shown that [12], if the tension satisfies the relation

$$\left|\frac{dt_{i-1}}{d\tau}\right| < \frac{\mu\, v_{r,i-1}}{R_{i-1}}\, t_{i-1}, \tag{21}$$

the entry region of slip disappears and only regions of adhesion and exist region of slip exist as shown in Figure 9. When equa-

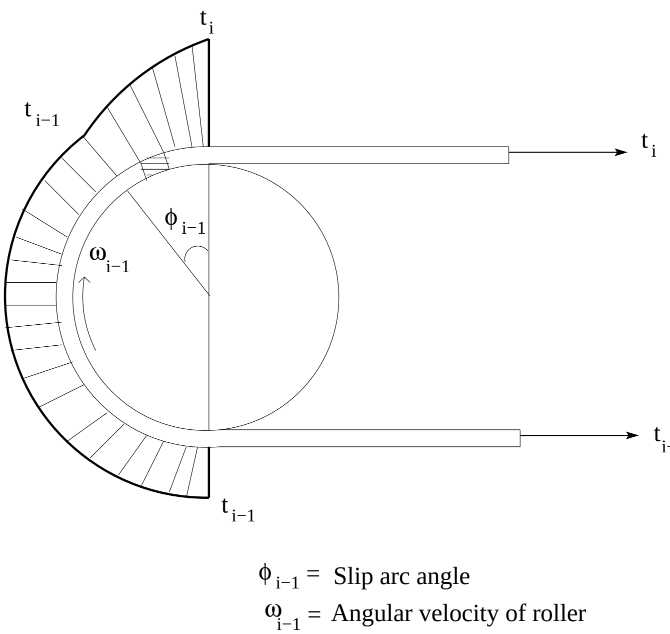


Figure 9. SLIP UNDER THE INFLUENCE OF SLOWLY VARYING TENSION

tion (21) is satisfied, equations (8)– (17) may still be used to find the velocity of the web; however, the angle ϕ_{i-1} needs to be used in place of α_{i-1}. The method of computing the angle ϕ_{i-1} is illustrated in [18].

When the span tension does not satisfy the condition (21), analyzing the contact area is a complicated problem and a closed form solution for the slip arc angles is difficult to obtain. If the tensions in spans adjacent to the roller vary rapidly to violate the condition (21), the entry and the exit regions may spread over the entire area of contact and the region of adhesion may disappear altogether. This situation corresponds to the case of web slipping throughout the region of contact as considered in this paper.

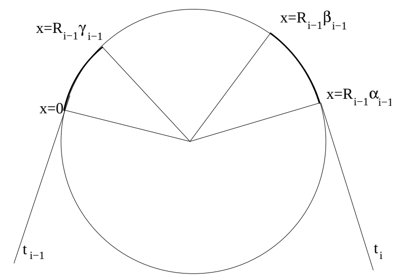


Figure 10. THREE REGIONS OF CONTACT

6 SUMMARY AND FUTURE WORK

This paper presented a model of span tension dynamics to include the slippage between the web and the roller. This model assumes that the web slips over the entire region of wrap on the roller. It is shown through the model that tension disturbances propagate not only downstream but also upstream if there is slippage on rollers. Also, simulation on a single span shows that the tension variations, due to a step input of roller reference velocity, are smaller when there is slippage between the web and the roller.

When the web tension is slowly varying, the web may not slip over the entire region of wrap; there will be a region of adhesion followed by a region of slip as shown in [12, 13]. In such cases also, the method shown in this paper can be applied by replacing the angle of wrap with the *slip arc angle*.

However, when the web span tensions on either side of the roller are varying fast, the contact between web and roller may be divided into three regions: (i) an entry region of slip, (ii) a region of adhesion, and (iii) an exit region of slip. Methods of computing the entry slip angle, exit slip angle, conditions under which each of the three regions exist, and the effect of these regions on the span tension dynamics is reserved for future work.

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