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An Application of Torsional Wave Analysis to Turbogenerator Rotor Shaft Response

In this paper transient torsional vibrations of a steam turbogenerator rotor shaft system due to high speed reclosing of the electric network are investigated. The analysis is performed using torsional elastic wave theory applied to a continuous model in the form of a stepped shaft. Wave solutions of the equations of motion are used in order to determine dynamic torsional elastic moments and vibratory angular velocities in cross-sections of the turbine shafts. The results are illustrated in the form of graphs.

Introduction

Transient torsional vibrations of a steam turbine rotor shaft system due to short circuits in generators and ground faults in the electric network are a source of severe dynamic loads on the turbine shafts and couplings. These loads can lead to material fatigue of the elements of the turbine system. Thus, for the last few years this important phenomenon, typical for modern high power steam turbogenerators, became a subject of investigation by many authors (Hizume, 1975; Bernasconi, 1986; Schwibinger et al., 1986; Schwibinger et al., 1987; Rubio et al., 1987).

At present, much effort is expended to develop sufficiently accurate and numerically efficient methods for the investigation of this problem. An essential factor is the selection of an appropriate mechanical model. The majority of authors used a discrete model of the turbogenerator rotor shaft system (Hizume, 1975; Schwibinger et al., 1986; Schwibinger et al., 1987; Rubio et al., 1987). But the turbogenerator rotor shaft system is characterized by large shaft and rotor masses continuously distributed along their axis of rotation. Thus, in order to obtain sufficiently accurate results, a discrete model with many degrees of freedom and appropriate parameter identification is applied (Schwibinger et al., 1986; Schwibinger et al., 1987).

Because of the mentioned continuous distribution of masses of the turbogenerator shafts and rotors, discrete-continuous models also were introduced (Bernasconi, 1986). But an application of methods of forced vibration analysis for discrete-continuous and continuous models used so far often lead to calculation difficulties. Thus, the majority of considerations usually were limited to free vibrations (Rao, 1978; Bernasconi, 1986).

An application of the one-dimensional elastic wave propagation theory to forced vibration analysis using continuous and discrete-continuous models simplifies the mathematical procedure and improves its numerical efficiency. For example, the torsional elastic wave propagation theory was applied by Szolc (1985) and Nadolski et al. (1986) for dynamic investigations of discrete-continuous models of crank mechanisms of the internal combustion engines.

In comparison with methods of torsional vibration analysis of the steam turbogenerator rotor shaft systems used so far, an alternative approach to this problem is proposed in the present paper. Elastic wave theory is used to analyze the onset of transient torsional vibration of a turbogenerator rotor shaft system due to high speed reclosing of the electric network.

Assumptions

The steam turbogenerator rotor shaft system is a complex set of elements with more or less complicated geometrical shapes. Turbine shafts usually have the form of stepped shafts; however, rotors are characterized by more complicated geometrical shapes (Schwibinger et al., 1987). Geometrical shapes of individual segments as well as their material constants, i.e., density ρ and shear modulus G, determine distributions of the mass moment of inertia and torsional flexibility of the considered system along its rotation axis. An important factor in dynamic modeling of mechanical systems for torsional vibration analysis is for the model to accurately approach the real distribution of the mass moment of inertia and torsional flexibility along the system rotation axis.

In the paper, a continuous model of the turbogenerator rotor shaft system in the form of a stepped shaft is proposed. In this model, turbine shafts and individual segments with more complicated geometrical shapes are represented by an appropriate number of equivalent torsionally deformable cylindrical elastic elements with continuously distributed parameters. These elements are characterized by lengths l_i and cross-sectional polar moments of inertia J_i, $i = 1, 2, \ldots, n$, where n denotes the total number of the elastic elements in the considered continuous model (Fig. 1).

Figure 1 represents a continuous model of the typical turbogenerator rotor shaft system with two low-pressure rotors.

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The elastic elements (m), $(i_a) \rightarrow (i_c)$ and $(i_d) \rightarrow (i_f)$ represent the generator and both of the low-pressure rotors, respectively. The elastic elements $(i_g) \rightarrow (i_j)$ correspond to the medium pressure rotor, and the elastic elements $(i_k) \rightarrow (i_n)$ correspond to the high pressure rotor. The remaining elements represent turbine shafts and couplings. The x-axis is parallel to the common symmetry axis of the elastic elements and its origin is to the left of elastic element (1). It is assumed that distributions of the model mass moment of inertia and torsional flexibility along the elastic element symmetry axis accurately represent their real distributions along the axis of rotation.

The excitation torque $T_m(t)$ due to high speed reclosing of the electric network is uniformly distributed along the elastic element (m) representing the generator rotor. In the case of high speed reclosing, the excitation torque function is assumed to be a superposition of the so-called "step and slow shake" function and attenuating sinusoidal functions at the system's rotational frequency Ω (Hizume, 1975). As it was mentioned by Hizume (1975), the "step and slow shake" function originates from the fact that an active electromagnetic torque in the generator comes about as a result of successive automatic openings and reclosings of the powered electric circuits due to ground faults. Superimposed on this an oscillation of the generator rotor angular velocity causes an additional electromagnetic torque fluctuation, since the mechanical torque transmitted from the turbine remains temporarily constant. Thus, the "slow shake" component of the excitation electromagnetic torque is generated in the form of a harmonic function of the frequency $\Omega_1 = 1.1 - 1.3$ [Hz]. However, attenuating sinusoidal components of the excitation torque function $T_m(t)$ originate from operating the generator armature current, which takes place in order to satisfy continuity of the electric current.

The resultant excitation torque function $T_m(t)$ for successive time intervals of the "step and slow shake" function can be assumed in the following analytical form:

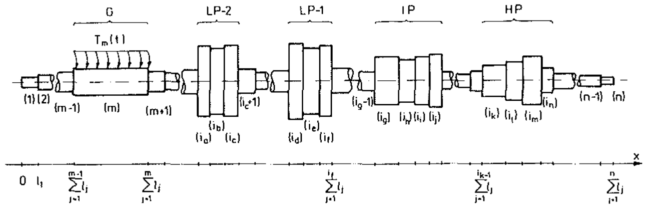

Fig. 1 Continuous model of the turbogenerator rotor shaft system

According to Hizume (1975), for high speed reclosing following a 3-phase ground fault in the power transmission line, t_1 and $t_3 - t_2$ correspond to 7 cycles each, and $t_2 - t_1$ corresponds to 18 cycles of the normal generator operation with the rated angular velocity Ω. Moreover, the maximum magnitude of the "step" function is 1.39 times as large as the rated torque transmitted to the generator, the maximum magnitude of the peak of "slow shake" function is 0.88 times, and the maximum amplitude of the attenuating sinusoidal function is 2.66 times. Figure 2 presents a plot of the assumed excitation torque function $T_m(t)$ at the rated torque M_{01} for $\Omega = 377$ [rd/s] (corresponding to the frequency $f = 60$ [Hz] of the produced electric current). Because a dynamic analysis is performed, constant torques transmitted through the turbine shafts, causing only their constant rated angular velocity to change, are not considered.

In the model presented here an equivalent damping of the viscous type is introduced in the form of damping moments imposed on the extreme cross-sections of the stepped shaft elastic elements (Szolc, 1985; Nadolski et al., 1986).

Formulation of the Problem

For further consideration it is convenient to introduce the following nondimensional quantities

$$T_m(t) = \begin{cases} c_1 + a_1 e^{-\delta t} \sin\Omega t & \text{for } 0 \leqq t < t_1, \\ c_2 + b \sin\Omega_1(t-t_1) + a_2 e^{-\delta(t-t_1)} \sin\Omega(t-t_1) & \text{for } t_1 \leqq t < t_2, \\ c_3 + a_3 e^{-\delta(t-t_2)} \sin\Omega(t-t_2) & \text{for } t_2 \leqq t < t_3, \\ b \cos\Omega_1(t-t_3) + a_4 e^{-\delta(t-t_3)} \sin\Omega(t-t_3) & \text{for } t \geqq t_3, \end{cases} \quad (1)$$

where t denotes time, t_j—time instants determining successive intervals of the "step and slow shake" function, c_j—constant values of the "step" function ($j = 1, 2, 3$), a_i—initial amplitudes of the attenuating sinusoidal functions ($i = 1, 2, 3, 4$), b—amplitude of the "slow shake" function, $\delta = \vartheta\Omega$, and ϑ denotes attenuation constant of the generator.

$$\bar{x} = \frac{x}{l_s},\ \lambda_i = \frac{l_i}{l_s},\ \tau = \frac{ct}{l_s},\ \bar{\Theta}_i(\bar{x},\tau) = \frac{\Theta_i(x,t)}{\Theta_s},\ i = 1,2,\ldots,n, \quad (2)$$

where $\Theta_i(x, t)$ denotes perturbation of the angular displacement of the ith elastic element cross-section from the shaft

Nomenclature

a_i = initial amplitudes of the attenuating sinusoidal functions
b = amplitude of the "slow shake function"
c_j = constant values of the "step function"
D_k = equivalent damping coefficients
f_i, g_i = functions representing torsional waves propagating in the individual elastic elements
G = shear modulus
$H(k)$ = Heaviside function
J_i = cross-sectional polar moments of inertia of the individual elastic elements
l_i = lengths of the model elastic elements
m = number of the model elastic element representing the generator rotor
M_{01} = rated torque transmitted from the turbine to the generator
n = total number of elastic elements in the continuous model
t = time
t_j = time instants determining time intervals of the "step and slow shake function"
$T_m(t)$ = excitation torque function
x = spatial coordinate
z = dimensionless argument
Ω = rated angular velocity of the rotor shaft system
Ω_1 = frequency of the "slow shake function"
ρ = material density
Θ = angular displacement of the elastic element cross-section
ϑ = attenuation constant of the generator
i, j, k, l = subscripts

uniform motion with the constant rated angular velocity Ω, $c = \sqrt{G/\rho}$ and
l_s [m], Θ_s [rd] are arbitrary values.
Equations of motion for cross-sections of the individual elements are partial differential equations of the wave propagation-type which, upon introducing (2), are obtained in the nondimensional form

$$\frac{\partial^2\overline{\Theta}_i(\overline{x},\tau)}{\partial\tau^2}-\frac{\partial^2\overline{\Theta}_i(\overline{x},\tau)}{\partial\overline{x}^2}=0,$$

$$i=1,2,\ldots,m-1,m+1,m+2,\ldots,n, \quad (3)$$

$$\frac{\partial^2\overline{\Theta}_m(\overline{x},\tau)}{\partial\tau^2}-\frac{\partial^2\overline{\Theta}_m(\overline{x},\tau)}{\partial\overline{x}^2}=q(\tau),$$

where

$$q(\tau)=\frac{T_m(t)l_s^2}{GJ_ml_m\Theta_s}.$$

Equations (3) are solved with homogeneous initial conditions

$$\overline{\Theta}_i(\overline{x},\tau)=0,\ \frac{\partial\overline{\Theta}_i(\overline{x},\tau)}{\partial\tau}=0 \quad \text{for } \tau=0,\ i=1,2,\ldots,n, \quad (4)$$

and with following boundary conditions

$$\frac{\partial\overline{\Theta}_i(\overline{x},\tau)}{\partial\overline{x}}=0 \quad \text{for } i=1,\ \overline{x}=0 \quad \text{and} \quad i=n,\ \overline{x}=\sum_{j=1}^{n}\lambda_j,$$

$$K_k\frac{\partial\overline{\Theta}_k(x,\tau)}{\partial\overline{x}}-K_{k-1}\frac{\partial\overline{\Theta}_{k-1}(\overline{x},\tau)}{\partial\overline{x}}-\overline{D}_k\frac{\partial\overline{\Theta}_k(\overline{x},\tau)}{\partial\tau}=0 \quad (5)$$

and $\overline{\Theta}_{k-1}(\overline{x},\tau) = \overline{\Theta}_k(\overline{x},\tau)$ for $\overline{x} = \sum_{j=1}^{k-1}\lambda_j$, $k = 2, 3, \ldots, n$,

where $K_k = \dfrac{GJ_kl_s}{c^2I_s}$, $\overline{D}_k = D_k\dfrac{l_s}{cI_s}$ and I_s [kgm^2] is an arbitrary value. Solutions of Eqs. (3) are sought in the form of wave solutions

$$\overline{\Theta}_i(\overline{x},\tau)=f_i\left(\tau-\overline{x}-\sum_{j=1}^{m-1}\lambda_j+2\sum_{j=1}^{i}\lambda_j\right)$$
$$+g_i\left(\tau+\overline{x}-\sum_{j=1}^{m-1}\lambda_j\right),\ i=1,2,\ldots,m-1,$$

$$\overline{\Theta}_m(\overline{x},\tau)=f_m\left(\tau-\overline{x}+\sum_{j=1}^{m-1}\lambda_j\right)+g_m\left(\tau+\overline{x}-\sum_{j=1}^{m-1}\lambda_j\right)+F(\tau),$$

$$\overline{\Theta}_k(\overline{x},\tau)=f_k\left(\tau-\overline{x}+\sum_{j=1}^{m}\lambda_j\right)$$
$$+g_k\left(\tau+\overline{x}-\sum_{j=1}^{m}\lambda_j-2H(k-m-1)\sum_{j=m+1}^{k-1}\lambda_j\right),$$
$$k=m+1,\ m+2,\ \ldots,\ n, \quad (6)$$

where $F(\tau) = \int_0^\tau (\tau-\nu)\ q(\nu)\ d\nu$ and $H(k)$ is the Heaviside function.

The functions f_j and g_j, $j = 1, 2, \ldots, n$, represent torsional waves caused by the excitation torque, where the function f_j represents a torsional wave propagating in the jth elastic element along the x-axis positive sense (Fig. 1); however, the function g_j represents a torsional wave propagating along the x-axis negative sense. According to the one-dimensional wave propagation theory, it is taken into account in (6) that the first perturbation in an arbitrary cross-section of the stepped shaft occurs after appropriate finite time instant. Furthermore, it is assumed that the functions f_j and g_j are continuous and are

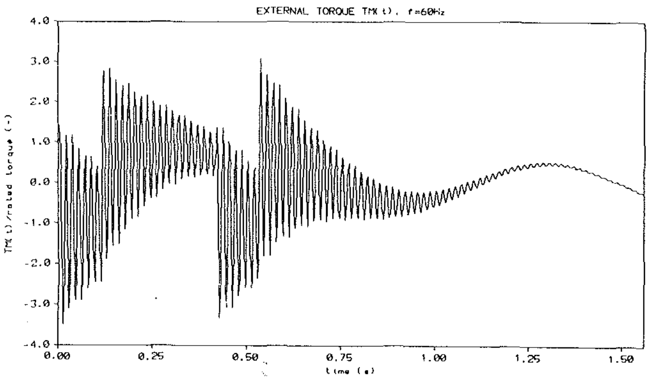

Fig. 2 Plot of the excitation torque function for Ω = 377 [rd/s]

null for negative arguments, i.e., before arriving the first perturbation.

By substituting the wave solutions (6) into the boundary conditions (5), denoting the largest argument in each equation by z, and by rearranging these equations in such a way that their right hand sides are always known, we obtain the following system of ordinary differential equations of the first order with a "shifted argument" for the functions f_j and g_j, $j = 1, 2, \ldots, n$

$$f_1'(z)=g_1'(z-2\lambda_1),$$

$$f_i'(z)=H(m-2)[-g_i'(z-2\lambda_i)+f_{i-1}'(z-2\lambda_i)+g_{i-1}'(z-2\lambda_i)],$$
$$i=2,3,\ldots,m-1,$$

$$g_j'(z)=-f_j'(z-2\lambda_j)+f_{j+1}'(z-2\lambda_j)+g_{j+1}'(z-2\lambda_j),$$
$$j=m+1,m+2,\ldots,n-1,$$

$$g_n'(z)=f_n'(z-2\lambda_n),$$

$$g_m'(z)=-f_m'(z-2\lambda_m)+f_{m+1}'(z-\lambda_m)+g_{m+1}'(z-\lambda_m)$$
$$-V(z-\lambda_m),$$

$$f_m'(z)=s_1V(z)+r_{2m}g_m'(z)+r_{3m}f_{m-1}'(z),$$

$$f_{m+1}'(z)=s_2V(z)+r_{2,m+1}g_{m+1}'(z)+r_{3,m+1}f_m'(z-\lambda_m),$$

$$g_{m-1}'(z)=-f_{m-1}'(z)+f_m'(z)+g_m'(z)+V(z),$$

$$g_l'=H(m-2)[r_{1l}f_l'(z)+r_{4l}g_{l+1}'(z)],\quad l=m-2,m-3,\ldots,1,$$

$$f_k'(z)=r_{2k}g_k'(z)+r_{3k}f_{k-1}'(z),\quad k=m+2,m+3,\ldots,n, \quad (7)$$

where

$$V(z)=\frac{d}{dz}F(z),\quad s_1=-\frac{K_{m-1}+\overline{D}_m}{K_{m-1}+K_m+\overline{D}_m},$$

$$s_2=\frac{K_m}{K_m+K_{m+1}+\overline{D}_{m+1}},\quad r_{1l}=H(m-2)\frac{K_l-K_{l+1}-\overline{D}_{l+1}}{K_l+K_{l+1}+\overline{D}_{l+1}},$$

$$r_{4l}=H(m-2)\frac{2K_{l+1}}{K_l+K_{l+1}+\overline{D}_{l+1}},\quad l=1,2,\ldots,m-2,$$

$$r_{2k}=\frac{K_k-K_{k-1}-\overline{D}_k}{K_k+K_{k-1}+\overline{D}_k},\quad r_{3k}=\frac{2K_{k-1}}{K_k+K_{k-1}+\overline{D}_k},$$

$$k=m,m+1,\ldots,n.$$

Solving the above system of equations and using (6), we can obtain dynamic angular displacements and vibratory velocities as well as torsional strains or torsional elastic moments in any cross-section of the stepped shaft elastic elements for arbitrary time instants. Because the right hand sides of Eqs. (7) are always known, it is possible to solve each of them numerically in the sequence determined in (7). This fact, in comparison with analogous systems of coupled ordinary differential equations for discrete models, essentially increases the numerical efficiency of the presented method. Moreover, when only vibratory angular velocities and torsional strains or torsional

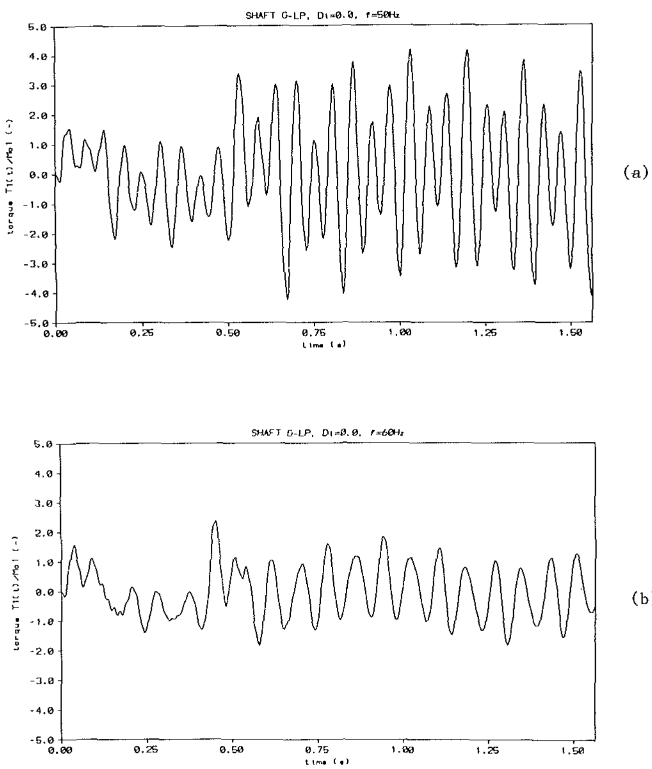

Fig. 3 Torsional elastic moment in the shaft connecting the generator with the low-pressure rotor for: (*a*) $\Omega = 314$ [rd/s] and (*b*) $\Omega = 377$ [rd/s]

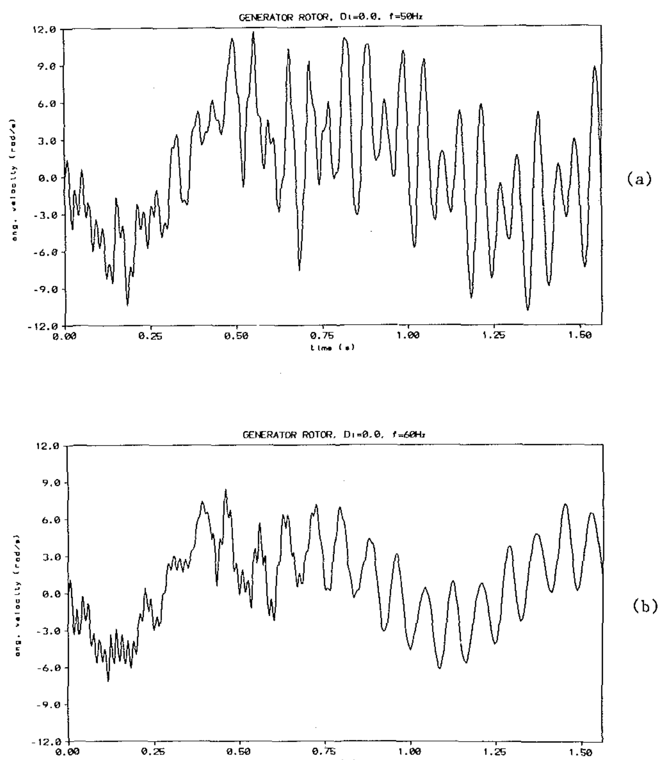

Fig. 4 Vibratory angular velocity of the generator rotor for: (*a*) $\Omega = 314$ [rd/s] and (*b*) $\Omega = 377$ [rd/s]

elastic moments are needed, Eqs. (7) are solved with regard to the first derivatives of the functions $f_j(z)$ and $g_j(z)$, $j = 1, 2, \ldots, n$. Thus, the problem reduces to solving $2n$ algebraic equations in the determined sequence simplifying the numerical procedure, even further.

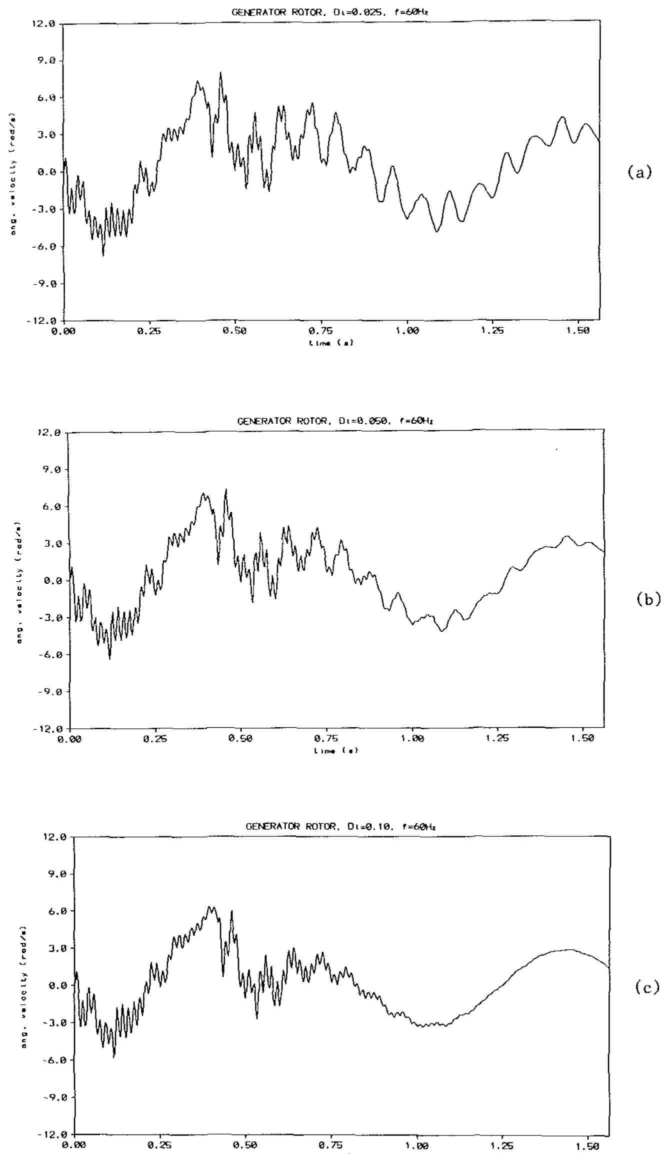

Fig. 5 Vibratory angular velocity of the generator rotor for $\Omega = 377$ [rd/s] and (*a*) $\overline{D}_i = 0.025$, (*b*) $\overline{D}_i = 0.05$, (*c*) $\overline{D}_i = 0.1$

Numerical Results

Numerical calculations were performed for a system consisting of a 500 MW steam turbogenerator with one low-pressure rotor. It was assumed, that the presented continuous model of this turbogenerator rotor shaft system consists of $n = 10$ elastic elements and $m = 2$.

In the first part of the numerical example, undamped transient vibrations were considered, where $D_i = 0$, $i = 2, 3, \ldots, 10$, for two values of the shaft rated angular velocity—$\Omega = 314$ [rd/s] and $\Omega = 377$ [rd/s]—corresponding to the frequency of the produced electric current $f = 50$ [Hz] and $f = 60$ [Hz], respectively. It was assumed, that in both cases rated torques transmitted through the individual turbogenerator rotor shafts were identical, and remaining parameters of the considered system were also the same. Thus, in these two cases of the value Ω, the system was forced to vibrate by different excitation torques due to high speed reclosing of the electric network. These excitation torques differ in the frequency Ω and the attenuating constant δ of the sinusoidal functions as well as in the time intervals t_1, $t_2 - t_1$ and $t_3 - t_2$ of the "step and slow shake" function.

Figure 3 presents plots of the torsional elastic moment versus time in the shaft connecting the generator with the low pressure rotor at a given rated torque M_{01} for $\Omega = 314$ [rd/s] and for $\Omega = 377$ [rd/s]. Figure 4 presents plots of vibratory angular velocity versus time of the left extreme cross-section of the elastic element (2) representing the generator rotor for the same values of Ω. From the presented plots as well as from analogous results of torsional elastic moments in the remaining turbine shafts, it follows that essentially greater peak values of torsional elastic moments and vibratory angular velocities were obtained for $\Omega = 314$ [rd/s].

In the second part of the numerical example, damped transient torsional vibrations due to high speed reclosing of the electric network were investigated. Calculations were performed for four assumed values of the dimensionless equivalent damping coefficients: $\bar{D}_i = 0.01, 0.025, 0.05$, and 0.1, $i = 2, 3, \ldots, 10$, for $\Omega = 377$ [rd/s]. Figure 5 presents plots of vibratory angular velocity versus time of the left extreme cross-section of the elastic element (2), representing the generator rotor, for $\bar{D}_i = 0.025, 0.05$, and 0.1. In comparison with the case of undamped vibrations [$\bar{D}_i = 0$, $i = 2, 3, \ldots, 10$, Fig. 4(*b*)], the equivalent damping in the model causes a successive reduction of local extreme values as well as an eventual extinction of the transient state components. As it follows from the plots in Fig. 5, the influence of damping is more evident for greater values of $\bar{D}_i$.

Final Remarks

The work presented here constitutes a preliminary study in which the torsional elastic wave propagation theory was applied for dynamic analysis of the steam turbogenerator rotor shaft system.

In the paper a continuous model of the steam turbogenerator rotor shaft system was considered. Using this model, transient torsional vibrations due to high speed reclosing of the electric network were investigated. In contrast to discrete models of the discussed system generally applied so far, the continuous model in form of stepped shaft as well as the interpretation of the investigated dynamic phenomenon in terms of propagating waves lead to better understanding of this problem. Moreover, relatively simple and clear mathematical relations were obtained. An application of the torsional wave propagation theory allows to solve sequentially the coupled differential equations with a "shifted argument." For torsional elastic moments and vibratory angular velocity calculations, the problem reduces to a set of algebraic equations. Thus, a great numerical efficiency of the presented procedure is achieved, making it particularly advantageous from a practical standpoint.

In the numerical examples, influence of the excitation torque parameters and of the equivalent damping coefficients on the system dynamic response was presented. It was shown that, in case of high speed reclosing, the turbogenerator rotor shaft system is more sensitive to torsional vibrations for the rated angular velocity $\Omega = 314$ [rd/s] than for $\Omega = 377$ [rd/s]. Nevertheless, the obtained results require experimental verification.

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