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CONSTRAINT REDUNDANCY IN MOBILITY OF PARALLEL MANIPULATORS

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Abstract: This paper presents a systematic approach to obtain the degrees of freedom (DOF) of the platforms of parallel manipulators. The paper begins with general Kutzbach criterion for mobility. With simple mathematical transformations this criterion is modified to incorporate number of parallel legs used in the parallel platform-type mechanism and the number of joints in the legs. The theory of screws is used to study the freedom of the joints in the individual legs and the mobility of the platform. It is established that the general Kutzbach mobility criterion does not cater for situations where the freedom screws (or constraint screws) of the joints in a leg become dependent on the freedom screws (or constraint screws) of one or more of the other legs; thus, altering the mobility of the platform. The general modified Kutzbach mobility formula is further modified to resolve the problem of redundant constraints. The paper then provides a systematic approach towards the number synthesis of parallel platform-type mechanims. The paper includes three examples of such mechanisms analyzed by this approach. Results agree with the existing studies carried out on the mechanism used in the examples. A numerical example of a three-degree-of-freedom parallel manipulator with three legs is used to show the enumeration of all possible parallel manipulators. This includes cases with and without redundant constraints.

Key words: mobility, parallel manipulators, constraints, redundant constraints, degrees of freedom

1. Introduction

Intrinsic to their design parallel manipulators have a higher rigidity, better accuracy, and more load carrying capacity than serial mechanisms. Thus these mechanism find wide spread practical applications; flight simulators, vehicle simulators, and entertainment simulators are just a few examples. A large number of references can be found in published literature [1,2,3]. A general six DOF motion simulator having one actuator in each leg possesses six legs with six freedoms (or six single DOF joints) in each leg. In a number of such platform-type simulators where full motion (i.e., six DOF) is not needed the number of legs is also reduced to desired level with appropriately reduced number of joints in legs.

This paper provides, first; a strategy to design a parallel manipulator for a given platform mobility: Second; to enumerate the total number of platform-type manipulators within the given restriction on the number of parallel legs and the defined degree of freedom of the platform. This theory is also applied for analyzing platform type mechanisms for finding out the mobility of the platform. A detailed review of the mobility of mechanisms is presented by Gogu [4]. He discussed 35 approaches followed by different researchers in the last 150 years. He considered mobility with and with out constraint equations. Rico Martinez, J. M., and Ravani [5] have also worked on parallel mechanism with emphasis on Paradoxical Linkages.

The present paper uses the concept of redundant constraints on some what similar lines to that of Dai, et al. [6,7] who use virtual and common constraints concept.

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The paper extends their approach further by integrating loops in mobility analysis. The approach begins with the case of non-redundant constraints and with changes in equation incorporates the element of redundant constraints and their effect on the mobility of the platform. It is observed that in such mechanism the legs are attached symmetrically about the centre vertical axis joining the base and the platform. Since there exists symmetry in leg designs, positioning and joint orientations about the vertical axis, there lies a strong possibility of linear dependency of the freedom (and constraint) screws of the joints of one leg to that of the other. This understandably affects the freedom of the platform. There is thus very little possibility for a mechanism to agree to the Kutzbach mobility criterion [8] in its original form. In line with a large number of published material, this is the main motivation behind this paper. Mechanisms that obey the Kutzbach mobility criterion in its original form are referred to as trivial mechanisms.

Numerous publications focus on the Kutzbach mobility criterion and its limitations to accurately predict the DOF of over constrained mechanisms [5,6,9,]. Grübler [10,11,12] mobility formula also suffered the same limitations. Specific cases of parallel manipulators and platform type manipulators are solved in [13-16]. This present work explores the subject of virtual and common constraints at the junction point of all the kinematic chains or loops of a platform-type manipulators. The number synthesis part discussed in this paper is similar to that presented by Kokkinis [13]. The procedure followed in this paper for the number synthesis of parallel manipulators can lead to various mechanisms that belong to the category of parallel manipulators. Type syntheis of such mechanisms was presented by Xianwen and Gosselin [15]. The method, used in this paper, for linear dependency or otherwise of freedoms and constraints depends heavily on screw theory or screw algebra. Screw algebra was developed by Ball in 1900 [17] but the theory remained dormant for quite sometime. The revival of the theory of screws is evidenced by the work of Dimentberg [18], Waldron [19], Hunt [20], Roth [21], Sugimoto and Duffy [22], and Phillips [23]. The theory has been applied to mechanisms by a large number of researchers, for example, Sugimoto and Duffy [22], Lipkin and Duffy [24], Rico and Duffy [25], Dai [26], Dai and Jones [27]. A comprehensive book by Davidson and Hunt [28] gives detailed insight into the theory of screws and its applications in mechanisms. The theory of screws elegantly describes the concept of freedoms and restraints in mechanisms.

The remainder of this paper is arranged as follows. Section 2 is a study of the DOF of the platform using the Kutzbach mobility criterion which is then modified to suite a platform-type manipulator. Section 3 discusses the problem of the orientations of the joints in the legs and linear dependencies of the constraints (and freedoms) within the legs in which the mobility criterion is further modified by incorporating the constraint dependencies. Section 4 presents a systematic approach, with the help of an example, to perform the number synthesis of this category of mechanism. Section 5 presents three examples incorporating the developed concept in the previous sections. The results of the examples agree with the work of others, for example Dai [6,7]. Section 6 gives number synthesis extended from the mobility study.

2. Parallel manipulators with In-Parallel Legs

A platform-type mechanism consists of a base which is the ground; a platform (whose degree of freedom is of interest in this study). The base and the platform are joined together with legs. The legs are further composed of joints and links. The number of joints in each leg vary depending upon the freedom equation dealt in depth in the proceeding lines. The number of legs are limited between three and six in this paper.

The study of the individual legs of the mechanism as the open kinematic chains provides the freedoms and constraints of the legs. When the legs are joined, the intersection of the constraints and mobility spaces of the legs at the intersection point (platform) give the constraints and degrees of freedom of the platform respectively.

Consider parallel mechanisms with legs having equal DOF. The freedom equation of one leg (expressed in terms of screws) is equal to that of the remaining individual legs. The necessary condition for the mechanism to successfully move can hence be written as:

$$\sum_{i=1}^{n} f_{1i} = \sum_{i=1}^{n} f_{2i} = \sum_{i=1}^{n} f_{3i} = \ldots = \sum_{i=1}^{n} f_{Li} \qquad (1)$$

where f_{1i} denotes the degrees of freedom of the ith joint of leg 1, **a**nd n is the number of single degree of freedom joints in the respective leg. The number of legs range from 1 to L. Since all these legs are connected in parallel, the degrees of freedom of the platform can not be more than the degree of freedom of the leg with least joints. However, there may exist freedoms of the legs, called local freedoms, which do not affect the mobility of other legs and, therefore, not the platform. For example, a leg of the Stewart Platform type rotating about its own axis.

In the most general case, the constraints constituted by all the legs are linearly independent and thus the total constraints at the platform are equal to the sum of all the constraints of the legs. However, when the constraint screw of one leg is dependent on the constraint screw of the other leg (s), it can not be considered a separate constraint. This, then increased the freedom of the platform.

Note that each leg can have more than one actuator. However, to maintain the intrinsic advantages of the parallel mechanism each leg will contain only one actuator. As a passing remark note that: *If all the actuators of the mechanism are locked the mechanism should behave like a rigid structure with mobility zero*. If one of the actuator moves then the freedom provided by that actuator at the platform must be supported by the passive joints of other legs and vice versa. *The passive joints, here means, the joints other than actuators.* At times these are considered as the joints that do not take part in the

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operation of the mechanism. This approach can also help in selection of the actuators in respective legs. The subject of appropriate selection of actuators is not part of this paper.

3. Loop Integrated Mobility Analysis

It is well-known that in some cases the Kutzbach mobility criterion cannot provide the right answer for the mobility of a mechanism. One reason is that it does not cater for over constrained mechanisms with common and virtual constraints. In this paper such mechanisms are categorized as the ones possessing redundant constraints. The Kutzbach mobility criterion will be modified in this section to cater for such redundancies in all their forms. According to the Kutzbach mobility criterion [8], the mobility of a spatial mechanism can be written as

$$M = 6\,(n - g - 1) + \sum_{i=1}^{g} f_i \tag{2}$$

where n is number of links, g is the number of joints, and $\sum_{i=1}^{g} f_i$ is the total DOF of all the joints. For parallel manipulators, Equation (2) can be modified as follows. The number of known bodies (platform and base) = 2, the number of parallel legs = L, and the DOF of the i-th leg = f_{li}. Thus the loop number is L. Therefore, the total degrees of freedom of all the legs = $\sum_{i=1}^{L} f_{li}$ which is equal to the number of single DOF joints = $\sum_{i=1}^{g} f_i = \sum_{i=1}^{L} f_{li}$, and the number of links in the i-th leg = $f_{li} - 1$ (excluding the platform and the base). Substituting these values into Equation (2) gives

$$M = 6\,[\{2 + (f_{l1}-1) + (f_{l2}-1) + \ldots + (f_{lL}-1)\} - \{f_{l1} + f_{l2} + \ldots + f_{lL}\} - 1] + \sum_{i=1}^{L} f_{li} \tag{3}$$

The term (-1) in the first curly brackets, appears L times so can be replaced by "- L" and Equation (3) can be written as:

$$M = 6(1 - L) + \sum_{i=1}^{L} f_{li} \tag{4}$$

The number 6 in this equation is replaced by the symbol "d", which denotes the dimension of the screw system.

4. Redundant Constraints Integrated Mobility Analysis

The presence of redundant constraints introduces additional terms into the mobility formula of Equation (4). These redundant constraints can be divided into two types: (i) the common redundant constraint C_c and; (ii) the virtual redundant constraint C_v. Also, there is a third condition that brings an additional change in the mobility equation and that is of linear dependencies of constraints. This condition will not be discussed in this paper because it involves a large variation in the linear dependencies. However, it will be the subject of a future research activity.

The common redundant constraint C_c is defined as a constraint present in all the legs of the mechanism. The additional term, due to the common redundant constraint, is

$$C_c\,(L - 1) \tag{5}$$

where C_c represents the number of common constraints. In the case of one common constraint, the number of redundant constraint is (L-1), that is, one less than the number of legs which is also conceptually verified. The presence of a virtual redundant constraint, defined as; *a constraint which is present in more than one legs but not in all*, adds another term to the mobility formula. One has to incorporate all the virtual redundant constraints separately, since; each virtual redundant constraint may be present in two or more than two legs and those legs may be different for different virtual constraint. The additional term in this case is

$$\sum_{i=1}^{N} C_{vi}\,(L_{vi} - 1) \tag{6a}$$

where, C_{vi} is the i-th virtual redundant constraint, L_{vi} is the total number of legs in which that i-th virtual redundant constraint is present and N is the total number of virtual redundant constraints. It is more convenient to hand each virtual redundant constraint separately with the additional term $C_{vi}(L_{vi}-1)$. These terms (i.e., virtual redundant constraints) can be added to give

$$\sum_{i=1}^{N} C_{vi}(L_{vi} - 1) \tag{6b}$$

The modified mobility formula can now be written as

$$M = 6(1-L) + \sum_{i=1}^{L} f_{li} + C_c(L-1) + \sum_{i=1}^{N} C_{vi}(L_{vi} - 1) \tag{7}$$

If the term redundant constraint is to replace the common and virtual constraints then the Equation (7) can be written as

$$M = 6\,(1 - L) + \sum_{i=1}^{L} f_{li} + C_r \tag{8}$$

where C_r represents the number of common and virtual redundant constraints. The number 6 in Equation (8) is the total dimension of the freedom screw system in free space with no constraints. If there exist some common constraints, then the number 6, will be reduced by that number.

Let the number of common constraints be 1. This means that the dimension of the freedom screw system is 5, that is, it is reduced by one dimension. If the dimension "d" is used in the equation then the term of the common constraint is omitted from Equation (5) and the mobility formula changes. If the number 6, in Equation (1), is replaced by "d", the dimension of the freedom screw system, then Equation (1) can be written as

$$M = d(1 - L) + \sum_{i=1}^{L} f_{li} + \sum_{i=1}^{N} C_{vi}(L_{vi} - 1) \tag{9}$$

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Equations (7) or (9) can be used to evaluate the mobility of a parallel platform-type manipulator with or without redundant constraints.

The following section presents an analytical treatment of the theory that was presented in previous sections.

5. Examples of Parallel Manipulators

The freedom provided by a joint in the leg of a parallel manipulator can be written in the form of a twist screw on the basis of the geometric arrangement of that joint in the mechanism. The twist screw of each joint of the leg must be linearly independent of others in that leg. And we know that the constraint screws of each leg are the reciprocal screws to the freedom screws of that leg. Three typical parallel mechanisms are chosen for case studies to illustrate the implementation of the theoretical analysis presented in the previous sections. The mechanisms chosen in these examples possess redundant constraints including the case of constraint which is common to all the legs and in some cases this commonality is not among all legs. It thus proves from a different angle the results that are proven by a large number of researchers.

Example 1. A platform-type manipulator with three symmetric legs and five revolute joints in each leg is shown in Fig. 1. The three legs (legs 1, 2 and 3) are attached at 120 degree intervals and rotated about the centre vertical line joining the base and the platform.

The joint screw of leg 1 can be written as

$$\$_{11} = [0 \quad 0 \quad 1 \quad p_{11} \quad q_{11} \quad 0]^T \tag{10a}$$
$$\$_{12} = [0 \quad 0 \quad 1 \quad p_{12} \quad q_{12} \quad 0]^T \tag{10b}$$
$$\$_{13} = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T \tag{10c}$$
$$\$_{14} = [l_{14} \quad m_{14} \quad n_{14} \quad 0 \quad 0 \quad 0]^T \tag{10d}$$

and

$$\$_{15} = [l_{15} \quad m_{15} \quad n_{15} \quad 0 \quad 0 \quad 0]^T \tag{10e}$$

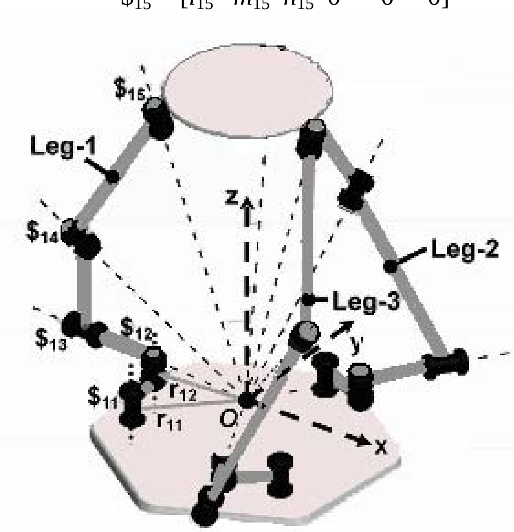

Fig. 1. Platform-Type Mechanism with Five Revolute Joints in each of the Three Legs.

where the subscript i in $\$_{ij}$ indicates the leg number and the subscript j refers to the joint number in the sequence starting from the base to the platform.

Since there is symmetry in the three legs then the joint screws for legs 2 and 3 can be obtained from a simple transformation of the screws for leg 1. Since there are five linearly independent joints in each leg, thus, each leg contains one constraint screw, as the reciprocal screw to the five twist screws. The legs being symmetric about the vertical axis, the simple transformation shows that the above reciprocal screw is common to all the three legs and can be written as

$$\$_{l1}^r = \$_{l2}^r = \$_{l3}^r = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]^T \tag{11}$$

where the subscript 1 in $\$_{l1}^r$ indicates the reciprocal screw to leg 1. There are a total of three constraints of the mechanism which are the same constraint screws of the three legs then the redundant constraints $C_r = 2$. This mechanism has one common constraint and no virtual constraints. Therefore, the degrees of freedom of the platform, from Equation (7), are

$$M = 6(1 - L) + \sum_{i=1}^{L} f_{li} + C_c (L - 1) + \sum_{i=1}^{N} C_{vi} (L_{vi} - 1) \tag{12a}$$

Substituting the values into this equation gives

$$M = 6(1 - 3) + 15 + 1(3 – 1) + 0 = 5 \tag{12b}$$

This result shows the effect of the common constraint. If the Kutzbach mobility criterion were used in their original form the answer for the mobility of the platform would be 3. Due to the common redundant constraints which are two that 2 more freedoms are obtained and, Equation (12b) gives the correct answer for the DOF which is 5.

Example 2. A spherical mechanism with three legs and three joints in each leg is shown in Fig. 2. The three legs (legs 1, 2 and 3) are attached at 120 degree intervals and rotated about the centre vertical line joining the base and the platform.
The joint axes of the joints in each leg are coincident with the centre, point O, as shown in Fig. 2.

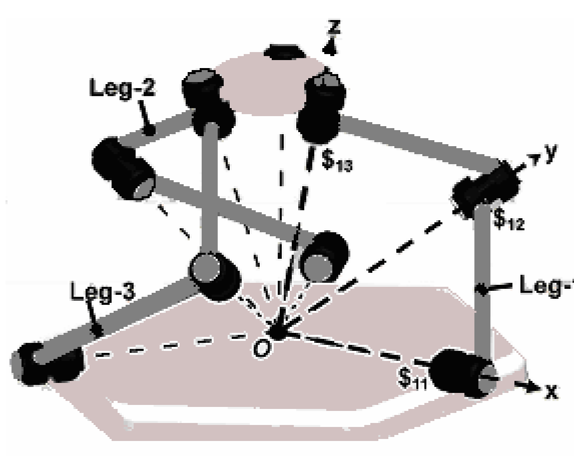

Fig. 2. A Spherical Mechanism Possessing Three Legs.

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If the fixed reference frame is located at the intersection point of the joint axes then the twist screws of the joints of the three legs can be written in a straightforward manner. The twist screws for leg 1, are

$$\$_{11} = [1 \quad 0 \quad 0 \;;\; 0 \quad 0 \quad 0]^T \tag{13a}$$
$$\$_{12} = [0 \quad 1 \quad 0 \;;\; 0 \quad 0 \quad 0]^T \tag{13b}$$

and

$$\$_{13} = [0 \quad 0 \quad 1 \;;\; 0 \quad 0 \quad 0]^T \tag{13c}$$

Since the three legs are symmetric then the twist screws of the other two legs will be similar. The three reciprocal screws to the three twist screws can be written as

$$\$^r_{11} = [1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T \tag{14a}$$
$$\$^r_{12} = [0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T \tag{14b}$$

and

$$\$^r_{13} = [0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0]^T \tag{14c}$$

where the 1 in $\$^r_{12}$ indicates the second reciprocal screw of leg 1. From Fig. 1, it can be verified that the three reciprocal screws are common to all three legs; i.e.,

$$\$^r_{11} = \$^r_{21} = \$^r_{31} = [1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T \tag{15a}$$
$$\$^r_{12} = \$^r_{22} = \$^r_{32} = [0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T \tag{15b}$$

and

$$\$^r_{13} = \$^r_{23} = \$^r_{33} = [0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0]^T \tag{15c}$$

Applying Equation (7), the degrees of freedom of the mechanism can be written as

$$M = 6(1 - L) + \sum_{i=1}^{L} f_{li} + C_c(L-1) + \sum_{i=1}^{N} C_{vi}(L_{vi} - 1) \tag{16a}$$

Substituting the values into this equation gives

$$M = 6(1 - 3) + 9 + 3(3 - 1) + 0 = 3 \tag{16b}$$

From Equation (7), the degrees of freedom of the mechanism can be written as

$$M = d(1 - L) + \sum_{i=1}^{L} f_{li} + \sum_{i=1}^{N} C_{vi}(L_{vi} - 1) \tag{17a}$$

Substituting the values into this equation gives

$$M = 3(1 - 3) + 9 + 0 = 3 \tag{17b}$$

Example 3. Consider the four leg mechanism shown in Fig. 3. Each leg is an RRR configuration of joints. With reference to leg1, legs 2, 3 and 4 are attached at an interval of 90, 180 and 270 degrees about the centre line connecting the base and the platform. All three joint axes of leg 1 are parallel to the X-axis. The orientation of the joints of the remaining three legs can then be easily ascertained.

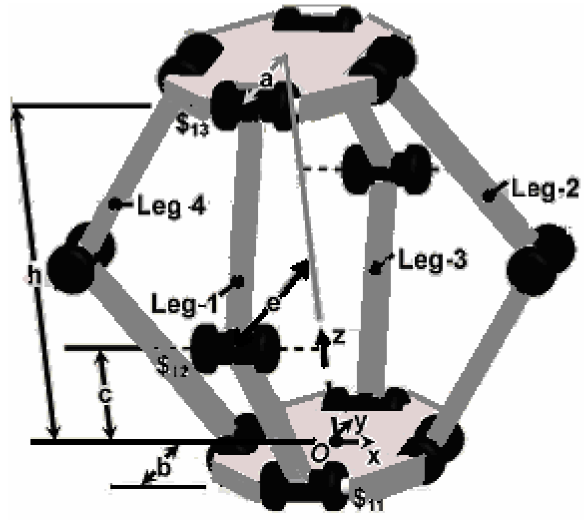

Fig. 3. The Four Leg RRR Platform-Type Mechanism.

The twist screws of leg 1 are

$$\$_{11} = [1 \quad 0 \quad 0 \;;\; 0 \quad 0 \quad b]^T \tag{18a}$$
$$\$_{12} = [1 \quad 0 \quad 0 \;;\; 0 \quad c \quad e]^T \tag{18b}$$

and

$$\$_{13} = [1 \quad 0 \quad 0 \;;\; 0 \quad h \quad a]^T \tag{18c}$$

The twist screws of leg 2 are

$$\$_{21} = [0 \quad 1 \quad 0 \;;\; 0 \quad 0 \quad b]^T \tag{19a}$$
$$\$_{22} = [0 \quad 1 \quad 0 \;;\; -c \quad 0 \quad e]^T \tag{19b}$$

and

$$\$_{23} = [0 \quad 1 \quad 0 \;;\; -h \quad 0 \quad a]^T \tag{19c}$$

The twist screws of leg 3 are

$$\$_{31} = [1 \quad 0 \quad 0 \;;\; 0 \quad 0 \quad -b]^T \tag{20a}$$
$$\$_{32} = [1 \quad 0 \quad 0 \;;\; 0 \quad c \quad -e]^T \tag{20b}$$

and

$$\$_{33} = [1 \quad 0 \quad 0 \;;\; 0 \quad h \quad -a]^T \tag{20c}$$

The twist screws of leg 4 are

$$\$_{41} = [0 \quad 1 \quad 0 \;;\; 0 \quad 0 \quad -b]^T \tag{21a}$$
$$\$_{42} = [0 \quad 1 \quad 0 \;;\; -c \quad 0 \quad -e]^T \tag{21b}$$

and

$$\$_{43} = [0 \quad 1 \quad 0 \;;\; -h \quad 0 \quad -a]^T \tag{21c}$$

The constraint screws of leg 1 are

$$\$^{r1}_{l1} = [0 \quad 0 \quad 0 \;;\; 0 \quad 0 \quad 1]^T \tag{22a}$$
$$\$^{r2}_{l1} = [0 \quad 0 \quad 0 \;;\; 0 \quad 1 \quad 0]^T \tag{22b}$$

and

$$\$^{r3}_{l1} = [1 \quad 0 \quad 0 \;;\; 0 \quad 0 \quad 0]^T \tag{22c}$$

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The constraint screws of leg 2 are

$$\$_{l2}^{r1} = [0 \quad 0 \quad 0 \;;\; 0 \quad 0 \quad 1]^T \tag{23a}$$

$$\$_{l2}^{r2} = [0 \quad 0 \quad 0 \;;\; 1 \quad 0 \quad 0]^T \tag{23b}$$

and

$$\$_{l2}^{r3} = [0 \quad 1 \quad 0 \;;\; 0 \quad 0 \quad 0]^T \tag{23c}$$

The constraint screws of leg 3 are

$$\$_{l3}^{r1} = [0 \quad 0 \quad 0 \;;\; 0 \quad 0 \quad 1]^T \tag{24a}$$

$$\$_{l3}^{r2} = [0 \quad 0 \quad 0 \;;\; 0 \quad 1 \quad 0]^T \tag{24b}$$

and

$$\$_{l3}^{r3} = [1 \quad 0 \quad 0 \;;\; 0 \quad 0 \quad 0]^T \tag{24c}$$

The constraint screws of leg 4 are

$$\$_{l4}^{r1} = [0 \quad 0 \quad 0 \;;\; 0 \quad 0 \quad 1]^T \tag{25a}$$

$$\$_{l4}^{r2} = [0 \quad 0 \quad 0 \;;\; 1 \quad 0 \quad 0]^T \tag{25b}$$

and

$$\$_{l4}^{r3} = [0 \quad 1 \quad 0 \;;\; 0 \quad 0 \quad 0]^T \tag{25c}$$

where, the number 2 in $\$_{l1}^{r2}$ indicates the second reciprocal (constraint) screw of leg 1.

Observe from Eqs. (22)-(25) of reciprocal screws, that, there is one screw which is common to all four equations, that is, Eqs. (22a), (23a), (24a), and (25a), are the same. Similarly, the second and third screws of leg 1 and leg 3 are the same and can therefore be considered as virtual constraints. Also, the second and third constraint screws of legs 2 and 4 are the same.

Substituting Equations (18)-(25) into Equation. (7), the DOF of the mechanism can be written as:

$$M = 6(1 - L) + \sum_{i=1}^{L} f_{li} + C_c(L-1) + \sum_{i=1}^{N} C_{vi}(L_{vi} - 1) \tag{26a}$$

Substituting the known values into this equation gives

$$M = 6(1 - 4) + 12 + 1(4 - 1) + [(2\text{-}1)+(2\text{-}1)+(2\text{-}1)+(2\text{-}1)] = 1 \tag{26b}$$

Therefore, the DOF of the platform of the mechanism is one.

Note that in the last entry of Equation (26a) each virtual constraint is handled separately. Therefore, there are four similar entries of "(2-1)", representing one virtual constraint in two legs separately. So in this case the limits of i goes from 1 to 4. Equation (9) will also give the same result; i.e:

$$M = d(1 - L) + \sum_{i=1}^{L} f_{li} + \sum_{i=1}^{N} C_{vi}(L_{vi} - 1) \tag{27a}$$

Substituting the known values into Equation (27a) gives

$$M = 5(1 - 4) + 12 + [(2\text{-}1)+ (2\text{-}1)+ (2\text{-}1)+ (2\text{-}1)] = 1 \tag{27b}$$

This gives the same result to that of Equation [26b].

6. Number Synthesis of Parallel manipulators

Given the limits on the mobility of the platform and the number of legs, the purpose here is not to enumerate all possible platform type mechanisms but to devise a simple strategy for the enumeration. This strategy can then be applied to the enumeration of all mechanisms within the given limits of the number of legs and the number of degrees of freedoms of the platform. A systematic approach, with the help of an example is applied to enumerate all possible mechanisms within the design constraints.

6.1 Design Example

Let the mobility of the platform M = 3, and the number of legs L = 3. The cases with no redundant constraints to the case of maximum possible redundant constraints are dealt for the exhaustive enumeration within the design limits. The trivial case with no redundant constraints agrees to the Kutzbach criteria. In the example, with M = 3 and L = 3, the extreme limits are from zero redundant constraint (15 single DOF joints), to a maximum possible of 6 redundant constraints (9 single DOF joints).

(i) No Redundant Constraints. Applying Equation (7), the number of single DOF joints, $\sum f_i = 15$. Note that this is the trivial case [8] where the mechanism obeys the Kutzbach criteria. The type of joints and their arrangement is generalized to be such that no special geometries are encountered. The distribution of the 15 joints in the three legs also follows a set rule. This is also based on Equation (7), applied on two legs and single leg cases separately. It may be noted that Equation (7) is not only applied on the complete mechanism with L legs, but it must be applied to all possible number of combination of legs called kinematics-sub-chains (KSC) here.

Applying Equation (7) on one leg (a serial mechanism with 3 freedom) gives us $\sum f_i = 3$. Thus, as obvious, any one leg should have minimum 3 linearly independent joints. Again using Equation. (7) on 2 leg case with M=3, the number of single DOF joints i.e., $\sum f_i$ equals 9. For three legs we have already calculated the number of joints to be 15. Arranging the number of joints in the three parallel legs, the possible mechanisms with mobility M = 3, possessing 3 legs are as shown in Table 1.

Mechanisim-I	(3 6 6)
Mechanisim-II	(4 5 6)
Mechanisim-III	(5 5 5)

Table 1. The Number of Possible Mechanisms possessing 3 legs considering freedoms

The number of entries in each row in Table 1 represents three legs in the respective mechanism. Each entry in every row represents the number of freedoms of that leg. In the spatial case, the number of freedoms and constraints of a leg or of any KSC or the whole mechanism should add up to 6. If the

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constraints of the legs are considered, instead of the degrees of freedoms, then Table 1 can be written as Table 2.

Mechanisim-I	(3 0 0)
Mechanisim-II	(2 1 0)
Mechanisim-III	(1 1 1)

Table 2. The Number of Possible Mechanisms possessing 3 legs considering constraints

The number of entries in each row, in Table 2, represents the number of legs (which are three in each case). In this case, unlike Table 1, each entry in every row represents the number of constraints of that leg. The total number of constraints in every mechanism in this example is 3. Since freedoms plus constraints should be equal to 6 then the freedoms at the platform are also 3. The important point to note here is, that, the constraints of every leg are linearly independent of the constraints of other legs individually. With the above procedure, the following proposition is suggested.

Proposition I. *In a platform type mechanism of mobility M at the platform, every sub-mechanism or kinematic sub-chain (KSC) must be capable of providing at least M, DOF to the platform.*

In the three mechanisms presented in Table 2 there are three constraints at the platform. For example in the case (3 0 0), all the 3 constraints of the mechanism are in leg 1, and the other two legs give no constraints. In case of (1 1 1), each leg gives one constraint at the platform and similarly case (2 1 0) shows that there are two constraints because of leg 1 and 1 constraint by leg 2, and the third leg does not give any constraint at the platform. And all these constraints are linearly independent of each other. By considering the freedoms of the legs and observing Table 1, another proposition can be put forth:

Proposition II. *In the absence of redundant constraints, a spatial KC (the whole mechanism of Figure-1) with mobility M, in which every KSC (any leg or combination of legs) has mobility M, must contain all KSCs which possess minimum of M+6 freedoms.*

However; in the presence of redundant constraints in the legs the formula is to be described in such a way so as to cater for the redundant constraints. The cases of one or more redundant constraints are discussed in the subsequent text.

(ii) One Redundant Constraint. *A parallel manipulator is said to have one redundant constraint if there is one constraint screw of any one of the legs which is same as any one (only one) constraint screw of any other leg.* More than one redundant constraint screws are handled in the same manner.

Most of the practical mechanisms do possess special geometries and thus redundant constraints are present in the legs of the mechanisms. The mobility of the platform in the presence of redundant constraints increases by the terms already discussed above.

In parallel legged mechanisms the redundant constraints occur mostly due to symmetries in leg designs. The symmetries in the legs are preferred so as to avoid unnecessary additional manufacturing costs. Also, since the presence of redundant constraint increases the DOF of the platform, they are a welcome feature.

A Design with One Redundant Constraint. With one redundant constraint in Equation (7), that is C_r =1, and M = 3, the number of single DOF joints reduce by one and we get $\sum_{i=1}^{L} f_{li}$ =14. In this case there are total of 4 constraints but one of these is a redundant constraint. The inclusion of this redundant constraint with all possibilities in the legs gives us three additional mechanisms. Note that the redundant constraints are to be added in already existing mechanisms of Table 2, which has no redundant constraints. The additional mechanisms are:

Mechanism I (3 1 0) redundant constraint between leg 1 and leg 2.

Mechanism II (2 2 0) redundant constraint between leg 1 and leg 2.

Mechanism III (2 1 1) redundant constraint between leg 1 and leg 2.

(iii) Two Redundant Constraints. Substituting C_r = 2 and M = 3 into Equation (7) gives $\sum_{i=1}^{L} f_{li}$ =13. Out of the total five constraints, shown in the following mechanisms, only three are independent. The distribution of the redundant constraints in the mechanisms of Table 2 gives the additional mechanisms:

Mechanism I (3 2 0). In this mechanism, two independent constraints of leg 2, are common to two independent constraints of leg 1.

Mechanism II (3 1 1). In this mechanism, there can be two possibilities. First, the only constraint of leg 2 is common to a constraint of leg1, and the constraint of leg 3, is common to the other constraint of leg 1. Second; there is common constraint in all the three legs. The structure of the mechanism remains same in both the cases, thus it is considered only one mechanism.

Mechanism III (2 2 1). There are many possibilities of the relationships of the redundant constraint in this case. First; one constraint of leg 1 is common to one constraint of leg 2, and the other constraint of leg 1, is common to the other constraint of leg 2. Second; One constraint of leg 1 is common to one constraint of leg 2, and the other constraint of leg 1 is common to the constraint of leg 3. Third; One constraint of leg 2 is common to one constraint of leg 1, and the other constraint of leg 2 is common to the only constraint of leg 3. Fourth; There is a common constraint to all the three legs. So we have three linearly independent constraints including 1 common to all legs.

(iv) Three Redundant Constraints. With C_r = 3, and M = 3, the number of single DOF joints is 12. With these three additional constraints and with the similar explanations of the distribution of these additional constraints, the additional mechanisms are:

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Mechanism I - (3 3 0), Mechanism II - (3 2 1) and Mechanism III - (2 2 2)

(iv) Four Redundant Constraints. The total number of joints in this case reduces from 15 to 11. With four redundant constraints the additional mechanisms are tabulated as Mechanism I - (3 3 1) and Mechanism II - (3 2 2)

(v) Five Redundant Constraints. In case of five redundant constraints there is one additional mechanism; namely Mechanism I - (3 3 2).

(vi) Six Redundant Constraints. In case of six redundant constraints there is one additional mechanism, which is Mechanism I - (3 3 3).

One such mechanism is the spherical mechanism that is studied in some detail in [2, 28, 5]. It may be noted here that the platform-type mechanism with minimum possible number of joints possessing 3 degrees of freedom at the platform with three legs has three common constraints. Similarly, a mechanism with the arrangement (5 5 5) with the mobility formula, Equation (5) but with a common constraint will give 5 degrees of freedom at the platform.

6.2 Total Number of Mechanisms with Platform Mobility M = 3

Summarizing the above tables, the total number of platform-type mechanism possessing three legs and three DOF at the platform are presented in the following table.

Mechanism I	(3	0	0)
Mechanism II	(2	1	0)
Mechanism III	(1	1	1)
Mechanism IV	(3	1	0)
Mechanism V	(2	2	0)
Mechanism VI	(2	1	1)
Mechanism VII	(3	2	0)
Mechanism VIII	(3	1	1)
Mechanism IX	(2	2	1)
Mechanism X	(3	3	0)
Mechanism XI	(3	2	1)
Mechanism XII	(2	2	2)
Mechanism XIII	(3	3	1)
Mechanism XIV	(3	2	2)
Mechanism XV	(3	3	4)
Mechanism XVI	(3	3	3)

Table - 3 The total number of mechanisms including redundant constraints – For M=3 and L=3

Therefore, for the case of three leg mechanisms with the requirement of 3 DOF at the platform, the total number of mechanism increases from 3 with no common constraints to 16 with maximum of six common constraints. The number of single DOF joints decreases from 15 to 9.

7. Conclusions

This research work had three major parts. First, it investigated and suggested a mobility formula specific to parallel manipulators, incorporating the mobility M, the number of parallel legs L, and the number of single DOF joints. It also studied the redundant constraints which were either common, or virtual in their relationships with other legs. On the basis of these relationships of freedoms and constraints, the paper suggested a formula that fully functions in designing platform-type mechanism and analyzing/evaluating the mobility of an existing mechanism. This was then substantiated with the help of numerical examples. Theory of screws was used in the analysis of the design examples. Freedom screws and their reciprocals (constraints screws) were used in the study of mechanisms. Based on the formula developed, the paper suggested a number synthesis of parallel manipulators. The paper also presented a numerical example of mobility 3 with three legs. The aspect which this paper did not touch but is an integral part of the subject is that of linear dependence of the linear combination of the freedoms (and constraints) of one leg to that of the other. This aspect of parallel manipulators will be the subject of a future research activity.

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