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On One Analytic Method of Constructing Program Controls

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Abstract

The article proposes an analytical method for constructing control function that ensures transferring linear inhomogeneous stationary system from an initial state to a given final state. Conditions under which the specified transfer is guaranteed are presented.

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1 Introduction

Among the most important and difficult aspects of the mathematical control theory are issues related to the development of methods for building control functions, wherein solutions of linear stationary systems of ordinary differential equations connect the given points in phase space. There is a wealth of research papers on the subject. Most closely this work is connected to the research presented in [1] – [3].

In [1] the linear time-invariant homogeneous system is considered. An algorithm for constructing the desired control function and the corresponding functions of phase coordinates presented in [1] is reduced to solving a system of linear algebraic equations. This system might be of quite high order. Therefore, an implementation of this algorithm involves computational difficulties. Methods of construction of control functions for linear inhomogeneous systems proposed in [2, 3] do not allow, in general, to find the required functions in an analytical form.

The main difference of the present article from the others is the simplicity of developed algorithm implementation. The latter is achieved due to the fact that the desired control function and the corresponding function of phase coordinates are being found in analytical form.

The object of the study is a controlled system of differential equations

$$\dot{x} = Px + Qu + f\ , \tag{1.1}$$

where $x = (x^1, ..., x^n)^T$, $x \in R^n$; $u = (u^1, ..., u^r)^T$, $u \in R^r$, $t \in [0,1]$;

$f \in R^n$, $f = (f^1, ..., f^n)^T$ – constant vector;

$P = \{p_i^j\}$, $i, j = 1, ..., n$; $Q = \{q_i^j\}$, $i = 1, ..., n$, $j = 1, ..., r$ – constant matrices;

$$\text{rank}(B, AB, ..., A^{n-1}B) = n\,. \tag{1.2}$$

Statement of the problem. Find functions $x(t) \in C^1[0,1]$, $u(t) \in C^1[0,1]$, satisfying system (1.1) and conditions

$$x(t_0) = x_0, \quad x(T) = x_T\,. \tag{1.3}$$

In (1.3) $x_T = (x_T^1, ..., x_T^n)^*$ is a fixed vector. Let us agree for the mentioned pair of functions to be called a solution to the problem (1.1), (1.3).

Let us make a change of variables in the system (1.1) replacing the dependent and independent variables x and t according to the formulas

$$x = y + x_0,\ t = \theta + t_0\,. \tag{1.4}$$

Then in the new variables system (1.1) and boundary conditions (1.3) will be as follows:

$$\frac{dy}{d\theta} = Py + Qu + Px_0 + f\ , \tag{1.5}$$

$$y(0) = 0, \quad y(T - t_0) = x_{T-t_0}\ , \tag{1.6}$$

where $x_{T-t_0} = x_T - x_0$.

Changing the independent variable θ to ω by the formula

$$\omega = \frac{\theta}{T - t_0} \tag{1.7}$$

brings the system (1.5) and the boundary conditions (1.6) to the form

$$\begin{gathered} \frac{dy}{d\omega} = P(T - t_0)y + Q(T - t_0)u + (T - t_0)(Px_0 + f), \\ y(0) = 0, \quad y(1) = x_1, \quad x_1 = x_T - x_0. \end{gathered} \tag{1.8}$$

We assume below that the transformations (1.4) and (1.7) are satisfied and the boundary conditions for system (1.1) after the substitution of variable y by x have the form of (1.8). Herein we assume $P = P_1$, $Q = Q_1$, $f = f_1$, $P_1 = P(T - t_0)$, $Q_1 = Q(T - t_0)$, $f_1 = (T - t_0)(Px_0 + f)$, $t = \omega$.

2 Problem solution

Theorem. Let the condition (1.2) be fulfilled. Then $\forall\, x_T \in R^n$ there exists a solution to the problem (1.1), (1.3), which can be obtained after solving the stabilization problem for linear time-dependent system of a special type and the subsequent solution to the Cauchy problem for the auxiliary linear system of ordinary differential equations.

Proof. We will look for a function $x(t)$, which is the solution to the considered problem, in the following form:

$$x(t) = a(t) + x_1 \,. \tag{2.1}$$

After substituting (2.1) in (1.1) we obtain the system

$$\dot{a} = Pa + Qu + Px_1 + f \,. \tag{2.2}$$

Let us seek functions $a(t) \in C^1[0,1]$, $u(t) \in C^1[0,1]$, satisfying (2.1) and conditions

$$a(0) = -x_1, \quad a(t) \to 0 \ \text{ as } \ t \to 1. \tag{2.3}$$

Replacing the variable t to τ by the formula

$$1 - t = e^{-\alpha\tau}; \quad \tau \in [0, +\infty), \tag{2.4}$$

where $\alpha > 0$ is a fixed number, converts the system (2.2) and conditions (2.3) into the form

$$\frac{dc}{d\tau} = \alpha e^{-\alpha\tau} Pc + \alpha e^{-\alpha\tau} Qd + \alpha e^{-\alpha\tau} Px_1 + \alpha e^{-\alpha\tau} f \,, \tag{2.5}$$

$$c(\tau) = a(t(\tau)), \quad d(\tau) = u(t(\tau)), \quad \tau \in [0, +\infty). \tag{2.6}$$

We will look for functions $c(\tau) \in C^1[0,\infty)$, $d(\tau) \in C^1[0,\infty)$ that satisfy (2.5) and conditions

$$c(0) = -x_1, \ c(\tau) \to 0 \ \text{ as } \ \tau \to \infty. \tag{2.7}$$

Let us make the change of variable c according to the formula

$$c = c^{(1)} - (Px_1 + f) e^{-\alpha\tau}. \tag{2.8}$$

In that case, system (2.5) and condition (2.7) take on the form

$$\frac{dc^{(1)}}{d\tau} = \alpha e^{-\alpha\tau} Pc^{(1)} + \alpha e^{-\alpha\tau} Qd - \alpha e^{-2\alpha\tau} P(Px_1 + f) \,, \tag{2.9}$$

$$c^{(1)}(0) = -x_1 + (f + Px_1)\,. \tag{2.10}$$

Next, we do the transformation of variable $c^{(1)}$ according to the formula

$$c^{(1)} = c^{(2)} + \frac{1}{2}e^{-2\alpha\tau}P(Px_1 + f) \tag{2.11}$$

Then the system (2.9) and conditions (2.8) take the form

$$\frac{dc^{(2)}}{d\tau} = \alpha e^{-\alpha\tau}Pc^{(2)} + \alpha e^{-\alpha\tau}Qd + \frac{1}{2}\alpha e^{-3\alpha\tau}P^2(Px_1 + f)\,, \tag{2.12}$$

$$c^{(2)}(0) = -x_1 + (f + Px_1) - \frac{1}{2}P(f + Px_1)\,. \tag{2.13}$$

In turn, the transformation

$$c^{(2)} = c^{(3)} - \frac{1}{3!}e^{-3\alpha\tau}P^2(Px_1 + f) \tag{2.14}$$

brings the system (2.12) and the initial condition (2.13) to the form

$$\frac{dc^{(3)}}{d\tau} = \alpha e^{-\alpha\tau}Pc^{(3)} + \alpha e^{-\alpha\tau}Qd - \frac{1}{3!}\alpha e^{-4\alpha\tau}P^3(Px_1 + f)\,,$$

$$c^{(3)}(0) = -x_1 + (f + Px_1) - \frac{1}{2}P(f + Px_1) + \frac{1}{3!}P^2(f + Px_1)\,.$$

Using the latter system, as well as the initial conditions of the system and the inductive approach, we obtain the transformation

$$c^{(j-1)} = c^{(j)} + \frac{(-1)^j}{j!}e^{-j\alpha\tau}P^{j-1}(Px_1 + f), \tag{2.15}$$

that results in the original system (2.5) and the initial condition taking the following form:

$$\frac{dc^{(j)}}{d\tau} = \alpha e^{-\alpha\tau}Pc^{(j)} + \alpha e^{-\alpha\tau}Qd + \frac{1}{j!}\alpha e^{-(j+1)\alpha\tau}P^j(Px_1 + f)\,, \tag{2.16}$$

$$c^{(j)}(0) = -x_1 + \sum_{k=1}^{j}(-1)^{k+1}\frac{1}{k!}P^{k-1}(f + Px_1)\,. \tag{2.17}$$

Together with (2.16) let us consider the system

$$\frac{dc^{(j)}}{d\tau} = \alpha e^{-\alpha\tau}Pc^{(j)} + \alpha e^{-\alpha\tau}Qd\,. \tag{2.18}$$

We will search for $d(c^{(j)}, \tau) = M(\tau)c^{(j)}$ that provide exponential stability for the system (2.18). Let $q_i, i = 1,...,r$ be the *i*-th column of matrix Q. Let us construct a matrix

$$S = \{q_1, ..., P^{k_i - 1} q_1, ..., q_r, ..., P^{k_r - 1} q_r\}, \tag{2.19}$$

where $k_i, i = 1, ..., r$ is the maximum number of columns of the form $q_i, Pq_i, ..., P^{k_i - 1} q_i$, $i = 1, ..., r$, so that the vectors $q_1, Pq_1, ..., P^{k_1 - 1} q_1, ..., q_r, ..., P^{k_r - 1} q_r$ are linearly independent.

Condition (1.2) implies that the rank of the matrix (2.19) equals n. Transformation

$$c^{(j)} = Sy \tag{2.20}$$

brings the system (2.18) to the form

$$\frac{dy}{d\tau} = \alpha S^{-1} P S e^{-\alpha\tau} y + \alpha S^{-1} Q e^{-\alpha\tau} d. \tag{2.21}$$

Based on [2], matrices $S^{-1}PS$ and $S^{-1}Q$ have the form

$$S^{-1}\overline{P}S = \{e_2, ..., e_{k_1}, g_{k_1}, ..., e_{k_{r-1}+2}, ..., e_{k_r}, g_{k_r}\},$$

$e_i = (0, ..., 1, ..., 0)^T{}_{n\times 1}$, where "1" is in the *i*-th place and

$$\begin{gathered} g_{k_i} = (-g_{k_1}^0, ..., -g_{k_1}^{k_1-1}, ..., -g_{k_i}^0, ..., -g_{k_i}^{k_i-1}, 0, ..., 0)^*_{n\times 1}; \\ P^{k_i} q_i = -\sum_{j=0}^{k_1-1} g_{k_1}^j P^j q_1 - ... - \sum_{j=0}^{k_i-1} g_{k_i}^j P^j q_i, i = 1, ..., r. \end{gathered} \tag{2.22}$$

In (2.22) $g_{k_1}^j, j = 0, ..., k_1 - 1, ..., g_{k_i}^j, j = 0, ..., k_i - 1$ are coefficients of the vector decomposition into vectors

$P^j q_1; j = 0, ..., k_1 - 1, ..., P^j q_i; j = 0, ..., k_i - 1$, $S^{-1}Q = \{e_1, ..., e_{k_i+1}, ..., e_{\gamma+1}\}$; $\gamma = \sum_{i=1}^{r-1} k_i$.

Let us consider the stabilization problem for the system

$$\begin{gathered} \frac{dy_{k_i}}{d\tau} = \{\overline{e}_2^{k_i}, ..., \overline{e}_{k_i}^{k_i}, \overline{g}_{k_i}\}\alpha e^{-\alpha\tau} y_{k_i} + \overline{e}_1^{k_i} \alpha e^{-\alpha\tau} d^i; i = 1, ..., r, \\ y_{k_i} = (y_{k_i}^1, ..., y_{k_i}^{k_i})^T_{k_i\times 1}; \overline{e}_i^{k_i} = (0, ..., 1, ..., 0)^T_{k_i\times 1}, \end{gathered} \tag{2.23}$$

where "1" is in the *i*-th place, and $\overline{g}_{k_i} = (-g_{k_i}^0, ..., -g_{k_i}^{k_i-1})^T_{k_i\times 1}$; $d = (d^1, ..., d^r)^T$.

In scalar form, the system (2.23) can be written as:

$$\frac{dy_{k_i}^1}{d\tau} = -\alpha g_{k_i}^0 e^{-\alpha\tau} y_{k_i}^{k_i} + \alpha e^{-\alpha\tau} d^i,$$
$$\frac{dy_{k_i}^2}{d\tau} = \alpha e^{-\alpha\tau} y_{k_i}^1 - \alpha g_{k_i}^1 e^{-\alpha\tau} y_{k_i}^{k_i},$$
$$\dots\dots\dots\dots\dots\dots\dots\dots \qquad (2.24)$$
$$\frac{y_{k_i}^{k_{i-1}}}{d\tau} = \alpha e^{-\alpha\tau} y_{k_i}^{k_i-2} - \alpha g_{k_i}^{k_i-2} e^{-\alpha\tau} y_{k_i}^{k_i},$$
$$\frac{dy_{k_i}^{k_i}}{d\tau} = \alpha e^{-\alpha\tau} y_{k_i}^{k_i-1} - \alpha g_{k_i}^{k_i-1} e^{-\alpha\tau} y_{k_i}^{k_i}.$$

Let $y_{k_i}^{k_i} = \alpha^{k_i}\psi$. Using the last equation from (2.24) and the inductive approach, we obtain

$$y_{k_i}^{k_i} = \alpha^{k_i}\psi,$$
$$y_{k_i}^{k_i-1} = \alpha^{k_i-1} e^{\alpha\tau}\psi^{(1)} + g_{k_i}^{k_i-1}\alpha^{k_i}\psi,$$
$$y_{k_i}^{k_i-2} = \alpha^{k_i-2} e^{2\alpha\tau}\psi^{(2)} + (\alpha^{k_i-1} e^{2\alpha\tau} + \alpha^{k_i-1} e^{\alpha\tau} g_{k_i}^{k_i-1})\psi^{(1)} + g_{k_i}^{k_i-2}\alpha^{k_i}\psi, \qquad (2.25)$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$y_{k_i}^1 = \alpha e^{(k_i-1)\alpha\tau}\psi^{(k_i-1)} + r_{k_i-2}(\tau)\psi^{(k_i-2)} + \dots + r_1(\tau)\psi^{(1)} + \alpha^{k_i} g_{k_i}^1\psi.$$

After differentiating the last equation (2.25), from the first equation of system (2.24) we get

$$\psi^{(k_i)} + \varepsilon_{k_i-1}(\tau)\psi^{(k_i-1)} + \dots + \varepsilon_0(\tau)\psi = e^{-k_i\alpha\tau} d^i; \quad i = 1,\dots,r. \qquad (2.26)$$

In (2.25) $r_{k_i-2}(\tau),\dots,r_1(\tau)$ are linear combinations of exponentials with exponents not greater than $(k_i - 1)\alpha\tau$. Expressions $\varepsilon_{k_i-1}(\tau),\dots,\varepsilon_0(\tau)$ in (2.26) are linear combinations of exponentials with exponents not greater than zero. Let

$$\upsilon^i = e^{-\alpha k_i\tau} d^i, \quad i = 1,\dots r. \qquad (2.27)$$

Let us assume that

$$\upsilon^i = \sum_{j=1}^{k_i} (\varepsilon_{k_i-j}(\tau) - \gamma_{k_i-j})\psi^{(k_i-j)}; \; i = 1,\dots,r, \qquad (2.28)$$

where $\gamma_{k_i-j}; \; j = 1,\dots,k_i$ are selected so that the roots $\lambda_{k_i}^1,\dots,\lambda_{k_i}^{k_i}$ of the equation

$$\lambda^{k_i} + \gamma_{k_i-1}\lambda^{k_i-1} + \dots + \gamma_0 = 0; \quad i = 1,\dots,r$$

satisfy the following conditions:

$$\lambda_{k_i}^i \neq \lambda_{k_i}^j, \quad i \neq j, \quad \lambda_{k_i}^j < -2n\alpha - 1, \quad j = 1,\dots,k_i, \quad i = 1,\dots,r. \qquad (2.29)$$

Using (2.20), (2.25), (2.27), and (2.28), we obtain

$$d^i = e^{k_i \alpha\tau} \delta_{k_i} T_{k_i}^{-1} S_{k_i}^{-1} c^{(j)}\ ;\ \ i = 1,...,r, \tag{2.30}$$

where $\delta_{k_i} = (\varepsilon_{k_i - 1}(\tau) - \gamma_{k_i - 1},...,\varepsilon_0(\tau) - \gamma_0)$; T_{k_i} is the matrix from (2.25), i.e. $y_{k_i} = T_{k_i}\bar{\psi}$; $\bar{\psi} = (\psi^{(k_i - 1)},...,\psi)^T$; $S_{k_i}^{-1}$ is the matrix consisting of the corresponding k_i-rows of S^{-1}. Let us substitute (2.30) into the right side of the system (2.18). Let $\Phi(\tau), \Phi(0) = \mathrm{E}$ be the fundamental matrix of the system (2.18) with a control (2.30). From the conditions (2.29), (2.25), and (2.20) we obtain

$$\left\|\Phi(\tau)\right\| \le Ke^{-\lambda\tau},\ \ \lambda > n\alpha,\ \ \tau \in [0,\infty)\,. \tag{2.31}$$

System (2.16) with the control (2.30) (for the case $j = 2n$) can be represented in the following form

$$\frac{dc^{(2n)}}{d\tau} = A(\tau)c^{(2n)} + \frac{1}{2n!}\alpha e^{-(2n+1)\alpha\tau} P^{2n}(Px_1 + f)\,, \tag{2.32}$$

where

$$A(\tau) = \alpha e^{-\alpha\tau} P + \alpha e^{-\alpha\tau} Q e^{k\alpha\tau} \delta_k T_k^{-1} S_k^{-1};$$

$$M(\tau) = e^{k\alpha\tau} \delta_k T_k^{-1} S_k^{-1} = (e^{k_1\alpha\tau} \delta_{k_1} T_{k_1}^{-1} S_{k_1}^{-1},...,e^{k_r\alpha\tau} \delta_{k_r} T_{k_r}^{-1} S_{k_r}^{-1})^T\,.$$

The solution to the system (2.32) with initial conditions (2.17) (for $i = 2n$) has the form

$$c^{(2n)}(\tau) = \Phi(\tau)c^{(2n)}(0) + \frac{\alpha}{n!} P^{2n}(Px_1 + f)\int_0^\tau \Phi(\tau)\Phi^{-1}(t)e^{-(2n+1)}dt,\ \ \tau \in [0,\infty)\,. \tag{2.33}$$

Conditions (2.25) and (2.31) ensure the existence of a constant $K_1 > 0$ so that

$$\left\|\Phi(\tau)\Phi^{-1}(t)\right\| \le K_1 e^{-\lambda(\tau - t)} e^{(n-1)\alpha\tau},\quad \lambda > n\alpha,\ \ \tau \in [0,\infty)\,. \tag{2.34}$$

Formulas (2.33), (2.34) imply that

$$\begin{aligned} \left\|c^{(2n)}(\tau)\right\| &\le Ke^{-\lambda\tau}\left\|c^{(2n)}(0)\right\| + \\ &+ \frac{\alpha}{2n!}\left\|P^{2n}(Px_1 + f)\right\|\int_0^\tau e^{-\lambda(\tau - t)} K_1 e^{-n\alpha t} dt,\ K_1 > 0,\ \ \tau \in [0,\infty). \end{aligned} \tag{2.35}$$

Based on (2.35) we get

$$\left\|c^{(2n)}(\tau)\right\| \le Ke^{-\lambda\tau}\left\|c^{(2n)}(0)\right\| + K_2 e^{-n\alpha t},\ \ K_2 > 0,\ \ \tau \in [0,\infty)\,. \tag{2.36}$$

Condition (2.36) guarantees that

$$c^{(2n)}(\tau) \to 0 \quad \text{as} \quad \tau \to \infty. \tag{2.37}$$

Substituting the function (2.33) in (2.30) with ($i = 2n$) and moving to the original variable $c(\tau)$ using formulas (2.15) (with $j = 2n$), (2.14), (2.11), and (2.8) will provide a pair of functions $c(\tau) \in C^1[0,\infty)$, $d(\tau) \in C^1[0,\infty)$ which, according to (2.17) (with $j = 2n, 2n-1, \ldots, 1$) and (2.37), satisfy the system (2.5) and the conditions (2.7). If in the obtained pair of functions we return to the initial dependent and independent variables using formulas (2.6), (2.4), (2.1), (1.7), (1.4) and move to the limit as $t \to 1$, then we obtain the solution to (1.1), (1.3). In turn, the transition to the initial dependent and independent variables by formulas (1.4) and (1.7) gives the solution to the original problem (1.1), (1.3). The theorem has been proved.

Modeling example. Let vector f, matrices P, Q, and system (1.1) with conditions (1.3) have the form:

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Q = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, f = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, x = (x^1, x^2)^T,$$

$$x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, x(1) = x_1, x_1 = (x_1^1, x_1^2), x_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}.$$

Using the developed algorithm, after ordinary calculations we obtain the desired control function and the corresponding functions of phase coordinates in the following form:

$$u(t) = \frac{1}{(1-t)^2}((1-t)(\alpha^2(1-t)^2 - 2)a_1(t) + \frac{\alpha-3}{\alpha^3}a_2(t));$$

$$a_1(t) = \frac{-(\alpha-2)}{\alpha}(1-t)^{\frac{1-\alpha}{\alpha}} - 2\frac{1-\alpha}{\alpha}(1-t)^{\frac{2-\alpha}{\alpha}} - \alpha(1-t)^{\frac{1-\alpha}{\alpha}}\frac{(1-t)^{\frac{3\alpha-1}{\alpha}} - 1}{(1-3\alpha)} +$$

$$+ 2\alpha(1-t)^{\frac{2-\alpha}{\alpha}}\frac{(1-t)^{\frac{3\alpha-2}{\alpha}} - 1}{(2-3\alpha)},$$

$$a_2(t) = -(1-t)^{\frac{1}{\alpha}} + \alpha^2(1-t)^{\frac{1}{\alpha}}\frac{(1-t)^{\frac{3\alpha-1}{\alpha}} - 1}{1-3\alpha} - \alpha^2(1-t)^{\frac{2}{\alpha}}\frac{(1-t)^{\frac{3\alpha-2}{\alpha}} - 1}{2-3\alpha};$$

$$x^1(t) = \frac{-(\alpha-2)}{\alpha}(1-t)^{\frac{1-\alpha}{\alpha}} - 2\frac{1-\alpha}{\alpha}(1-t)^{\frac{2-\alpha}{\alpha}} - \alpha(1-t)^{\frac{1-\alpha}{\alpha}}\frac{(1-t)^{\frac{3\alpha-1}{\alpha}} - 1}{(1-3\alpha)} +$$

$$+ 2\alpha(1-t)^{\frac{2-\alpha}{\alpha}}\frac{(1-t)^{\frac{3\alpha-2}{\alpha}} - 1}{(2-3\alpha)} - 1,$$

$$x^2(t) = -(1-t)^{\frac{1}{\alpha}} + \alpha^2 (1-t)^{\frac{1}{\alpha}} \frac{(1-t)^{\frac{3\alpha-1}{\alpha}} - 1}{1-3\alpha} - \alpha^2 (1-t)^{\frac{2}{\alpha}} \frac{(1-t)^{\frac{3\alpha-2}{\alpha}} - 1}{2-3\alpha} + 1 - t\,;$$

$$\alpha = \frac{1}{4}.$$

3 Conclusion

The analysis of the proof shows that the method proposed in the article allows the possibility of finding the required control in analytical form. This fact significantly simplifies the implementation of the developed algorithm.

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