Viscoelastic models for ligaments and tendons

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Ligaments and tendons serve a variety of important functions in the human body. Many experimental studies have focused on understanding their mechanical behavior, mathematical modeling has also contributed important information. This paper presents a brief review of viscoelastic models that have been proposed to describe the nonlinear and time-dependent behavior of ligaments and tendons. Specific attention is devoted to quasi-linear viscoelasticity (QLV) and to our most recent approach, the single integral finite strain model (SIFS) which incorporates constitutive modeling of microstructural change. An example is given in which the SIFS model is used to describe the viscoelastic behavior of a human patellar tendon.

INTRODUCTION

Ligaments and tendons serve important functions throughout the body. Although individual ligaments and tendons differ in their specific structure, many aspects of their structure, composition, and function are similar. These soft connective tissues are primarily composed of closely packed collagen fiber bundles oriented in parallel fashion along the length of the tissue. The microstructural organization of the collagen fibers consists of several levels beginning with procollagen molecules, which self assemble into microfibrils. These then aggregate to form subfibrils, which organize into the structural unit referred to as the fibril, the elemental component of fibers. Histologically, fibrils appear in microstructural form in a wave pattern that is referred to as crimp (Viidik and Ekholm, 1968). Crimping is thought to have a significant influence upon the biomechanical behavior of ligaments and tendons. In addition to collagen fibrils, ligaments and tendons also contain elastin, proteoglycans, glycolipids, water and cells (Woo and Young, 1991). Although these solid constituents make up only a small percentage of the total weight of a ligament or tendon, they significantly influence the tissue behavior because of their inherent stiffness and their association with water, which comprises 60 to 70% of the total weight of ligaments and tendons. In addition, collagen fibrils themselves also exhibit viscoelastic properties. The interactions of these solid components and the movement of water within the tissue result in the time and history dependent behavior of ligaments and tendons.

One conceptual approach to formulating mathematical descriptions of ligaments and tendons is to focus on the observed structure. Models developed in this manner utilize generalizations of known mechanical responses of individual components of the ligament or tendon to predict gross mechanical behavior. These models generally include parameters which are directly related to the structure of the tissue and are particularly well suited for exploring the connection between structure and mechanical properties. A simplified structural representation of nonlinear behavior was presented by Frisen et al. (1969). It depicts ligaments and tendons as consisting of many parallel, individual linearly elastic components, each representing a fibril of different initial length in its unloaded and crimped form. Under relatively small tensile loads, crimped fibrils begin to straighten. Initially, there is little resistance to tension as the fibrils uncrimp, but as elongation progresses an increasing number of fibrils become taut. This recruitment of additional fibrils results in the nonlinear behavior characteristic of the "toe", or initial, region of the stress-strain curve. As elongation continues at higher loads, all the fibrils become taut and the ligament displays a more linear response.

A second conceptual approach focuses on the behavior of the tissue, and attempts to mathematically describe it in the simplest possible terms without explicit parameters related to the microstructure of the tissue. These phenomenological models have the advantage of being amenable to generalization, and are particularly useful

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for predicting behavior under a variety of testing conditions.

Although early models neglected the time dependent components of tissue behavior and concentrated on describing the nonlinear aspects of the stress-strain response, recent attention has focused on incorporating viscoelastic effects into these nonlinear models. In this brief review, we will present a summary of viscoelastic models that have been used, followed by a discussion of our current approach which involves constitutive modeling of microstructural change and its use in formulating mathematical descriptions of tendons and ligaments.

VISCOELASTIC MODELS

Mathematical models of ligaments and tendons face the dual challenge of describing both time-dependent and nonlinear behaviors. Although the time-dependent aspects of ligament and tendon behavior are due partly to the inherent viscoelasticity of collagen fibers, the primary influence is thought to be the movement of water within the solid tissue components. A number of viscoelastic models, often based on earlier elastic models, have been proposed. Viidik (1968) formulated a rheological model of parallel fibered viscoelastic tissues based on mechanical analogs (i.e. spring and dashpot combinations). In a subsequent investigation, this model was modified to account for nonlinearity of the elastic response (Frisen et al., 1969). Barbenel et al. (1973) utilized a generalization of the discrete element (i.e. spring and dashpot) models which incorporated a logarithmic relaxation spectrum. DeHoff (1978) and Bingham and DeHoff (1979) adapted a continuum-based approximate constitutive equation, which had been used to characterize non-linear viscoelastic behavior of polymers, to the description of soft biological tissues. Their constitutive equation assumed that ligaments could be modeled as isotropic viscoelastic media with fading memory. Lanir (1979) proposed a structural elastic model of skin in which the crimp of collagen fibrils is induced and then sustained by the contraction of an elastic filament attached to an isolated fibril, at random intervals. The distribution of straightening strains for the numerous buckled loops governs the low stress deformation behavior of the structure. By assuming that the individual collagen fibers were linearly viscoelastic, Lanir (1980) extended this elastic model to include viscoelastic behavior. His model was formulated for two special cases; one incorporated a high density of crosslinks between collagen and elastin fibers and the other assumed a low density of crosslinks. Subsequently, the model was generalized to incorporate three-dimensional viscoelastic theory (Lanir, 1983).

Another structural viscoelastic model was based on ideas proposed in formulating the elastic sequential straightening and loading model (Kastelic et al., 1980). This model includes the assumption that resistance arises only from the elasticity of already straightened fibrils, with crimped fibrils assumed to have negligible resistance to extension. A morphologically based range of crimp angles is assumed for the fibrils within the fascicle. The resulting equation related crimp angle and crimp angle distribution to the stress generated by elongation. A similar elastic model was proposed by Decraemer et al. (1980a), who assumed that soft tissue could be modeled as a large number of purely elastic fibers embedded in a gelatin-like liquid. All fibers were assumed to have the same modulus and cross-sectional area but different lengths, being normally distributed about some known mean. Fibers were more or less folded based on their initial length. Decraemer et al. (1980b) extended this structural elastic model to the viscoelastic case by incorporating internal friction between fibers, and between fibers and the material in which they are embedded. Damping was introduced into the model by assuming that all fibers display identical linear viscoelastic properties.

QUASI-LINEAR VISCOELASTIC MODELS

Fung (1968) first formulated Quasi-Linear Viscoelasticity (QLV), which continues to be commonly used for viscoelastic modeling of ligaments and tendons as well as a wide variety of other tissues. The formulation of QLV combines elastic and time dependent components of a tissue's mechanical response using a hereditary integral formulation. An exponential form of the stress-strain relation for uniaxial tension has most often been incorporated into this general viscoelastic model.

Quasi-linear Viscoelasticity has been used to describe collagen fibers (Haut and Little, 1972), as well as a variety of ligaments and tendons including the medial collateral ligament (Woo et al., 1981; Woo, 1982), the anterior cruciate ligament (Lin et al., 1987; Lyon et al., 1988), and the patellar tendon (Lin et al., 1987). A curve-fit of QLV to stress-strain curves from canine medial collateral ligaments is shown in Figure 1. Because of its extensive use in biomechanics, a number of investigations have also concentrated on refining numerical methods for estimating QLV parameters (Dortmans et al., 1984; Myers er al., 1991; Nigul and Nigul, 1987; Sauren and Rousseau, 1983).

A similar model was developed by Pradas and Calleja (1990) who proposed a one-dimensional, nonlinear viscoelastic model to describe the creep behavior of a human flexor tendon. They employed an assumption of quasi-linear behavior, similar to that used in QLV, in formulating a constitutive equation.

SINGLE INTEGRAL FINITE STRAIN (SIFS) MODEL

Recently, we have used a general single integral finite strain (SIFS) viscoelastic theory to model the human patellar tendon (Johnson et al., 1992). This structurally motivated continuum model is founded on two basic concepts. First, the nonlinear viscoelastic constitutive equa-

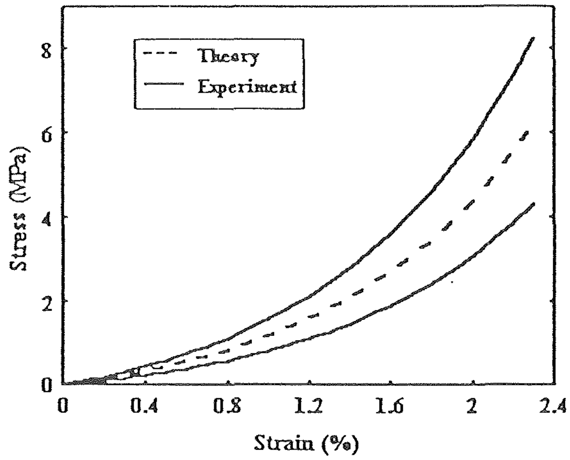

Figure 1: Stress-strain curves for canine medial collateral ligaments (n=8). Upper and lower bounds shown with QLV curve-fit. Adapted from Woo et al., 1981.

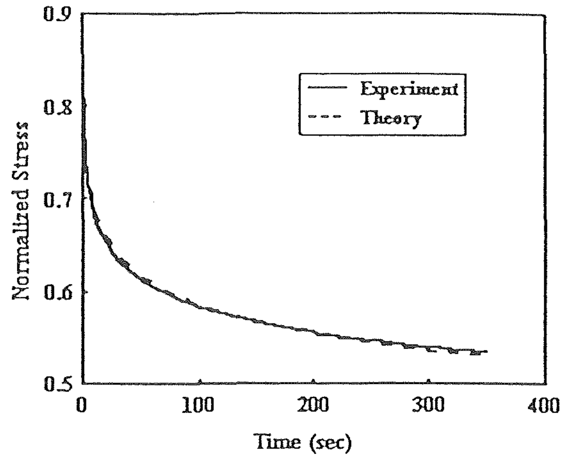

Figure 2: Single integral finite strain (SIFS) curve-fit to stress-relaxation data from a human patellar tendon.

tion incorporates a general nonlinear, three-dimensional description of a tissue's mechanical behavior. Second, the idea of microstructural change is incorporated in the uniaxial problem to describe the nonlinear stress-strain relationship resulting from recruitment. The concept of fading memory is also incorporated in the viscoelastic response. The specific form for the three-dimensional equation is obtained by truncating an integral series representation of viscoelastic behavior, and assuming finite deformation and fading memory (Wineman, 1972). This equation may be written as:

$$\mathrm{T} = -p\mathrm{I} + C_0\left\{[1+\mu I(t)]\mathrm{B}(t) - \mu \mathrm{B}^2(t)\right\} - (C_0 - C_\infty)$$
$$\int_0^t \dot{G}(t-s)\left\{[1+\mu I(s)]\mathrm{B}(t) - \mu \mathrm{F}(t)\mathrm{C}(s)\mathrm{F}^T(t)\right\} ds, \quad (1)$$

where T is the Cauchy stress, p is the indeterminate part of the stress arising due to the constraint of incompressibility, I is the identity tensor, B is the left Cauchy-Green strain tensor, $G(t)$ is the time-dependent relaxation function, C_0 is the instantaneous modulus, C_∞ is the long time modulus, μ is the shear modulus, and $I(s) = \mathrm{tr}\,\mathrm{C}$, where C is the right Cauchy-Green strain tensor. This constitutive equation reduces to the appropriate case of finite elasticity at very short times and, if linearized, yields classical linear viscoelasticity.

Having obtained an equation describing finite viscoelasticity, nonlinear stress-strain effects can be incorporated in the form of microstructural change. The idea that deformation imposed on a material can alter the micromechanism responsible for generating stress in that material was introduced by Tobolsky and Andrews (1945) and has since been extended and generalized for use in describing a variety of inelastic behaviors (Wineman and Rajagopal, 1990; Rajagopal and Wineman, 1992). These ideas may also be applied to the mechanical description of tissues. Chu and Blatz (1972) noted that simple viscoelasticity was insufficient to describe hysteresis of living tissue, as it predicts relaxation times to be the same for both loading and unloading. These investigators formulated a one-dimensional model for hysteresis based on a theory of cumulative microdamage. In their model, the constitutive equation describing the stress-strain response of a tissue changed with deformation to account for differences in loading and unloading behavior.

In the single integral finite strain viscoelastic model different constitutive equations are used for different levels of strain and "patched" together mathematically. Within the context of a continuum theory the specific change taking place within the ligament substance is not of paramount concern, but the form of the proposed constitutive equation must still reflect the different mechanisms involved. Thus, an expression for stress, $\sigma_1(\lambda)$, is used in the "toe" region and a second expression, $\sigma_2(\lambda)$ in the linear region. The model stretch parameter, $\hat{\lambda}$, marks the transition from non-linear to linear response on the stress-strain curve. For $\lambda \le \hat{\lambda}$ the stress is given by $\sigma = \sigma_1$ and for $\lambda \ge \hat{\lambda}$ the stress is given by $\sigma = f(\sigma_1, \sigma_2)$:

$$\sigma = \left\{ \begin{array}{ll} \sigma_1(\lambda) & \lambda \le \hat{\lambda} \\ \sigma_2(\frac{\lambda}{\hat{\lambda}}) + \sigma_1(\hat{\lambda}) & \lambda \ge \hat{\lambda} \end{array} \right\} \quad (2)$$

It is important to note that for the second region a new stretch should be defined so that $\hat{\lambda}$ is the reference stretch.

Stress-strain equations for the two regions were curve-fit to data obtained from experiments on two human patellar tendons. The value of $\hat{\lambda}$ was determined from the stress-strain curve and the model parameters were then determined by fitting the SIFS model equations to

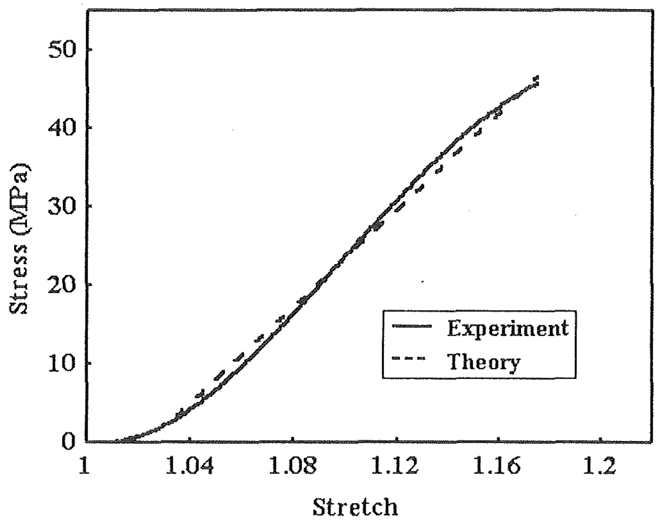

Figure 3: Single integral finite strain (SIFS) curve-fit to stress-strain data from a human patellar tendon.

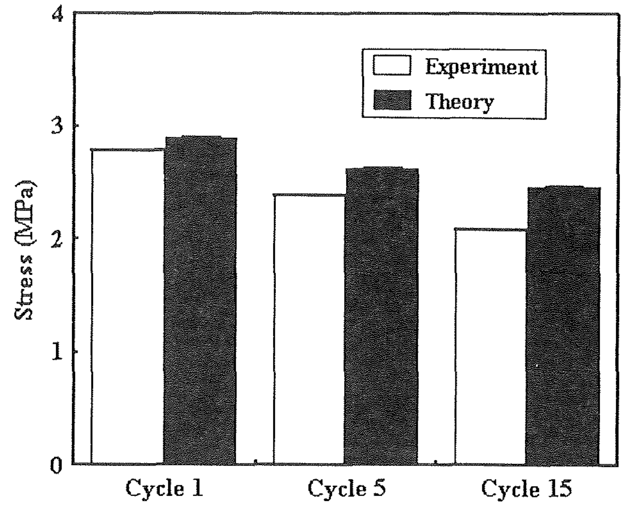

Figure 4: Experimental peak cyclic stresses and SIFS theory predictions for a human patellar tendon specimen.

stress-relaxation and stress-strain data (Figures 2 and 3). For confirmation, the parameters determined from the curve fits were used to predict peak stresses in the patellar tendons resulting from a cyclic elongation. Figure 4 shows the predicted stress and the experimentally measured peak stress resulting from two cyclic tests.

Based on the success of this one-dimensional application, the single integral finite strain (SIFS) viscoelastic model shows promise as a general theoretical framework that includes nonlinear, three-dimensional, finite viscoelasticity. It is also possible to extend this representation of viscoelastic behavior to describe anisotropic properties of ligaments and tendons. It will then be possible to use this model in describing deformations of ligaments and tendons that are more complex than uniaxial extension, thus providing a valuable tool for further understanding and more accurate prediction of the mechanical behavior of ligaments and tendons under complex loading conditions.

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