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Sources of error in road safety scheme evaluation: a method to deal with outdated accident prediction models

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Abstract

This paper considers the errors that arise in using outdated accident prediction models in road safety scheme evaluation. Methods to correct for regression-to-mean (RTM) effects in scheme evaluation normally rely on the use of accident prediction models. However, because accident risk tends to decline over time, such models tend to become outdated and the estimated treatment effect is then exaggerated. A new correction procedure is described which can effectively eliminate such errors.

Keywords: Road safety; Regression-to-mean; Trend in risk; Correction for bias

1. Introduction

The task of estimating the effect of a road safety scheme on the mean frequency of accidents is not straightforward. While observations of accidents before and after treatment can establish the change in mean accident frequency, it is unlikely that all of the observed change can be attributed to the effects of the scheme. The primary task in scheme evaluation is then that of separating scheme effects, S, from the changes that would have occurred without the scheme, N. In a recent paper (Hirst et al., in press) the authors considered in detail the various factors that can have a confounding effect in the evaluation of road safety schemes and suggested a simple additive model to describe these.

The three main non-scheme sources of change in observed accident frequencies are regression-to-mean (RTM) effects; trends in accidents; and local changes in flow (due to transport or land use changes unrelated to the scheme under study). The observed change in annual accidents, B, can be written as

$$B = S + N$$

The non-scheme effects are then

$$N = N_T + N_F + N_R$$

where N_T is the change due to national trends in accidents over the period of observation arising as a result of the combined effect of trends in risk and in flow; N_F the change in accidents due to local changes in flow other than those attributable to trend but unrelated to the study scheme and N_R is the change in accidents due to the RTM effect.

The change in accidents attributable to the scheme may be in part due to the effect of the scheme on accident risk (accidents per unit of exposure), S_R, and in part due to the effect of the scheme on flow, S_F. Thus

$$S = S_R + S_F$$

and

$$B = S_R + S_F + N_T + N_R + N_F$$

The authors (Hirst et al., in press) have proposed a modification to current methods which allows the reduction in accidents attributable to each of the five causal factors to be separately evaluated. The proposed approach, in common with others that include a correction for RTM effects (see, for example, Hauer, 1997; Elvik, 1997), relies on the availability of suitable predictive accident models. These are assumed to represent the relationship between mean accident frequency and various explanatory variables (typically traffic flow and site characteristics) during the scheme evaluation period. The problem is that, in practice, this assumption will rarely be satisfied because of the effects of trends in accidents.

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2. Outdated accident prediction models

To appreciate the problem, it is useful to briefly consider the nature of the evaluation process. In order to estimate the true scheme effect, it is necessary to estimate what the expected accident frequency in the period after treatment would have been had the scheme not been implemented. A common approach is to use an empirical Bayes (EB) method (see, for example, Maher and Summersgill, 1996; Hauer, 1997; Elvik, 1997). In this the mean accident frequency in the before period is estimated as a weighted average of observed accidents before treatment, X_B, and a predictive model estimate of expected accidents given the nature of the site and the level of traffic flow. The general form of predictive accident models is

$$\hat{\mu} = Cq_B^{\beta}$$

where C is a constant for each site (incorporating the relevant site characteristics for the particular model used), q_B a measure of traffic flow in the period before treatment and β is the predictive model coefficient for flow. The predictive model estimate of *total* accidents in a before period of t_B years is then

$$\hat{\mu}_B = t_B\hat{\mu}$$

Generally such predictive models assume that the random errors are from the negative binomial (NB) family. If K is the shape parameter for the NB distribution, the EB estimate of total accidents in the before period, $\hat{M}_B$, is calculated as

$$\hat{M}_B = \alpha\hat{\mu}_B + (1-\alpha)X_B$$

where

$$\alpha = \left(1 + \frac{\hat{\mu}_B}{K}\right)^{-1}$$

The EB estimate of expected accidents in the after period in the absence of the scheme, $\hat{M}_A$, can then be estimated. The effects of general trends in risk and flow on accidents during the study period can be accounted for by using a comparison group ratio of accidents

$$\frac{A_{A\_NAT}}{A_{B\_NAT}}$$

where A_{B_NAT} is the total national (or regional) accidents in the before period of t_B years and A_{A_NAT} is the total national (or regional) accidents in the after period of t_A years.

The use of a comparison group ratio implicitly assumes that flows at the study site have changed in line with national or regional trends. To take account of the effects of any local flow changes, while avoiding double counting, it is necessary to have a representative measure of traffic flow at the scheme in the after period, q_A, together with flow data for the comparison group. If Q_{B_NAT}: total national (or regional) flow in the before period, Q_{A_NAT}: total national (or regional) flow in the after period, then the expected flow in the after period if flows at the study site had changed in line with general trends, q'_A, can be estimated using

$$q'_A = \left(\frac{Q_{A\_NAT}/t_A}{Q_{B\_NAT}/t_B}\right) q_B$$

If the observed flow in after period, q_A, differs from q'_A then there have been local changes in flow at the site other than those attributable to trend. If, on the basis of local knowledge, these are judged to be due to transport or land use changes unrelated to the scheme under study, then the expected accidents in the after period in the absence of the scheme is

$$\hat{M}_A = \hat{M}_B \left(\frac{A_{A\_NAT}}{A_{B\_NAT}}\right)\left(\frac{q_A}{q'_A}\right)^{\beta}$$

If, on the other hand, the local flow changes are judged to be a consequence of the scheme itself, then

$$\hat{M}_A = \hat{M}_B \left(\frac{A_{A\_NAT}}{A_{B\_NAT}}\right)$$

If X_A accidents are observed at the scheme site in the after period, the scheme effect is estimated as

$$\hat{S} = \frac{(X_A/t_A) - (\hat{M}_A/t_A)}{X_B/t_B}$$

and the non-scheme effects as

$$\hat{N} = \frac{(\hat{M}_A/t_B) - (X_B/t_B)}{X_B/t_B}$$

It is clear that the EB approach implicitly assumes that the predictive model represents the relationship between accidents and flows in the before period at the study site. Equally, the comparison group approach implicitly recognises that there can be an underlying trend in risk within the study period. However, no allowance is made for the effects of trend in risk between the time period used for modelling and the time period used for scheme assessment: this in spite of the fact that available models are typically derived using historical data, often for a period of time many years prior to the study period used for scheme assessment.

The standard form of the available predictive models assumes that the risk of accidents, C, per unit of exposure, q^{β}, is constant over time. The value of C represents the average risk per unit of exposure during the modelled period. In practice we do not expect accident risk per unit of exposure (C) to remain constant over time: the whole purpose of many road safety initiatives is to reduce risk at a regional or national level. Measures such as improvements in road user training, national road safety awareness initiatives, and speed enforcement campaigns are all believed to reduce accident risk per unit of exposure. In the UK there is evidence to suggest that accident risk as a function of exposure has been declining over time. For example, for the years 1975–1995, based on national data, the average rate of decline in accident risk was found to be 2% per year while for a subset of roads in six English counties over the period 1980–1991 the rate of decline was estimated to be 5% per year on link sections and 6% per year at major junctions (Mountain et al., 1997, 1998). It has recently become recommended practice in the UK (DfT, 2002) to allow for trends in accident risk, with the predicted annual change depending on the location. For most urban roads (speed limit ≤ 40 mph) the predicted decrease in risk is 1.6% per year, with a decrease of 0.09% at major urban junctions and 2.4% at minor junctions.

If it is accepted that there are trends in risk over time then it must also be recognised that predictive models that do not allow for trend in risk will rapidly become outdated: they represent the average accident risk per unit of exposure only over the modelled period. As a consequence, if the before period for the scheme to be evaluated is not contained within the modelled period, the estimates of accidents in the before period will be biased. Since predictive models are generally based on historical data, the elapsed time between the modelled period and the before period (and hence the effects of trend) may well be large. For example, a typical model for UK urban single carriageway roads was derived using accident data for a 5-year-period from April 1983 to March 1988 (Summersgill and Layfield, 1996). The models routinely used to predict accidents at UK intersections (Binning, 1996, 2000) are based on accident data for the 6-year-period 1974–1979 in the case of four-arm roundabouts and for the period 1984–1989 in the case of urban priority intersections. While it would, of course, be theoretically possible to update predictive accident models at regular intervals, this is not normally done in practice because of the high cost of carrying out such studies.

A more appropriate form of predictive model would be one which allows for trend in risk. One such model (Maher and Summersgill, 1996) takes the form

$$\hat{\mu}_t = C_0 \gamma^t q_t^\beta$$

where $\hat{\mu}_t$ is the expected number of accidents in year t; C_0 the risk in year 0; γ the factor by which risk changes from year to year and q_t is the flow in year t.

This model is a marginal model that avoids modelling the year-to-year variation but allows for trend in risk based on an annual change factor (γ). The merits of various trend models are discussed by Lord and Persaud (2000) but this form of model is perhaps the most fruitful to consider here since the change in risk from year to year is fixed, allowing predictions beyond the modelled period.

While models which allow for trend have been fitted to accident data (Mountain et al., 1997, 1998; Lord and Persaud, 2000) such models are not widely available: for most site types in most regions the only available predictive accident models do not include a trend term. This is in part because suitable data are not readily available: ideally accident and traffic counts for many years are needed, with the traffic counts for each year treated as separate observations. In addition, the disaggregation of the data presents difficulties for traditional model fitting procedures (Maher and Summersgill, 1996, Lord and Persaud, 2000). The aim in this study was therefore to produce a correction for the bias introduced by using the more commonly available form of model: an outdated accident prediction model with no trend term.

3. Bias arising from using the model without trend

The underlying assumption is that the trend model outlined above is the correct form of model. If a predictive accident model of the form $\hat{\mu}_t = Cq_t^\beta$ is fitted when there is actually a trend in risk, the model is mis-specified. It is necessary to consider what implications this may have for estimates of expected accidents.

It is assumed, for a sample of sites, that accident and flow data are available for each year of an n year modelling period. Accidents will have a mean of $\mu_0 = C_0 q_0^\beta$ in the first year of the study period ($t = 0$) and in the final year ($t = n-1$) a mean of $\mu_{(n-1)} = C_0 \gamma^{(n-1)} q_{(n-1)}^\beta$. The model without trend is normally derived using a single estimate of the mean observed flow in the model period, $\bar{q}$, and thus, for the total n-year-period, the fitted model is

$$C\bar{q}^\beta n \sim NB\left(\sum_{t=0}^{n-1} \mu_i, K\right), \quad \text{where } \sum_{t=0}^{n-1} \mu_i = C_0 \sum_{t=0}^{n-1} \gamma^t q_t^\beta$$

A simple rearrangement of the model equation and the total true accident mean gives

$$C = \frac{C_0 \sum_{t=0}^{n-1} \gamma^t q_t^\beta}{\bar{q}^\beta n} = \frac{\text{mean accidents}}{(\text{mean flow})^\beta}$$

Thus C could be estimated as a function of mean accidents and flows. It can be assumed that the mean of accidents and the mean of flows occur at approximately the middle of the modelled period (at time $t = (n-1)/2$). This is illustrated for a specific example in Fig. 1. In line with the results of Mountain et al. (1997), the example is for a 12-year modelled period (1980–1991) for a site with typical flows with $C_0 = 3$, $\beta = 0.61$ and $\gamma = 0.95$. It can be seen that the mean of accidents and of flows both occur close to the mid-point of the modelled period ($t = 5.5$ in this example).

In practice, the mean flow will only occur at the mid-point of the modelled period if flows follow an arithmetic progression but this assumption should not be unreasonable if flows are not changing too dramatically over time. The assumption that the mean of accidents occurs in the middle year is also not likely to be strictly true since it is assumed that the decline in risk follows a geometric progression while flows are increasing: again if flows are not changing too dramatically over time, and γ is reasonably close to 1, this assumption should not be unreasonable. Under these assumptions,

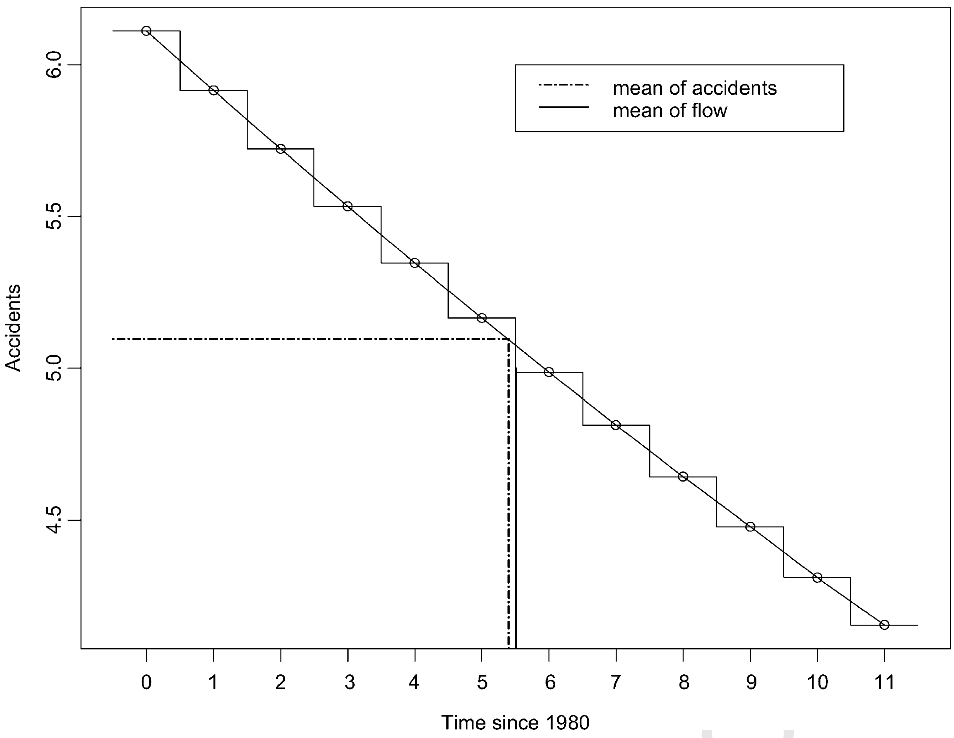

Fig. 1. Accidents for 1980–1991 (typical UK link flow with $C_0 = 3$, $\gamma = 0.95$ and $\beta = 0.61$).

it is possible to equate the models at the middle of the modelling period ($t = (n - 1)/2$). If it is also assumed that the power of flow (β) is the same for both models (not necessarily true since available models have a range of values for β and estimates of β and C are not independent) then

$$C \approx \frac{C_0 \gamma^{(n-1)/2} \bar{q}^{\beta}}{\bar{q}^{\beta}} = C_0 \gamma^{(n-1)/2}$$

Assuming that $C = C_0 \gamma^{(n-1)/2}$, Fig. 2 shows how the predicted before mean accident frequency ($\hat{\mu}_{\mathrm{B}}$) for a study site some years after the modelled period would be affected by trend in risk. In this hypothetical example, the scheme site has a before period of 3 years (1997–1999) and the modelled period is 12 years (1980–1991) as before. There is thus a gap of 5 years (1992–1996) between the end of the modelled period and the start of the before period. Traffic flows are assumed to increase arithmetically over time (in line with the actual growth in traffic flow in the UK over the period 1980–1999). Thus the model without a trend in risk term shows an increase in expected accidents in each year, in line with the increase in flow. The model with a trend term reflects the combined effects of the increasing traffic flows together with the declining accident risk ($\gamma = 0.95$). The overall effect in this case is a decrease in expected accidents over time.

The two models, under these assumptions, are equivalent at the mid-point of the modelled period. Assuming that, for the 3-year before period at the scheme, the mean of flows also occurs in the middle year, the effects of trend between the middle of the modelled period and the middle of the before period can be estimated. For this it is convenient to shift the time datum point ($t = 0$) to the middle of the modelling period. With this time datum, at $t = 0$, $\mu_0 = C q_0^{\beta}$ and for subsequent years $\mu_t = C \gamma^t q_t^{\beta}$. The last year of the modelled period occurs at $t = 5.5$ (i.e. $t = (n - 1)/2$), the last year of the gap between the end of the modelled period and the start of the before period will be at $t = 10.5$ (i.e. $t = ((n-1)/2) + g$, where g is the duration of the gap). The middle of the before period will occur in the second year of the 3-year-period at $t = 12.5$. More generally, if t_{B} is the duration of the before period as before,

$$t = \left(\frac{n-1}{2}\right) + g + \left(\frac{t_{\mathrm{B}}+1}{2}\right) = g + \left(\frac{n+t_{\mathrm{B}}}{2}\right)$$

For this example, the estimated means ($\hat{\mu}_{\mathrm{B}}$ or $\hat{\mu} t_{\mathrm{B}}$) obtained using the models with and without trend would differ by a factor of $\gamma^{12.5}$ (the trend model giving the smaller estimate).

This result leads to the possibility of a correction procedure which could be applied to any mis-specified model. Thus, more generally, if $\hat{\mu}_{\mathrm{B}}$ is estimated using a mis-specified predictive model which makes no allowance for trend, the estimate ($\hat{\mu}_{\mathrm{B\,NO\,TREND}}$) can be corrected using

$$\hat{\mu}_{\mathrm{B\,CORRECTED}} = \gamma^t \hat{\mu}_{\mathrm{B\,NO\,TREND}}$$

where γ is the factor by which risk changes from year to year and t the elapsed time between the middle of the modelling and study periods $= g + (n + t_{\mathrm{B}})/2$.

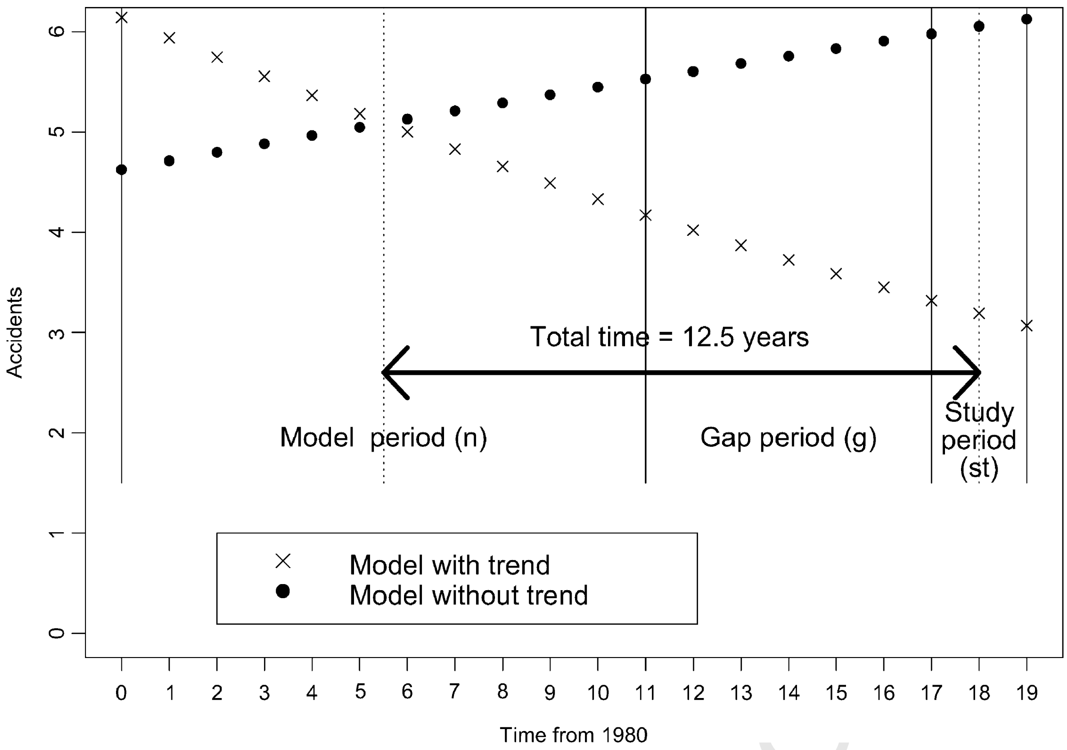

Fig. 2. Accidents for 1980–1999 (typical UK link flow with $C_0 = 3$, $\gamma = 0.95$ and $\beta = 0.61$).

This definition of the expected bias arising when fitting a model without a trend in risk term to data which exhibits trend relies on a number of assumptions. No attempt has been made to mathematically derive these suggested results and instead justification is now sought via simulation.

4. Simulation studies to determine the magnitude of bias

Simulations were carried out to assess the relationships suggested above. The aim in the simulations was to reflect the conditions that might be encountered in a typical accident study. It was thus necessary to select typical time periods; typical accident model parameters; and typical accident trends. It was also necessary to generate observed accident data for typical safety scheme study sites: sites which are normally selected (at least partially) on the basis of a high accident frequency in a particular time period and thus subject to a RTM effect in a subsequent time period.

Each simulation study followed a pre-defined time period. This comprised a modelling period of either 5 years or 12 years ending in 1991, a gap of 3 years between the end of the modelling period and the study period, and a 7-year study period for new sites under investigation. The 5-year modelling period is typical of the periods used to derive models with no trend term; the 12-year-period was that used by Mountain et al. (1997) to derive a model with trend. The 7-year study period comprised a 3-year before period (1995–1997), a 1-year investigation and treatment period, and a 3-year after period (1999–2001). The underlying population characteristics for the trend model (C_0, β, γ and K) were fixed in advance. The true parameters were chosen so that $C_0 = 3$ (reflecting an average value for treated sites currently under investigation in a research project at the University of Liverpool), with $\beta = 0.61$ and $K = 1.92$ (in line with the Mountain et al. (1997) model for link data). The annual change in risk was set at 2.5 and 5% ($\gamma = 0.975$ and 0.95): in line with the UK national trend in risk over the period 1980–2001 (3%) and with the Mountain et al. (1997, 1998) model for link data for 1980–1991 (5%). The number of sites (nmod) in the sample used to estimate the model parameters was also fixed at 100 (chosen to represent a typically sized data set such as that used by Summersgill and Layfield (1996)) and at 1000 (roughly the size of the data set used by Mountain et al. (1997) to fit trend models for link data). The different combinations of time period, number of sites and values of γ meant that eight individual simulation studies were carried out.

Each simulation consisted of 500 realisations. For each of the 500 realisations, nmod sites were generated from the true underlying population characteristics C_0, β, γ and K. Each of the nmod sites followed a randomly generated subset of the model period.

In order to calculate the mean accidents at each site it was necessary to simulate traffic counts. This was done so that overall flows followed an arithmetic progression (the best fitting model to UK national flow data for the hypothetical study period) and so that the overall total flows for the nmod sites increase by a factor of 1.9 from 1975 to 2000 (again in line with UK national flow data), although annual flows at individual sites could vary from this relationship from year to year. The distribution of flows across sites was generated to reflect the observed flows used by Layfield and Summersgill (1996) to derive a model for urban single carriageway roads. Once a flow vector for each of the nmod sites had been generated, the true underlying mean accidents for that site was known. This, together with the NB shape parameter K, was used to generate observed accidents at the site from a NB distribution.

The models with and without a trend term were then fitted to the observed data for the nmod sites, giving estimates $\hat{C}_0$, $\hat{\beta}_{\mathrm{TREND}}$ and $\hat{\gamma}$ for the trend model and $\hat{C}$ and $\hat{\beta}_{\mathrm{NO\,TREND}}$ for the model without trend. Estimation for the trend model was achieved via the algorithm outlined by Maher and Summersgill (1996). This is an approximate fit based on linearising the predictors using constructed variables (see, for example, Atkinson, 1985; Cook and Weisberg, 1982).

For each of the eight simulations (consisting of 500 model realisations), 100 study sites were generated following an overall average (but not individually fixed) observed change in accidents of either −50% or −75%. Observed accidents in the before period were generated from the true mean, μ_{TRUE} for each study site. An unknown, but definite RTM effect was achieved by rejecting any generated before period accidents less than twice the true mean and re-sampling (i.e. sites with $X_{\mathrm{B}} < 2\mu_{\mathrm{TRUE}}$ rejected, as might typically be the case in selecting candidate sites for safety schemes).

For both the correctly specified trend model and the mis-specified model without trend, the bias in the estimate of the true mean was defined as τ, where

$$\tau\mu_{\mathrm{TRUE}} = \hat{\mu}_{\mathrm{B}}$$

For the model without trend

$$\tau = \frac{\hat{\mu}_{\mathrm{B\,NO\,TREND}}}{\mu_{\mathrm{TRUE}}} = \frac{t_{\mathrm{B}}\hat{C}\bar{q}^{\hat{\beta}\,\mathrm{NO\,TREND}}}{C_0\sum_{t\in\mathrm{BEFORE\,PERIOD}}\gamma^t q_t^{\beta}}$$

For the model with trend

$$\tau = \frac{\hat{\mu}_{\mathrm{B\,TREND}}}{\mu_{\mathrm{TRUE}}} = \frac{\hat{C}_0\sum_{t\in\mathrm{BEFORE\,PERIOD}}\hat{\gamma}^t q_t^{\hat{\beta}\,\mathrm{TREND}}}{C_0\sum_{t\in\mathrm{BEFORE\,PERIOD}}\gamma^t q_t^{\beta}}$$

For the trend model (if the parameter estimates are unbiased) it would be expected that the mean of τ would be 1 while, for the model without trend (for a study period after the modelled period), it would be expected that τ >1. The main reason for examining any bias resulting from a correctly specified trend model was to examine the stability of the approximation in estimating the model parameters.

It is important to examine the biases that may arise, not only in the predictive model estimates ($\hat{\mu}_{\mathrm{B}}$), but also in the EB estimates ($\hat{M}_{\mathrm{B}}$). This is used to estimate $\hat{M}_{\mathrm{A}}$ and hence the scheme and non-scheme effects (S_{R}, S_{F}, N_{T}, N_{R} and N_{F}) (Hirst et al., in press). The bias in the EB estimate is

$$\rho = \frac{\hat{M}_{\mathrm{B}}}{M_{\mathrm{B\,TRUE}}} = \frac{(K_{\mathrm{TRUE}} + \mu_{\mathrm{TRUE}})(\hat{K} + X_{\mathrm{B}})\hat{\mu}_{\mathrm{B}}}{(\hat{K} + \hat{\mu}_{\mathrm{B}})(K_{\mathrm{TRUE}} + X_{\mathrm{B}})\mu_{\mathrm{TRUE}}}$$

$$= \frac{(K_{\mathrm{TRUE}} + \mu_{\mathrm{TRUE}})(\hat{K} + X_{\mathrm{B}})}{((\hat{K}/\tau) + \mu_{\mathrm{TRUE}})(K_{\mathrm{TRUE}} + X_{\mathrm{B}})}$$

if $\hat{K} \approx K_{\mathrm{TRUE}}$ then

$$\rho \approx \frac{(K_{\mathrm{TRUE}} + \mu_{\mathrm{TRUE}})}{((\hat{K}/\tau) + \mu_{\mathrm{TRUE}})}$$

The bias in the EB estimates for individual sites, and in the estimates of the effects of regression-to-mean (N_{R}), trend (N_{T}) and treatment effects (S_{R} and S_{F}) were examined for each of the 500 studies of 100 sites. (It was assumed in this study that $N_{\mathrm{F}} = 0$.)

5. Results from the simulation studies

The simulation studies demonstrated that the relationship between C_0 and C was consistent with that suggested ($C \approx C_0\gamma^{(n-1)/2}$) and the estimate of β from both models was unbiased. The bias in the predictive model estimate of mean accidents in the before period was thus also consistent with that suggested previously. Thus

$$E(\tau) = \gamma^{-t}, \quad \text{where } t = g + \left(\frac{n + t_{\mathrm{B}}}{2}\right)$$

A simple correction to the estimate from the model without trend is therefore to multiply the estimated before mean from the mis-specified model by the inverse of the expected bias

$$\hat{\mu}_{\mathrm{B\,CORRECTED}} = \hat{\mu}_{\mathrm{B\,NO\,TREND}}\,(E(\tau)^{-1})$$

which is equivalent to the correction procedure proposed, namely

$$\hat{\mu}_{\mathrm{B\,CORRECTED}} = \gamma^t\hat{\mu}_{\mathrm{B\,NO\,TREND}}$$

Clearly this correction requires an estimate of γ. If total annual flows (Q_{NAT_i}) and accidents (A_{NAT_i}) are available for an appropriate comparison group over the relevant time period, then an estimate of γ can be obtained by fitting a model of the form

$$A_{\mathrm{NAT}\_i} = A_0\gamma^i Q_{\mathrm{NAT}\_i} \quad \text{for } i = 0, \ldots, ((n-1) + g + st)$$

Table 1 summarises the bias in the predictive model estimates of mean accidents in the before period ($\hat{\mu}_{\mathrm{B}}$) and the bias in the EB estimates ($\hat{M}_{\mathrm{B}}$) obtained using the three approaches: the trend model, the mis-specified model without trend and the proposed correction procedure. Using a data set of 1000 sites and a modelling period of 12 years, the estimates obtained using the trend model were as expected, with the mean and median of the bias (τ_{TREND}) close to 1.

Table 1
Bias in the predictive model estimates of mean accidents in the before period (τ) and the EB estimates (ρ)

γ, model period (years), n	τ_{TREND}			$\tau_{\text{NO TREND}}$			$\tau_{\text{CORRECTED}}$			ρ_{TREND}			$\rho_{\text{NO TREND}}$			$\rho_{\text{CORRECTED}}$		
	Mean	Median	S.D.	Mean	Median	S.D.	Mean	Median	S.D.	Mean	Median	S.D.	Mean	Median	S.D.	Mean	Median	S.D.
0.95, 5, 100	3.97	1.07	11.6	1.44	1.43	0.16	1	1	0.11	0.92	1.01	0.27	1.05	1.04	0.03	1	1	0.03
0.95, 5, 1000	1.14	1.01	0.58	1.43	1.43	0.05	1	1	0.03	0.99	1	0.08	1.05	1.04	0.03	1	1	0.01
0.95, 12, 100	1.16	0.98	0.70	1.72	1.71	0.19	1	1	0.11	0.97	0.99	0.14	1.09	1.07	0.05	0.99	1	0.03
0.95, 12, 1000	1.02	1.01	0.18	1.72	1.72	0.06	1	1	0.03	1	1	0.04	1.09	1.07	0.05	1	1	0.01
0.975, 5, 100	3.31	0.93	7.9	1.2	1.19	0.13	1	1	0.11	0.9	0.99	0.26	1.02	1.01	0.03	1	1	0.03
0.975, 5, 1000	1.14	1.02	0.59	1.2	1.19	0.04	1	1	0.04	0.99	1	0.07	1.02	1.02	0.01	1	1	0.01
0.975, 12, 100	1.18	1.01	0.79	1.31	1.3	0.15	1	1	0.11	0.98	1	0.11	1.03	1.03	0.03	0.99	1	0.03
0.975, 12, 1000	1.02	1	0.17	1.3	1.3	0.04	1	1	0.03	1	1	0.03	1.04	1.03	0.02	1	1	0.01

Mean: mean of bias; med: median of bias; S.D.: standard deviation of the bias. Results are shown to two decimal places. τ_{TREND}: bias in predictive model estimates using trend model; $\tau_{\text{NO TREND}}$: bias in predictive model estimates using model without trend; $\tau_{\text{CORRECTED}}$: bias in predictive model estimates using correction procedure; ρ_{TREND}: bias in EB estimates using trend model; $\rho_{\text{NO TREND}}$: bias in EB estimates using model without trend; $\rho_{\text{CORRECTED}}$: bias in EB estimates using correction procedure.

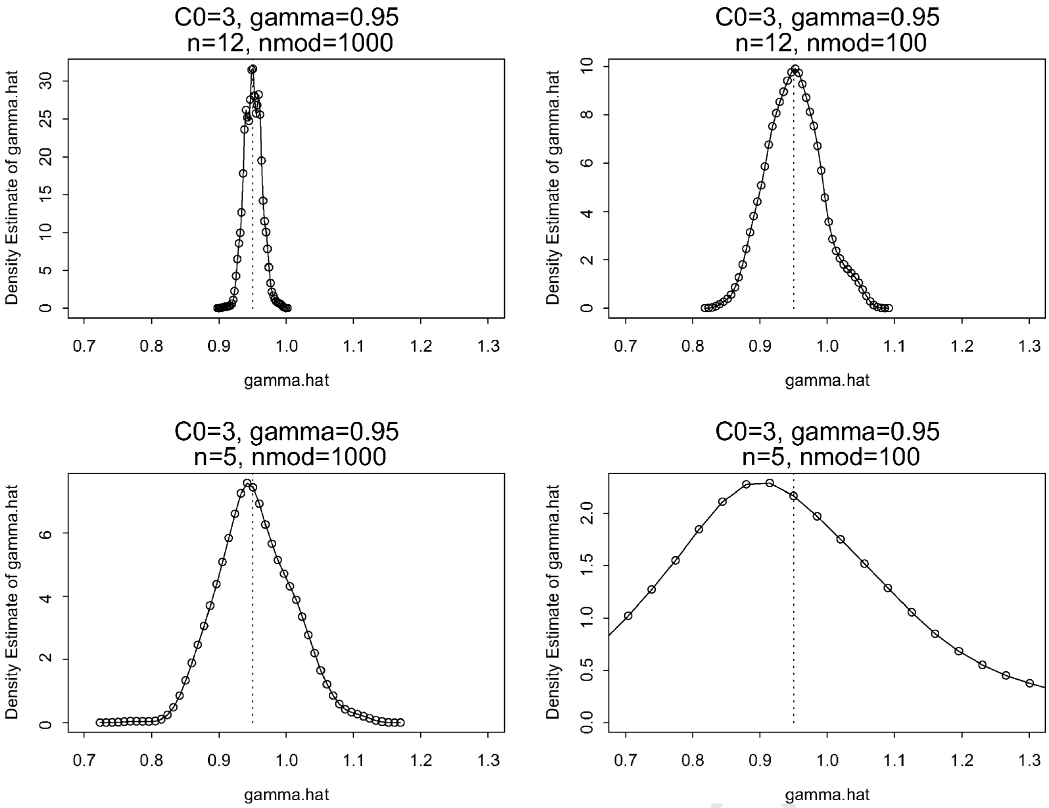

Fig. 3. Density of 500 estimates of γ for four cases in the simulation study (where $C_0 = 3$, $\beta = 0.61$ and $\gamma = 0.95$). The dashed lines represent the true value of $\gamma = 0.95$.

However, the algorithm for fitting the trend model proved inefficient using a data set of only 100 sites or a modelling period of only 5 years: the distribution of bias was skew, with the mean bias tending to be much greater than 1. This is illustrated in Fig. 3. It can be seen that, with $n = 5$ and nmod = 100, in the extremes of the distribution the before mean can be greatly under- or over-estimated. This result would suggest that the successful fitting of a trend model of the type used here requires data for a large number of sites over many years.

As expected, the bias in the model without trend ($\tau_{\text{NO TREND}}$) is substantial, particularly when γ is appreciably less than 1 and n (and hence t) is large. For the case of $\gamma = 0.95$ and $n = 12$ ($t = 10.5$), the mean over-estimate of $\hat{\mu}_{\text{B}}$ using the model without trend was 72%. The correction procedure proved extremely effective in estimating the before mean: both the mean and median of $\tau_{\text{CORRECTED}}$ are 1 for all cases.

The results for the distribution of bias in the EB estimates (Table 1) show that, using the model without trend, the before mean ($\hat{M}_{\text{B}}$) was consistently over-estimated ($\rho_{\text{NO TREND}} > 1$) although the bias was much closer to 1 than that in the estimates of $\hat{\mu}_{\text{B}}$ ($\tau_{\text{NO TREND}}$). In the most extreme case, with $\gamma = 0.95$ and $n = 12$, the model without trend over-estimated $\hat{M}_{\text{B}}$ by 9%. Although the model with trend (τ_{TREND}) performed well when the model period was 12 years, the trend models derived from 5 years data for 100 sites introduced more bias than the model without trend. For example, in the case of $\gamma = 0.95$ (with $n = 5$ and nmod = 100), the model with trend led to a mean under-estimate of $\hat{M}_{\text{B}}$ of 8% ($\tau_{\text{TREND}} = 0.92$) compared with a mean over-estimate of 5% using the model without trend ($\tau_{\text{NO TREND}} = 1.05$). Again the correction procedure proved extremely effective in estimating the before mean ($\hat{M}_{\text{B}}$), with $\tau_{\text{CORRECTED}} \approx 1$ in all cases.

The distribution of estimates of scheme and non-scheme effects for studies of nmod = 1000 are shown in Table 2 for $\gamma = 0.95$ and Table 3 for $\gamma = 0.975$. The use of the model without trend tended to result in under-estimates of regression-to-mean effects (N_{R}) and over-estimates of treatment effects ($S_{\text{R}} + S_{\text{F}}$), although the bias is not particularly large. The correction procedure was successful in eliminating bias in all cases: even when the underlying trend in risk was large, the correction consistently estimated the true treatment effect.

6. Application of correction method to real data

The uncorrected and corrected models without trend were also applied to a group of 50 real sites at which a variety of speed management measures had been applied. Total personal injury accidents and fatal and serious accidents were analysed. All of the sites were in 30 mph speed limits and

Table 2
The distribution of estimates of scheme and non-scheme effects for studies of nmod = 1000 with $\gamma = 0.95$

Properties	Model type	$B = -0.5$				$B = -0.75$			
		N_R	N_T	S_F	S_R	N_R	N_T	S_F	S_R
Model time = 5 years, size of model data set = 1000	True data	−0.07 {−0.07} [0]	−0.13 {−0.13} [0]	−0.03 {−0.03} [0]	−0.26 {−0.26} [0.04]	−0.07 {−0.07} [0]	−0.13 {−0.13} [0]	−0.03 {−0.03} [0]	−0.51 {−0.51} [0.02]
	Trend model	−0.08 {−0.07} [0.05]	−0.13 {−0.13} [0.01]	−0.03 {−0.03} [0]	−0.25 {−0.26} [0.06]	−0.08 {−0.07} [0.05]	−0.13 {−0.13} [0.01]	−0.03 {−0.03} [0]	−0.5 {−0.51} [0.05]
	Without trend	−0.04 {−0.04} [0]	−0.14 {−0.14} [0]	−0.03 {−0.03} [0]	−0.29 {−0.29} [0.04]	−0.04 {−0.04} [0]	−0.14 {−0.14} [0]	−0.03 {−0.03} [0]	−0.54 {−0.54} [0.02]
	Corrected model	−0.07 {−0.07} [0.01]	−0.13 {−0.13} [0]	−0.03 {−0.03} [0]	−0.26 {−0.26} [0.04]	−0.07 {−0.07} [0.01]	−0.13 {−0.13} [0]	−0.03 {−0.03} [0]	−0.51 {−0.51} [0.02]
Model time = 12 years, size of model data set = 1000	True data	−0.1 {−0.1} [0]	−0.13 {−0.13} [0]	−0.03 {−0.03} [0]	−0.24 {−0.24} [0.04]	−0.1 {−0.1} [0]	−0.13 {−0.13} [0]	−0.03 {−0.03} [0]	−0.49 {−0.49} [0.03]
	Trend model	−0.1 {−0.1} [0.03]	−0.13 {−0.13} [0]	−0.03 {−0.03} [0]	−0.23 {−0.24} [0.05]	−0.1 {−0.1} [0.03]	−0.13 {−0.13} [0]	−0.03 {−0.03} [0]	−0.49 {−0.49} [0.03]
	Without trend	−0.04 {−0.04} [0]	−0.14 {−0.14} [0]	−0.03 {−0.03} [0]	−0.29 {−0.29} [0.04]	−0.04 {−0.04} [0]	−0.14 {−0.14} [0]	−0.03 {−0.03} [0]	−0.54 {−0.54} [0.03]
	Corrected model	−0.1 {−0.1} [0.01]	−0.13 {−0.13} [0]	−0.03 {−0.03} [0]	−0.24 {−0.24} [0.04]	−0.1 {−0.1} [0.01]	−0.13 {−0.13} [0]	−0.03 {−0.03} [0]	−0.49 {−0.49} [0.03]

Cells contain the arithmetic mean, {median} and [standard deviation] of the distribution of each estimate to two decimal places. B: observed proportional change in annual accidents; N_R: RTM effect; N_T: trend in accidents within study period; S_F: scheme effect attributable to a change in flow; S_R: scheme effect attributable to a change in risk.

Table 3
The distribution of estimates of scheme and non-scheme effects for studies of nmod = 1000 with $\gamma = 0.975$

Properties	Model type	$B = -0.5$				$B = -0.75$			
		N_R	N_T	S_F	S_R	N_R	N_T	S_F	S_R
Model time = 5 years, size of model data set = 1000	True data	−0.08 {−0.08} [0]	−0.05 {−0.05} [0]	−0.03 {−0.03} [0]	−0.33 {−0.34} [0.05]	−0.08 {−0.08} [0]	−0.05 {−0.05} [0]	−0.03 {−0.04} [0]	−0.58 {−0.59} [0.03]
	Trend model	−0.09 {−0.08} [0.05]	−0.05 {−0.05} [0]	−0.03 {−0.03} [0]	−0.33 {−0.33} [0.07]	−0.09 {−0.08} [0.05]	−0.05 {−0.05} [0]	−0.03 {−0.03} [0]	−0.58 {−0.58} [0.05]
	Without trend	−0.07 {−0.07} [0]	−0.05 {−0.05} [0]	−0.04 {−0.04} [0]	−0.35 {−0.35} [0.05]	−0.07 {−0.07} [0]	−0.05 {−0.05} [0]	−0.04 {−0.04} [0]	−0.6 {−0.6} [0.03]
	Corrected model	−0.08 {−0.08} [0.01]	−0.05 {−0.05} [0]	−0.03 {−0.03} [0]	−0.33 {−0.34} [0.05]	−0.08 {−0.08} [0.01]	−0.05 {−0.05} [0]	−0.03 {−0.03} [0]	−0.58 {−0.58} [0.03]
Model time = 12 years, size of model data set = 1000	True data	−0.1 {−0.1} [0]	−0.05 {−0.05} [0]	−0.03 {−0.03} [0]	−0.33 {−0.33} [0.05]	−0.1 {−0.1} [0]	−0.05 {−0.05} [0]	−0.03 {−0.03} [0]	−0.57 {−0.57} [0.03]
	Trend model	−0.1 {−0.1} [0.02]	−0.05 {−0.05} [0]	−0.03 {−0.03} [0]	−0.32 {−0.33} [0.05]	−0.1 {−0.1} [0.02]	−0.05 {−0.05} [0]	−0.03 {−0.03} [0]	−0.57 {−0.57} [0.03]
	Without trend	−0.07 {−0.07} [0]	−0.05 {−0.05} [0]	−0.04 {−0.04} [0]	−0.35 {−0.35} [0.05]	−0.07 {−0.07} [0.01]	−0.05 {−0.05} [0]	−0.04 {−0.04} [0]	−0.59 {−0.59} [0.03]
	Corrected model	−0.1 {−0.1} [0.01]	−0.05 {−0.05} [0]	−0.03 {−0.03} [0]	−0.33 {−0.33} [0.05]	−0.1 {−0.1} [0.01]	−0.05 {−0.05} [0]	−0.03 {−0.03} [0]	−0.57 {−0.57} [0.03]

Cells contain the arithmetic mean, {median} and [standard deviation] of the distribution of each estimate to two decimal places. B: observed proportional change in annual accidents; N_R: RTM effect; N_T: trend in accidents within study period; S_F: scheme effect attributable to a change in flow; S_R: scheme effect attributable to a change in risk.

the schemes included both speed cameras and a variety of traffic calming measures. There were a total of 733 personal injury accidents in the before period, with 434 in the after period, and the mean durations of the before and after periods were 2.98 and 2.75 years, respectively. There were 131 fatal and serious accidents in the before period with 67 in the after period. The mean of the before period for the 50 sites occurred in September 1997.

The predictive accident models used were the models without trend presented by Mountain et al. (1997) with a modelling period of 12 years (1980–1991). Hence the mean time difference from the mid-point of the modelling period to the mid-point of the before periods was roughly 12 years. Correcting for the effects of trend in risk from the model period to the study period was therefore desirable. The estimate of γ used in the correction procedure was obtained from a comparison group consisting of UK accidents and flows for the years 1980–2001: the entire study period for modelled sites and scheme sites. This gave $\gamma = 0.97$ for all accidents and $\gamma = 0.94$ for fatal and serious accidents. Calculation of traditional confidence intervals for the scheme and non-scheme effects was achieved by the *bootstrap* (Efron and Tibshirani, 1993). This is a Monte-Carlo technique where samples (of the same size as the original sample) are taken from the data with replacement and the statistic of interest (say S_R) is calculated for each sample. The distribution of the estimates from (say 1000 samples) is then used to calculate the standard error of the estimate and the 2.5th and 97.5th percentiles give an empirical 95% confidence interval. The results for the 50 sites are summarised in Table 4.

As was predicted by the simulation studies, ignoring the effects of trend in risk between the modelling period and the study period leads to under-estimates of the regression-to-mean effect (N_R), with over-estimates of the scheme effects (S). The impact of the correction procedure was particularly important for fatal and serious accidents: the estimated effect of treatment on fatal and serious accidents using the correction (−22%) is only half that obtained assuming a constant risk (−43%). The estimates of the regression-to-mean effect with and without the correction were −20.2 and +3.42% respectively. This is a rather greater impact than might have been anticipated from the simulation results. The simulations, however, were based on a representative value of C_0 for total accidents. As fatal and serious accidents represent only a proportion of all accidents, the value of C_0 for fatal and serious accidents will be smaller than for total accidents (with correspondingly smaller values of $\hat{\mu}_B$ and X_B). The models presented by Mountain et al. (1997) also give an estimate of the negative binomial shape parameter (K) of 2.65 for fatal and serious accidents compared with 1.92 for total accidents. These factors will clearly affect the EB estimation process and may indicate that for fatal and serious accidents the need for the correction procedure is greater. Further simulation studies (with $C_0 = 0.75$, i.e. only a quarter of the value used in the original simulation studies) have indeed shown this to be true.

7. Discussion

The majority of available models assume that the underlying risk of accidents per unit of exposure is constant over time and yet, if road safety programmes are effective,

Table 4
Estimates of scheme effects at 50 sites

Accident type	Method	Estimate	
		Scheme effect, $\hat{S}$ (standard error) {95% empirical bootstrap CI}	Non-scheme effect, $\hat{N}$ (standard error) {95% empirical bootstrap CI}
All accidents	Simple before and after comparison	$S = -36.0\%$ (5.8) {−46.3, −24.4}	
	EB with comparison group and flow correction—model without trend	$S = -32.1\%$ $S_R = -27.1\%$ (5.3) {−36.6, −15.8} $S_F = -5.0\%$ (1.3) {−7.8, −2.7}	$N_R = -4.2\%$ (1.2) {−6.5, −1.8}, $N_T = 0.3\%$ (2.0) {−3.5, 4.4}
	EB with comparison group and flow correction—corrected model ($\gamma = 0.97$)	$S = -28.3\%$ $S_R = -23.4\%$ (5.6) {−33.5, −11.4} $S_F = -4.9\%$ (1.3) {−7.5, −2.5}	$N_R = -8.3\%$ (1.5) {−11.5, −5.6}, $N_T = 0.6\%$ (1.9) {−2.9, 4.6}
Fatal and serious accidents	Simple before and after comparison	$S = -48.8\%$ (9.3) {−65.1, −28.3}	
	EB with comparison group and flow correction—model without trend	$S = -42.8\%$ $S_R = -37.9\%$ (7.4) {−51.5, −23.2} $S_F = -4.9\%$ (1.3) {−7.6, −2.5}	$N_R = +3.4\%$ (6.3) {−7.3, 17.8}, $N_T = -9.5\%$ (1.8) {−13, −6}
	EB with comparison group and flow correction—corrected model ($\gamma = 0.94$)	$S = -22.2\%$ $S_R = -18.0\%$ (7.4) {−31.6, −1.9} $S_F = -4.2\%$ (1.2) {−6.7, −2}	$N_R = -20.2\%$ (5.3) {−29.6, −9.4}, $N_T = -6.4\%$ (1.6) {−9.2, −3.1}

S: scheme effect; S_R: scheme effect attributable to a change in risk; S_F: scheme effect attributable to a change in flow; N_T: trend in accidents within study period; N_R: RTM effect.

a decline in risk per unit of exposure would be expected. The results of simulation studies show that trend in risk can lead to substantial errors in predictive model estimates of mean accident frequencies if the period for which estimates are required is several years after the modelling period (as is typically the case). The simulation studies also show that, if there is a trend in accident risk, the use of a model which ignores trend will result in errors in estimates of both the regression-to-mean effect and the treatment effect. The size of these errors will depend on the size of the factor by which risk changes from year to year (γ) and on the elapsed time between the mid-points of modelling period and the study period (t). The errors also tend to be larger for sub-groups of accidents (such as fatal and serious accidents) for which the observed and predicted accident frequencies are smaller, and the NB shape parameter is larger.

Given a reliable estimate of the factor by which risk changes from year to year (γ), the correction procedure outlined in this paper allows an appropriate adjustment for trend in risk to be made to any accident prediction model. Indeed, for models derived from data for a relatively small number of sites over a short time period (say 100 sites over 5 years), it could be preferable to use the correction procedure rather than attempting to fit a model incorporating a trend term: the simulations show that it is not possible to reliably fit a trend model of the type considered here to such data. Since the majority of existing models are derived from data for relatively small number of sites over short time periods, this is an important result.

Clearly the quality of the estimates obtained using the correction for trend will rely on the quality of the estimate of γ. The trend models presented by Mountain et al. (1997) for the period of 1980–1991 for link accidents estimate γ as 0.95 and 0.98 for total accidents and fatal and serious accidents, respectively. This was based on data for 1268 sites and hence the simulations presented here suggest these estimates should be stable. There is clearly a discrepancy, however, between these estimates and those obtained using national data for the period 1980–2001 which gave estimates of γ of 0.97 and 0.94 for all accidents and fatal and serious accidents, respectively (and which were used in the correction for the 50 real sites). Discrepancies between the trend estimates for individual links and the national data could be due to various factors: the national data may not be representative of link sites (the accident totals include all accidents not just those on links); the sample of link sites used by Mountain et al. (1997) may not be representative of national trends (the data were for only six of the English counties); the factor by which risk changes from year to year (γ) may not be constant over time. There is a need for this to be addressed in future research.

In the simulation studies presented in this paper, overall mean flows were assumed to follow an arithmetic progression. This was a strong assumption as it meant the mean of flows occurred at the middle of the study period. Some further investigations involving other possible representations of flow (such as a geometric progression or a sigmoid curve for flows over the study period) have shown that the correction is still valid.

It is perhaps also worth noting that if the true value of γ is close to 1 (i.e. trend in accident *risk* is negligible) then observed trends in accidents will be entirely attributable to trend in *flow*. In this case it could be preferable to estimate expected accidents in the after period using the actual before and after flows at the study site rather than observed accidents for a comparison group in the before and after periods (which might not be truly representative of the site under investigation). However, if the true value of γ is close to 1 it would raise questions about the effectiveness of current road safety strategies.

8. Conclusions

This paper has considered the problems of bias when using a mis-specified predictive model in the estimation of confounding factors in before and after studies of road safety schemes. Under the assumption of a genuine change in risk over time simulations showed that, if this is ignored, the estimation of RTM and treatment effects can be biased. However, the nature of the bias in the predictive model was established and a simple correction procedure outlined. The correction procedure was effective in eliminating bias and was also shown to be easily applicable to real data in an analysis of 50 treated sites.

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