# Influence of Chemoreflexes on Respiratory Variability in Healthy Subjects

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The background of this study was the hypothesis that respiratory variability is influenced by chemoreflex regulation. In search for periodicities in the variability due to instability of the respiratory control system, spectral analysis was applied to breath-to-breath variables in 19 healthy subjects at rest. During room-air breathing, coherent oscillations in end-tidal CO<sub>2</sub> (PETCO<sub>2</sub>) and mean inspiratory flow (VI/TI) were found in 15 subjects with frequencies mostly below 0.15 cycles per breath. Coherent oscillations in  $\mathsf{Pet}_{\mathsf{CO}_2}$  and VI/TI were expressed by gain (0.13 to 0.34 L/second · kPa) and phase  $(-170^{\circ} \text{ to } +8^{\circ})$ . The oscillations in V<sub>I</sub>/T<sub>I</sub> were in phase with inspiratory volume (Vi). A model that describes the effects of chemoreflex feedback to noise in the system could explain these gains and phases, whereas a model without chemoreflex could not. During 100% O2 breathing, only eight subjects had coherent oscillations in  $Pet_{CO_2}$  and VI/TI. The coherent oscillations in  $Pet_{CO_2}$ and VI/TI were interpreted as a manifestation of chemoreflex activity. We conclude that respiratory variability is not a random process but contains information on chemoreflex properties, such as the chemoreflex gain. The analysis of respiratory variability therefore provides a new tool to study the action of the chemoreflexes without application of external stimuli.

#### Keywords: chemoreceptors; respiratory variability; spectral analysis

Is the normal variability of respiratory parameters from breathto-breath a random process? This would imply that, for example, inspiratory volume (V1) or inspiratory and expiratory time (T1 and TE, respectively) are independent of previous breaths. However, due to the circulatory delay from the lungs to the systemic arteries, an "accidental" change in V1 causes a change in arterial Po<sub>2</sub> and Pco<sub>2</sub> that only becomes manifest during the following breaths at a normal breathing frequency (1, 2). An adaptation of V1 to such a change therefore inevitably leads to a dependency between successive breaths. Conversely, purely random variability of V1 implies that V1 does not take part in the feedback control of Pa<sub>O2</sub> and Pa<sub>CO2</sub> through the chemoreflexes.

Several authors have found evidence for a nonrandom breath-to-breath variability of respiratory parameters in the normal steady state (3–5). Significant (auto)correlations have been found between successive values of V<sub>I</sub>, T<sub>I</sub>, and T<sub>E</sub> (4, 6). Specific variability patterns have also been found, mainly in the form of subtle oscillations with a cycle time of approximately 25 seconds to more than 3 minutes (6–10). Clear periodic breathing is seldom observed in healthy subjects during wakefulness (3, 7, 11, 12), but often occurs during sleep or at high altitude (13, 14). The cause of frank periodic breathing

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table of contents online at www.atsjournals.org Am J Respir Crit Care Med Vol 165. pp 1041–1047, 2002 DOI: 10.1164/rccm.2104100 Internet address: www.atsjournals.org like Cheyne-Stokes breathing in cardiac failure is probably an instability of the chemoreflex-feedback control system (15–17). It has been supposed that spontaneous changes in breathing pattern can induce (dampened) oscillations due to chemoreflex feedback in healthy subjects as well (8, 18, 19).

The aim of the present study was to derive information on respiratory regulation from the normal breathing pattern in the steady state. The hypothesis was that because of the delays and time constants of the chemoreflexes, continuous regulation tends to induce oscillations in ventilatory drive (represented by mean inspiratory flow, VI/TI) with a certain coherency with oscillations in end-tidal PCO<sub>2</sub> (PET<sub>CO2</sub>). To identify such oscillatory components and their mutual relationships, power and cross-spectral analysis was applied to breath-tobreath respiratory variables in 19 healthy subjects at rest. To test the hypothesis that the features of coherent oscillations in PET<sub>CO2</sub> and VI/TI are compatible with the characteristics of chemoreflex-feedback regulation, experimental spectra were compared with theoretical spectra derived from a chemoreflex model. The breathing pattern was also analyzed during 100% O<sub>2</sub> breathing to estimate the contribution of the peripheral chemoreflex (20).

# **METHODS**

#### Subjects and Measurements

Nineteen healthy nonsmoking medical students were studied with a history free of cardiopulmonary disease and normal physical examination (9 male and 10 female, aged 23  $\pm$  3 years, body mass index 22.2  $\pm$  3.1 kg/m<sup>2</sup>, mean  $\pm$  SD). The hospital ethical committee approved the protocol. Informed consent was obtained. The subjects knew which measurements were performed. To prevent "conscious" breathing, they were told that the study involved blood pressure regulation. They sat in a comfortable chair in a quiet room and breathed through a cushion-sealed face mask fitted with elastic bands around the head (dead space  $\sim$  70 ml).

A Lilly type pneumotachograph (Siemens pressure transducer, Munich, Germany) was connected to the mask and hung with an elastic cord to the ceiling. A two-way nonrebreathing valve (S and W, Copenhagen, Denmark) was connected to the pneumotachograph. The inspiratory limb was connected to a stopcock (through a 1-m spirometer tube) which could be switched from room air to 100%  $O_2$  from a 100 L bag. The stopcock was hidden behind a curtain so that the subject did not know which gas was inspired. The experiments began between 9:00 and 10:00 A.M. Recordings started after a 5-minute acclimatizion period. There were two episodes of 30 minutes with more than 5 minutes in between, performed in a random order, during which the subjects breathed either air or 100% O2 (starting when endtidal Po<sub>2</sub> exceeded 85 kPa). Also measured were Po<sub>2</sub> and Pco<sub>2</sub> in the facemask (partial pressures in dry air, Centronic 200 MGA mass spectrometer, Croydon, United Kingdom), arterial  $O_2$  saturation (Sa<sub>O2</sub>, Ohmeda Biox ear pulse oximeter, Madison, WI), finger arterial pressure (Finapres BMI-TNO, Amsterdam, Netherlands) and a single-channel chest-lead ECG. All signals were recorded on a Bell and Howell T4 recorder (Durham, NC) with airflow, ECG and blood pressure on FM channels and the other signals on a direct record channel using a Kayser Threde K 1180 pulse code modulator (Munich, Germany). The frequency response was 0-625 Hz for FM channels and 0-105 Hz for pulse code modulated channels.

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After digitizing at 200 Hz, breath-to-breath variables were derived from the pneumotachogram. To reduce a disproportionate influence of isolated deep breaths, the values of breaths with VI greater than 1.5 times the mean of adjoining breaths were linearly interpolated.  $PeT_{CO_2}$ and end-tidal PO<sub>2</sub> (PeT<sub>O2</sub>) were derived as the maximal PCO<sub>2</sub> (and minimal PO<sub>2</sub>) during the last second of expiratory flow. Beat-to-beat mean blood pressure and R-R interval were derived from the finger pressure and ECG. Means and SDs of each variable were compared between air and O<sub>2</sub> breathing (paired *t* test) with a prior log-transformation of SDs of ventilatory variables (10). Group values are means ± SEM.

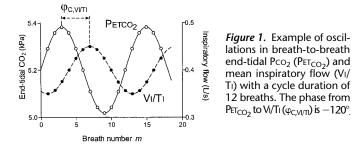
#### Spectral Analysis

Power spectra. In search of oscillatory components in respiratory variability, power spectra were derived for each variable. The underlying concept of spectral analysis is that each variability pattern can be written as the sum of a number of oscillations with a certain frequency, amplitude and phase (21). The power is proportional to the squared amplitude of such an oscillation. The reason to use power instead of amplitude is that the mean power, as it is defined here, equals the variance. Thus, the power spectrum shows the extent to which oscillation at each frequency contributes to the overall variance. For a variable that changes from breath to breath (with breath number M), the oscillation frequencies  $(f_m)$  are in cycles per breath. For example, when f<sub>m</sub> is 0.25 cycles per breath, the oscillation has a cycle duration of four breaths. In a series of M breaths, only a limited number of frequencies can be discerned. The slowest oscillation has a frequency of 1/M cycles per breath. The other frequencies are all multiples of this basic frequency. The highest possible frequency is 0.5 cycles per breath.

Cross spectra. With spectral analysis it is also possible to "dissect" two variables into oscillatory components and to analyze the relationship between these oscillations. Figure 1 shows an example of two oscillations that occur in the breath-to-breath variability of PET<sub>CO2</sub> and VI/TI. The cycle duration is 12 breaths ( $f_m$  is 0.083 cycles per breath). Both oscillations have a constant amplitude here, 0.36 kPa for  $P_{ET_{CO_2}}$  and 0.10 L/second for VI/TI. The relationship between the oscillations is expressed by the gain and phase. The gain from PETCO2 to VI/TI ( $G_{C,VI/TI}$ ) is the ratio of the amplitude of VI/TI to the amplitude of PET<sub>CO2</sub>. In the example,  $G_{C,VI/TI} = 0.10/0.36 = 0.28$  L/second  $\cdot$ kPa. The phase from  $P_{ET_{CO_2}}$  to VI/TI ( $\varphi_{C,VI/TI}$ ) is defined as negative when a maximum in  $P_{ET_{CO_2}}$  occurs less than half a cycle before a maximum in VI/TI. In the example, a maximum in PETCO2 occurs four breaths before a maximum in VI/TI, so that  $\phi_{C,VI/TI}$  =  $-(\bar{4}/12)\cdot 360^\circ$  =  $-120^{\circ}$ . When the relationship between PET<sub>CO2</sub> and VI/TI is linear, an increase in the amplitude of  $\ensuremath{\mathsf{PET}}_{\ensuremath{\mathrm{CO}}_2}$  is accompanied by a proportional increase in the amplitude of VI/TI. A perfectly linear relationship between two oscillations is completely described by gain and phase. In reality, some "noise" always occurs so that an exact determination of gain and phase is never possible. The gain and phase can, however, be estimated with a certain level of confidence, depending on the number of degrees of freedom of the data and the degree of linearity of the relationship. For the relationship between  $\ensuremath{\text{Pet}_{\text{CO}_2}}$  and  $\ensuremath{\text{VI/TI}}$  , the degree of linearity is described by the squared coherency  $k^2_{CVI/TI}$ which ranges from zero (no linear relationship) to one (perfectly linear relationship). The (estimated) squared coherency can be interpreted as a squared correlation coefficient for variations with a specific frequency. Similarly, the estimated gain can be seen as a frequencyspecific linear regression coefficient. The squared coherency, gain, and phase spectra describe these estimates for all frequencies that occur within the variability of the two variables. Together they are derived from the "cross spectrum" (21). Cross-spectral analysis thus amounts to a linear regression in the frequency domain.

Cross spectra were determined for  $Pet_{CO_2}$  and VI/TI, VI/TI and VI, VI and TE, and for  $Pet_{CO_2}$  and  $Pet_{O_2}$ . We were particularly interested in the relationship between  $Pet_{CO_2}$  and VI/TI as these variables are related to the input and output of the chemoreflexes (1, 22, 23). "Coherent oscillations" in  $Pet_{CO_2}$  and VI/TI were defined as oscillations with (1) a significant coherency between  $Pet_{CO_2}$  and VI/TI and (2) a power of both  $Pet_{CO_2}$  and VI/TI that is significantly higher than the mean power.

*Details of spectral analysis.* A trend was removed from the data to reduce the influence of very low frequencies. The data were multiplied by a Tukey window and transformed from the "breath number" (m) domain to the "frequency" ( $f_m$ ) domain with the discrete Fourier



transform (21). The spectra were smoothed by a triangular running window (width  $\sim 0.01$  cycles per breath) to increase the number of degrees of freedom of each spectral estimate (24). The power spectral estimate was considered as significantly higher than the mean power when the lower limit of the 90% confidence interval was higher than the mean. The centroid frequency, defined as the frequency below which 50% of the power occurs, was determined for each variable (6). The expected centroid frequency for white noise is 0.25 cycles per breath (21). The centroid frequencies were tested for deviation from white noise with the Kolmogoroff-Smirnov test (21). Cross-spectral estimates were considered significant if the lower limit of the 90% confidence interval of the squared coherency was larger than 0.3 (21).

To calculate group averages for cross-spectral estimates, the frequencies between zero and 0.5 cycles per breath were divided into bins of 0.01 cycles per breath. The reason was that the frequencies (multiples of 1/M) differed between the subjects because of differing M. For each subject, the mean cross-spectral estimate for each frequency bin was derived, only including cross-spectral estimates with a significant coherency. These mean values were used to calculate the group means. Coherent oscillations in  $Petr_{CO2}$  and Vt/Tt were compared between air and 100%  $O_2$  breathing as to occurrence (25) and oscillation frequency (Wilcoxon rank sum test). p < 0.05 was taken as significant.

## RESULTS

## Air Breathing

The subjects breathed regularly during the experiments without falling asleep. Only in Subject 19 the last 120 breaths were discarded because of relative hyperventilation. Group averages for breath-to-breath means and SDs are given in Table 1. The steady state was confirmed by the relatively small SDs of blood pressure and R-R interval. Sa<sub>O2</sub> was above 94% during air breathing. The number of breaths was 442  $\pm$  25, of which 1.3  $\pm$  0.3% were interpolated as sighs.

In the example of Figure 2, periodicities in the breath-tobreath variability are not obvious. Neither is there a clear relationship between changes in  $Pet_{CO2}$  and VI/TI. Only the variations in  $Pet_{CO2}$  and  $Pet_{O2}$  seem to be related and appear out-of-phase. The corresponding power spectra of  $Pet_{CO2}$  and VI/TI are shown in Figure 3. The power of both variables was concentrated at low frequencies (below 0.20 cycles per breath). This also appears from the low centroid frequencies in most subjects (Table 2).

The *bold lines* in Figure 3 represent the part of the power spectra that meet the criteria for coherent oscillations in  $PET_{CO_2}$  and VI/TI. This occurs at a small peak in both spectra at approximately 0.09 cycles per breath, corresponding to a cycle of about 11 breaths or roughly 50 seconds, as the mean breath duration ( $\overline{T}$ roT) was 4.4 seconds for this subject. Coherent oscillations in  $PET_{CO_2}$  and VI/TI were found in 15 of the 19 subjects. Figure 4 shows the averaged gain and phase for coherent oscillations, most of which occurred below 0.15 cycles per breath. The phase  $\varphi_{C,VI/TI}$  was mostly negative, on the average about  $-90^{\circ}$ . This means that a maximum in the oscillation in  $PET_{CO_2}$  occurred less than half a cycle before a maximum in VI/TI (as in Figure 1). Table 3 gives a summary of the

TABLE 1. MEANS AND STANDARD DEVIATIONS OF RESPIRATORY AND CIRCULATORY VARIABLES

	Air $(n = 19)$		100% O <sub>2</sub> ( <i>n</i> = 18)	
	Mean	SD	Mean	SD
Vı/Tı, L/second	0.403 ± 0.011	0.036 (0.023, 0.075)	0.459* ± 0.019	0.043 (0.022, 0.67)
Vi, L	0.611 ± 0.025	0.075 (0.040, 0.201)	$0.700^{*} \pm 0.037$	0.091 (0.029, 1.03)
Ti, seconds	$1.52 \pm 0.06$	0.16 (0.082, 0.40)	$1.53 \pm 0.06$	0.15 (0.058, 0.42)
TTOT, seconds	$4.34 \pm 0.31$	0.63 (0.18, 1.85)	$4.09 \pm 0.22$	0.52 (0.16, 2.31)
Pet <sub>CO2</sub> , kPa	$5.59\pm0.09$	0.16 (0.10, 0.37)	5.38* ± 0.11	0.18 (0.12, 0.42)
PETO2, kPa	$15.12 \pm 0.20$	0.40 (0.20, 0.95)	> 85	
BP, mm Hg	78.3 ± 1.0	4.6 (3.2, 7.2)	$80.6 \pm 2.0$	4.5 (3.1, 6.9)
RRI, ms	$0.881 \pm 0.026$	0.066 (0.045, 0.089)	0.934* ± 0.028	0.078* (0.041, 0.151)

Definition of abbreviations: BP = blood pressure (mm Hg);  $P_{ET_{CO_2}}$ ,  $P_{ET_{O_2}}$  = end-tidal  $P_{CO_2}$  and  $P_{O_2}$  (kPa); RRI = R-R interval from the ECG;  $T_{TOT}$  = total breath duration (seconds); VI,  $T_I$  = inspiratory volume (L) and time (seconds); VI/ $T_I$  = mean inspiratory flow (L/second).

Values are group means ± SEM (for breath-to-breath means) or geometric means and range in parentheses (for breath-to-breath SD).

\* Significant difference between air and 100%  $O_2$  breathing (p < 0.05).

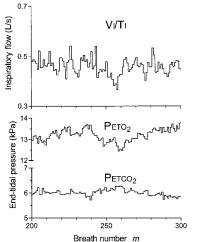
other cross-spectral estimates at the frequencies of coherent oscillations in  $Pet_{CO_2}$  and VI/TI. It shows that in 14 of the 15 subjects, the oscillation in VI/TI was tightly coupled to an inphase oscillation in VI. Almost every coherent oscillation in  $Pet_{CO_2}$  and VI/TI was accompanied with an out-of-phase oscillation in  $Pet_{O_2}$ .

# 100% O<sub>2</sub> Breathing

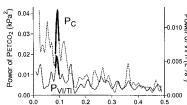
Means and SDs of breath-to-breath variables during air breathing are compared with 100% O<sub>2</sub> breathing in Table 1. The number of breaths was  $425 \pm 12$  with  $1.5 \pm 0.4\%$  sighs. The smaller number of breaths compared with air breathing was due to the criterion that  $P_{ET_{O_2}}$  had to exceed 85 kPa. In Subject 1 a reliable pneumotachogram was not obtained. The mean VI was significantly higher than during air breathing (difference 0.089  $\pm$  0.028 L), while the mean PET<sub>CO2</sub> was significantly lower (difference  $-0.21 \pm 0.04$  kPa). None of the respiratory variables showed a significant difference in breathto-breath SD between air and  $O_2$  breathing (Table 1). The centroid frequencies were not significantly different from air breathing. Coherent oscillations in PETCO2 and VI/TI were only found in eight subjects, significantly less often than during air breathing (p < 0.01, Table 3). The coherency occurred at a mean frequency of 0.055 cycles per breath (range 0.023-0.096, not significantly different from air breathing).

### **Model Simulation**

The spectra can be interpreted by showing that they are compatible with a supposed mechanism. This approach is strengthened when the spectra cannot be explained by alternative hy-



*Figure* 2. Breath-to-breath variability in Subject 4 during air breathing.  $V_I/T_I$ , mean inspiratory flow;  $P_{ET_{CO_2}}$  and  $P_{ET_{O_2}}$ , end-tidal  $P_{CO_2}$  and  $P_{O_2}$ .



Frequency f<sub>m</sub> (cycles/breath)

potheses. It is, however, difficult to deduce, even from a simple hypothesis, what the spectra would precisely look like. We therefore expressed our hypotheses in mathematical equations and algebraically derived the spectra from these equations. This also improves the insight into the way the various physiologic mechanisms can influence the spectra.

The main hypothesis was that the variability of  $P_{ET_{CO_2}}$  and  $V_I/T_I$  is influenced by chemoreflex feedback. Suppose that these variables would, instead, vary at random from breath to breath according to a normal distribution with a given mean and variance. Then the variability would contain all possible oscillations to the same extent (if the series of breaths is infinitely long). The power would thus be constant as a function of frequency, equal to the variance ("white noise") (21). If, however, chemoreflexes continuously react to such noise, the noise becomes filtered so that the power becomes significantly higher or lower than the mean power at particular frequencies. Such a filter mechanism can also influence the relationship between otherwise independent variables, resulting in a significant coherency.

Because breath-to-breath variability is a discontinuous process in the course of time, we used "discontinuous" equations (difference equations). The input to the model is white noise, which is mainly filtered by the chemoreflex. Provided that the difference equations are linear, the resulting spectra can be derived in a straightforward manner (21). We therefore applied linearized equations for the chemoreflex response to changes in  $PET_{CO2}$  and for the effects of ventilation on  $PET_{CO2}$ . This is supported by the relatively high coherencies between  $PET_{CO2}$  and VI/TI (Table 3). The model is analogous to earlier models of hemodynamic variability (24, 26).

## **Description of the Chemoreflex-Feedback Model**

A schematic of the model is shown in Figure 5. The variability is "driven" by two sources of noise,  $\varepsilon_1$  and  $\varepsilon_2$ . The noise  $\varepsilon_1$  is noise that directly affects  $P_{ET_{CO_2}}$ , such as changes in breathing frequency, cardiac output, or mixed venous  $P_{CO_2}$ . The noise  $\varepsilon_2$ 

> **Figure 3.** Power spectra of end-tidal  $P_{CO_2}$  ( $P_C$ , *dotted line*) and mean inspiratory flow ( $P_{VI/TV}$ , *continuous line*) as a function of frequency in cycles per breath (from Subject 4). *Bold lines* indicate the part of the spectra where the relationship between oscillations in  $P_{ET_{CO_2}}$ and  $V_I/T_I$  is coherent.

TABLE 2. CENTROID FREQUENCIES FOR BREATH-TO-BREATH VARIABLES

Variable	Centroid frequency (cycles per breath)	n
VI/TI	0.160 ± 0.011	16
VI	$0.105 \pm 0.009$	19
Ті	$0.133 \pm 0.012$	10
TE	$0.122 \pm 0.011$	18
Pet <sub>CO2</sub>	$0.097 \pm 0.012$	13

Definition of abbreviations:  $Pet_{CO_2}$  = end-tidal  $PcO_2$  (kPa); TE = expiratory time; TI = inspiratory time (seconds); VI = inspiratory volume (L); VI/TI = mean inspiratory flow (L/seconds).

Centroid frequencies are group means ± SEM. n, number of subjects in whom the centroid frequency implied a significant difference from white noise (from 19 subjects).

has a direct influence on VI/TI and can be due to any changes in respiratory drive that are not caused by chemoreflex feedback (22, 27). It is assumed that the chemoreflex response only consists of an increase in VI/TI after an increase in  $PET_{CO_2}$ (1, 22). It is assumed that a change in VI/TI leads to a proportional change in VI. The variability of TI is neglected, so that TI is considered equal to the mean value (TI). In the online data supplement (Section A) it is shown that this implies that the ratio of a change in VI to a change in VI/TI equals TI. The constant  $c_2$  in Figure 5 expresses the ventilatory influence of VI on  $PET_{CO_2}$ . The minus sign for  $c_2$  indicates that an increase in VI leads to a decrease in  $PET_{CO_2}$ . The constant  $c_1$  expresses the dependency of  $PET_{CO_2}$  on the previous breath, mainly because of the buffer of the FRC.

Development of respiratory oscillations by chemoreflex feedback. The supposed principle of the development of respiratory oscillations by chemoreflex feedback has been described by a number of authors (e.g., 14, 15, 16, 17, 18, 28). Essential is the delay of the response. Figure 6 shows the simplest version of the present model that can explain the occurrence of respiratory oscillations. The dependency of  $P_{ET_{CO_2}}$  on the previous breath is neglected here. It is assumed that the chemoreflex response to an "accidental" increase in PET<sub>CO2</sub> (arrow in Figure 6) leads to an increase in VI/TI after a delay of two breaths. The correction (a decrease in  $P_{ET_{CO2}}$ ) thus occurs two breaths after the initial increase in PETCO2. This decrease in  $PET_{CO_2}$  in turn leads to a corrective increase in  $PET_{CO_2}$ after two breaths, and so on. The result is a dampened oscillation in PET<sub>CO2</sub> and VI/TI with a cycle duration of four breaths, or a frequency of 0.25 cycles per breath. The cycle duration increases if the respiratory drive persists during following breaths ("short-term potentiation") (1, 29) or the dependency of Pet<sub>CO2</sub> on the previous breath increases.

### Power and Cross-Spectra according to the Model

When the situation of Figure 6 is extended to repetitive accidental disturbances in  $P_{ET_{CO_2}}$ , multiple dampened oscillations are generated in  $P_{ET_{CO_2}}$  and  $V_I/T_I$ . This implies a filter mechanism that selectively amplifies random disturbances in  $P_{ET_{CO_2}}$  around a "resonance" frequency. Figure 7 shows an example of a purely peripheral chemoreflex model. Without chemoreflex, the two sources of noise would initially result in white noise in  $P_{ET_{CO_2}}$  with a constant power (*see* Figure 7A for  $c_1 = 0$ ). Due to the dependency of  $P_{ET_{CO_2}}$  on the previous breath, the power is amplified below  $\sim 0.20$  cycles per breath and suppressed at higher frequencies ("low-pass filter", *see* Figure 7A for  $c_1 = 0.5$ ). Chemoreflex feedback further amplifies the power around 0.12 cycles per breath and suppresses the rest of the spectrum (resonance or band-pass filter phenomenon). The power of VI/T1 also shows a resonance around this fre-

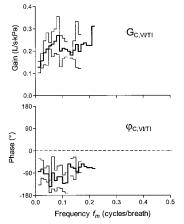


Figure 4. Gain ( $G_{C,VI/T}$ ) and phase ( $\varphi_{C,VI/T}$ ) for coherent oscillations in  $P_{ET_{CO_2}}$  and  $V_I/T_I$  during air breathing. Bold lines are group means and thin lines are means  $\pm$  SD for each frequency bin of 0.01 cycles per breath. Only significant gain and phase estimates are included (from 15 subjects with coherent oscillations).

quency (*open circles* in Figure 7). For lower frequencies, the power of V<sub>I</sub>/T<sub>I</sub> is also amplified, as opposed to  $PeT_{CO2}$ . These opposing effects of chemoreflex activity reflect an adequate action of the reflex, which tends to suppress spontaneous changes in  $PeT_{CO2}$  by increasing the variability of V<sub>I</sub>/T<sub>I</sub>. Without chemoreflex, the total power of V<sub>I</sub>/T<sub>I</sub> would be considerably lower.

The squared coherency  $(k^2_{C,VI/TI})$  is maximal at the resonance frequency (Figure 7C). Without chemoreflex, the coherency would be low for all frequencies. The relationship would then exclusively be due to the ventilatory influence of VI/TI on PET<sub>CO2</sub>. The coherency would thus be determined by the contribution of noise in VI/TI to the noise in PET<sub>CO2</sub>.

The gain ( $G_{C,VI/TI}$ ) in Figure 7D refers to the "closed-loop" situation (as in Figure 6).The chemoreflex gain ( $G_{\chi}$ ) is the "open-loop" gain from  $Pet_{CO2}$  to VI/TI (neglecting the ventilatory influence of VI/TI on  $Pet_{CO2}$ ). When the frequency is zero,  $G_{\chi}$  equals the peripheral chemoreflex sensitivity ( $S_p$ ). At the resonance frequency,  $G_{C,VI/TI}$  is almost equal to  $G_{\chi}$  (only slightly higher). Without chemoreflex, the gain would be low ( $G_0$ ).

Figure 7E shows that the closed-loop phase  $\varphi_{C,VI/TI}$  is closely related to the open-loop chemoreflex phase  $\varphi_{\chi}$  at the resonance frequency. The phase  $\varphi_{\chi}$  is almost linearly related to the frequency, which is mainly determined by the chemoreflex delay. Without chemoreflex,  $\varphi_{C,VI/TI}$  would lie approximately in the range between  $-150^{\circ}$  and  $-170^{\circ}$  ( $\varphi_{0}$ ). A more extensive analysis is given in Section D of the online data supplement. An alternative model is also analyzed where short-term potentiation of respiratory drive occurs after other stimuli (27, 30).

#### Interpretation of Experimental Data with the Model

- 1. The variability of PET<sub>CO2</sub>, VI/TI, and VI is not random, but is subject to low-pass filter mechanisms. Possible filter mechanisms are the dependency of PET<sub>CO2</sub> on the previous breath, chemore-flex feedback, and short-term potentiation of respiratory drive.
- 2. Chemoreflex feedback can explain coherent oscillations in  $P_{ET_{CO_2}}$  and VI/TI. The reflex delay causes resonance at a frequency where the squared coherency is high. This frequency is mainly determined by the reflex delay, but shifts to lower frequencies when the dependency of  $P_{ET_{CO_2}}$  on the previous breath becomes more important. Experimentally observed frequencies are compatible with peripheral chemoreflex feedback, with an unknown contribution of the central response. Gains and phases are compatible with the chemoreflex model.
- 3. Chemoreflex feedback through central chemoreceptors can explain coherent oscillations in  $Pet_{CO_2}$  and Vi/Ti during 100%  $O_2$  breathing.
- 4. Without chemoreflex, coherent oscillations in  $Pet_{CO_2}$  and VI/TI are unlikely as there is no resonance at a specific fre-

TABLE 3. CROSS-SPECTRAL ESTIMATES FOR COHERENT OSCILLATIONS IN END-TIDAL  $\mathrm{CO}_2$  AND MEAN INSPIRATORY FLOW

Relation	n	Squared Coherency	Gain	Phase
Air				
$P_{ET_{CO_2}} \rightarrow V_I/T_I$	15	0.76 (0.53, 0.90)	0.24 (0.13, 0.34)	-85° (-170°, +8°)
$V_I/T_I \rightarrow V_I$	14	0.72 (0.52, 0.90)	1.88 (0.92, 3.87)	-1° (-57°, +38°)
$V_I \rightarrow T_E$	8	0.69 (0.55, 0.88)	6.0 (1.6, 15)	-5° (-86°, +120°)
$P_{ET_{CO_2}} \rightarrow P_{ET_{O_2}}$	14	0.90 (0.81, 0.99)	2.09 (1.33, 2.77)	-179° (-200°, -150°)
100% O <sub>2</sub>				
$P_{ET_{CO_2}} \rightarrow V_I/T_I$	8	0.79 (0.65, 0.93)	0.26 (0.12, 0.51)	-105° (-146°, -34°)
$V_I/T_I \rightarrow V_I$	5	0.85 (0.67, 0.92)	2.86 (0.86, 5.86)	+4° (-48°, +47°)
$V_I \rightarrow T_E$	4	0.77 (0.69, 0.84)	4.0 (2.1, 9.3)	-24° (-63°, +17°)

Definition of abbreviations:  $P_{ET_{CO_2}}$ ,  $P_{ET_{O_2}}$  = end-tidal  $P_{CO_2}$  and  $P_{O_2}$  (kPa);  $T_E$  = expiratory time;  $T_{TOT}$  = total breath duration (seconds);  $V_I$ ,  $T_I$  = inspiratory volume (L) and time (seconds);  $V_I/T_I$  = mean inspiratory flow (L/second).

All values are group means from individual spectral estimates at the frequency of maximal coherency between  $Pet_{CO_2}$  and  $V_1/T_1$ . Ranges are in parentheses.  $Pet_{CO_2} \rightarrow V_1/T_1$  refers to cross-spectra with  $Pet_{CO_2}$  as input and  $V_1/T_1$  as output variable, etc. n, number of subjects with significant coherency.

\* To obtain the smallest phase range, 360° was subtracted from the lower limit of the range (the estimated phase cannot be distinguished from the phase plus or minus a multiple of 360°).

quency, and the expected coherency is low. The corresponding gain is low compared with experimental data. The phase is confined to a range from about  $-150^{\circ}$  to  $-170^{\circ}$  and cannot explain most experimental values.

5. Short-term potentiation of VI/TI without chemoreflex-feedback can explain the relatively high coherencies and gains between PET<sub>CO2</sub> and VI/TI below approximately 0.08 cycles per breath. The phase is, however, equal to the situation of conclusion 4.

#### DISCUSSION

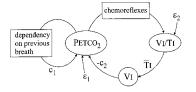
# Interpretation of Coherent Oscillations in End-Tidal $\rm CO_2$ and Mean Inspiratory Flow

The coherent oscillations in PET<sub>CO2</sub> and VI/TI that were found in 15 subjects were interpreted as the result of spontaneous "noise" in the respiratory system, which is filtered by the chemoreflex. Sighs, coughs, and changes in CO<sub>2</sub> production or cardiac output may all add to such noise. This mechanism was proposed by Modarreszadeh and colleagues (8), who found that the highest power of PET<sub>CO2</sub> and minute volume occurred below 0.10 cycles per breath in quietly breathing subjects. They applied an end-tidal CO<sub>2</sub> buffering technique which allowed an artificial reduction of the power of  $P_{ET_{CO2}}$  below 0.10 cycles per breath (under hyperoxic conditions). The consequence was a significant reduction in the power of minute volume. This is the most convincing evidence that a considerable part of the ventilatory variability can result from a response to fluctuations in PET<sub>CO2</sub>. Another indication that part of the respiratory variability is due to chemoreflex activity comes from a study in rats where the variability of phrenic nerve activity was studied (31).

The frequencies of coherent oscillations were mostly below 0.20 cycles per breath. Most of these frequencies could be explained by the delays of the peripheral chemoreflex, with a shift to lower frequencies due to the dependency of  $PET_{CO2}$  on

the previous breath. The cycle durations were 5 to 12 breaths ( $\sim$  20–60 seconds). This contrasts with the cycle time of frank periodic breathing during high-altitude hypoxia, which is mostly 18 to 25 seconds (13, 32). A similar difference in cycle time has been observed between normoxia and isocapnic hypoxia (10). The "fast" periodic breathing during hypoxia is probably caused by peripheral chemoreflex activity as well (20). This does not preclude, however, a role for peripheral chemoreceptors in the development of the relatively "slow" oscillations in the present study. The model predicts a difference in cycle time between normoxia and hypoxia if hypoxia is considered as an "amplifier" of the peripheral chemoreflex sensitivity to  $Pa_{CO_2}$  (33). As explained in Section D.2.3 of the online data supplement (Figure E8), a shift to a shorter cycle time due to an increased chemoreflex sensitivity can be expected if there is a certain dependency of  $P_{ET_{CO_2}}$  on the previous breath. In the model, it is actually this dependency that adds to the "slowness" of the respiratory system during normoxia. The contribution of the peripheral feedback loop in our study is further supported by the reduction of coherent oscillations by 100% O<sub>2</sub>. A contribution of the central feedback loop to the variability can also be expected below approximately 0.10 cycles per breath. However, because such slow oscillations can occur less often during a recording time of 30 minutes, the chance of finding them is smaller than for faster oscillations. The inhalation of 100% O<sub>2</sub> has probably also altered the central drive, as suggested by the increased mean ventilation. This may be secondary to a reduction of the Haldane effect or a suppression of central hypoxic inhibition (34).

The squared coherency between  $Pet_{CO_2}$  and VI/TI has a double meaning here. First, it is a measure of linearity of the relationship between  $Pet_{CO_2}$  and VI/TI at a specific frequency, regardless of its physiologic interpretation. Second, according to the model, it is a direct estimate of the "loop gain" of the feedback system. The loop gain reflects the amplification in



**Figure 5.** Schematic of the chemoreflex-feedback model. The variability of  $P_{ET_{CO_2}}$  and  $V_i/T_i$  is driven by two sources of noise,  $\varepsilon_1$  and  $\varepsilon_2$ . The noise is filtered by the chemoreflex-feedback loop and the dependency of  $P_{ET_{CO_2}}$  on the previous breath.  $c_1$ , co-

efficient of the dependency of  $P_{ET_{CO_2}}$  on the previous breath;  $c_2$ , coefficient of the influence of Vi on  $P_{ET_{CO_2'}}$ ,  $\overline{T}_1$ , mean Ti.

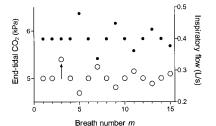
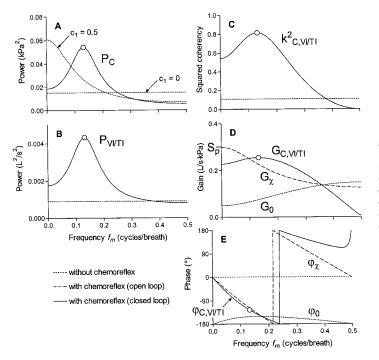


Figure 6. Dampened oscillation in  $P_{ET_{CO_2}}$  and  $V_I/T_I$  caused by delayed chemoreflex feedback after an accidental increase in  $P_{ET_{CO_2}}$ . *Open circles,* breath-tobreath  $P_{ET_{CO_2}}$  (*left axis*), *arrow* indicates accidental increase in  $P_{ET_{CO_2}}$ ; *closed circles,*  $V_I/T_I$  (*right axis*).



**Figure 7.** Hypothetical spectra for the model without chemoreflex (*dotted lines*) or with only a peripheral chemoreflex (*continuous lines*). P<sub>C</sub>, power of PET<sub>CO2</sub>; c<sub>1</sub>, coefficient of the dependency of PET<sub>CO2</sub> on the previous breath; P<sub>VITI</sub>, power of VI/TI; k<sup>2</sup><sub>C,VI/TI</sub>, squared coherency between PET<sub>CO2</sub> and VI/TI; G<sub>C,VI/TI</sub> and  $\varphi_{C,VI/TI}$ , closed-loop gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti; G<sub>0</sub> and  $\varphi_{0}$ , gain and phase from PET<sub>CO2</sub> to VI/Ti for the situation without chemoreflex; S<sub>p</sub>, peripheral chemoreflex sensitivity (in L/second · kPa). *Open circles* refer to the resonance frequency.

the feedback loop and is a major determinant of the tendency to oscillate (15). Other more indirect estimation procedures for loop gain have been described for pseudorandom  $CO_2$  inhalation (35) or transient changes in  $PET_{CO2}$  (16).

The model analysis revealed a close relationship between the closed-loop gain from  $P_{ET_{CO_2}}$  to VI/TI and the open-loop chemoreflex gain for coherent oscillations. This makes an estimation of the chemoreflex gain possible during the steady state without additional stimulation (e.g., in sleep studies or during continuous monitoring in the ICU). We did not, however, compare this method with a standard technique. The approach is not essentially different from the analysis of dampened respiratory oscillations induced by spontaneous sighs (18), acute changes in inspiratory  $P_{CO_2}$  (16), or pseudorandom  $CO_2$  stimulation (35). A drawback is that the facemask could have influenced the breathing pattern (12), although we did not find coherent oscillations in every subject. This probably depends on the ratio between the different sources of noise. The variability of arterial Po2 may also play a role because  $P_{ET_{CO_2}}$  and  $P_{ET_{O_2}}$  oscillated out-of-phase, so that an additive effect on peripheral chemoreceptors is possible (36).

The finding that the phase between VI/TI and VI was about zero for coherent oscillations (Table 3) indicates that VI/TI during a given breath was positively related to VI of the same breath. Because the gain of this relationship was about equal to the mean TI (see also Table 1), this could be explained by the model as a direct influence of changes in VI/TI on VI at a relatively constant T1. Apparently, there was no strong mechanism that counteracted the evoked changes in respiratory drive (VI/TI) by an adaptation of TI (5). The relationship between VI and TE was relatively weak, with a rough clustering of the phase between VI and TE around zero (Table 3). This may be related to a mechanism that lengthens TE in response to high V<sub>1</sub>, although the Hering-Breuer reflex is probably only operating for VI above 1.2 L (37). Anyway, a zero phase between VI and TE argues against a strong influence of chemoreflexes on TE, because chemoreflexes would simultaneously increase VI/TI (and thus increase VI) and shorten TE (23).

# Modeling of Chemoreflex Feedback

A number of mathematical models have already been implemented to explain periodic breathing from chemoreflex characteristics in heart failure or during hypoxia (e.g., 15, 16, 17). Several models also give a good description of dampened oscillations after sudden changes in the system such as a sigh or an increase in inspiratory  $CO_2$  (16, 18, 35). The present model is actually a simplification of existing models where the relationship between respiratory variables is described from breath to breath (18, 35, 38, 39). What is new is the analytical derivation of power and cross-spectra from such simplified linear difference equations. The reason to explore respiratory variability in the frequency domain is that the supposed linear interactions constitute a time-invariant filter to noise in the system. We tried to explain the spectral features by a model as simple as possible. The major advantage is that this gives insight into the main mechanisms of the variability. A drawback is that nonlinear interactions can occur that are not always well approximated by a linear model. For example, a highly variable TTOT would change the chemoreflex response in terms of number of breaths, so that the response function becomes nonlinear. The same holds true for variations in chemoreflex sensitivity or cardiac output. The ventilatory influence of VI/TI on  $P_{ET_{CO_2}}$  is probably more nonlinear for larger variations (40).

In conclusion, we found that coherent oscillations in  $Pet_{CO2}$ and VI/TI occur in the normal respiratory variability. These oscillations could be explained as a chemoreflex response to spontaneous changes in the respiratory system, indicating that respiration is continuously adjusted by the chemoreflexes so that the respiratory variability is not random. Previously, the function of the chemoreflexes was mostly studied with external stimuli (e.g., inhaled CO<sub>2</sub>). While the response to such stimuli shows the capability of the reflexes to respond, the analysis of respiratory variability makes it possible to derive information on the actual performance of the chemoreflexes.

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## References

- Bellville JW, Whipp BJ, Kaufman RD, Swanson GD, Aqleh KA, Wiberg DM. Central and peripheral loop gain in normal and carotidbody resected subjects. *J Appl Physiol* 1979;46:843–853.
- 2. Cunningham DJC. A model illustrating the importance of timing in the regulation of breathing. *Nature* 1975;253:440–442.
- 3. Priban IP. An analysis of some short-term patterns of breathing in man at rest. *J Physiol (Lond)* 1963;166:425–434.
- Tobin MJ, Yang KL, Jubran A, Lodato RF. Interrelationship of breath components in neighboring breaths of normal eupneic subjects. *Am J Respir Crit Care Med* 1995;152:1967–1976.
- Newsom Davis J, Stagg D. Interrelationships of the volume and time components of individual breaths in resting man. J Physiol (Lond) 1975;245:481–498.
- Jubran A, Grant BJB, Tobin MJ. Effect of hyperoxic hypercapnia on variational activity of breathing. Am J Respir Crit Care Med 1997;156: 1129–1139.
- 7. Lenfant C. Time-dependent variations of pulmonary gas exchange in normal man at rest. J Appl Physiol 1975;22:675–684.
- Modarreszadeh M, Bruce EN. Ventilatory variability induced by spontaneous variations of Pa<sub>CO2</sub> in humans. J Appl Physiol 1994;76:2765–2775.
- Hlastala MP, Wranne B, Lenfant CJ. Cyclical variations in FRC and other respiratory variables in resting man. J Appl Physiol 1973;34:670–676.
- Jubran A, Tobin MJ. Effect of isocapnic hypoxia on variational activity of breathing. Am J Respir Crit Care Med 2000;162:1202–1209.
- Gillam PMS. Patterns of respiration in human beings at rest and during sleep. Bull Eur Physiopath Resp 1972;8:1059–1070.
- Van den Aardweg JG, Karemaker JM. Respiratory variability and associated cardiovascular changes in adults at rest. *Clin Physiol* 1991;11: 95–118.
- Brusil PJ, Waggener TB, Kronauer RE, Gulesian P. Methods for identifying respiratory oscillations disclose altitude effects. J Appl Physiol 1980;48:545–556.
- Chapman KR, Bruce EN, Gothe B, Cherniack NS. Possible mechanisms of periodic breathing during sleep. J Appl Physiol 1988;64:1000–1008.
- Khoo MCK, Kronauer RE, Strohl KP, Slutsky AS. Factors inducing periodic breathing in humans: a general model. J Appl Physiol 1982;53: 644–659.
- Carley DW, Shannon DC. A minimal mathematical model of human periodic breathing. J Appl Physiol 1988;65:1400–1409.
- Longobardo GS, Cherniack NS, Fishman AP. Cheyne-Stokes breathing produced by a model of the human respiratory system. *J Appl Physiol* 1966;21:1839–1846.
- Khoo MCK, Marmarelis VZ. Estimation of peripheral chemoreflex gain from spontaneous sigh responses. Ann Biomed Eng 1989;17:557–570.
- Jensen JI, Mosekilde E, Ward DS. A system dynamics model of human respiratory regulation. In: Whipp BJ, Wiberg DM, editors. Modelling and control of breathing. New York: Elsevier; 1983. p. 316–321.
- Lahiri S, Hsiao C, Zhang R, Mokashi A, Nishino T. Peripheral chemoreceptors in respiratory oscillations. J Appl Physiol 1985;58:1901–1908.
- Jenkins GM, Watts DG. Spectral analysis and its applications. San Fransisco: Holden-Day; 1968.

- Whitelaw WA, Derenne JP, Milic-Emili J. Occlusion pressure as a measure of respiratory center output in conscious man. *Respir Physiol* 1975;23:181–199.
- Ledlie JF, Nielsen SG, Cherniack NS, Fishman AP. Effects of hypercapnia and hypoxia on phrenic nerve activity and respiratory timing. J Appl Physiol 1981;51:732–738.
- DeBoer RW, Karemaker JM, Strackee J. Hemodynamic fluctuations and baroreflex sensitivity in humans: a beat-to-beat model. Am J Physiol 1985;253 (Heart Circ Physiol 22):H680–H689.
- Armitage P, Berry G. Statistical methods in medical research. Oxford: Blackwell; 1987. 123–125.
- Van den Aardweg JG, Van Steenwijk RP, Karemaker JM. A chemoreflex model of relation between blood pressure and heart rate in sleep apnea syndrome. Am J Physiol 1995;268 (Heart Circ Physiol 37): H2145–H2156.
- Swanson GD, Ward DS, Bellville JW. Posthyperventilation isocapnic hyperpnea. J Appl Physiol 1974;37:723–735.
- Carley DW, Shannon DC. Relative stability of human respiration during progressive hypoxia. J Appl Physiol 1988;65:1389–1399.
- Georgopoulos D, Bshouty Z, Younes M, Anthonisen NR. Hypoxic exposure and activation of the afterdischarge mechanism in conscious humans. J Appl Physiol 1990;69:1159–1164.
- Benchetrit G, Bertrand F. A short-term memory in the respiratory centres: statistical analysis. *Respir Physiol* 1975;23:147–158.
- Khatib MF, Oku Y, Bruce EN. Contribution of chemical feedback loops to breath-to-breath variability of tidal volume. *Respir Physiol* 1991;83: 115–128.
- Waggener TB, Brusil PJ, Kronauer RE, Gabel RA, Inbar GF. Strength and cycle time of high-altitude ventilatory patterns in unacclimatized humans. J Appl Physiol 1984;56:576–581.
- 33. Honda Y, Tani H. Chemical control of breathing. In: Altose MD, Kawakami Y, editors. Control of breathing in health and disease. In: Lenfant C, editor. Lung biology in health and disease, Vol. 135. New York: Marcel Dekker; 2000. p. 41–88.
- Holtby SG, Berezanski DJ, Anthonisen NR. Effect of 100% O<sub>2</sub> on hypoxic eucapnic ventilation. J Appl Physiol 1988;65:1157–1162.
- Ghazanshahi SD, Khoo MCK. Estimation of chemoreflex loop gain using pseudorandom binary CO<sub>2</sub> stimulation. *IEEE Trans Biomed Eng* 1997;44:357–366.
- 36. Akiyama Y, Kawakami Y. Clinical assessment of the respiratory control system. In: Altose MD, Kawakami Y, editors. Control of breathing in health and disease. In: Lenfant C, editor. Lung biology in health and disease, Vol. 135. New York: Marcel Dekker; 2000. p. 251–287.
- Iber C, Simon S, Skatrud JB, Halowald MW, Dempsey JE. The Breuer-Hering reflex in humans. Am J Respir Crit Care Med 1995;152:217–224.
- Busso T, Liang PJ, Robbins PA. Breath-to-breath relationships between respiratory cycle variables in humans at fixed end-tidal Pco<sub>2</sub> and P<sub>O2</sub>. *J Appl Physiol* 1996;81:2287–2296.
- Modarrheszadeh M, Kump KS, Chizeck HJ, Hudgel DW, Bruce EN. Adaptive buffering of breath-by-breath variations of end-tidal CO<sub>2</sub> of humans. J Appl Physiol 1993;75:2003–2012.
- Chilton AB, Stacy RW. A mathematical analysis of carbon dioxide respiration in man. *Bull Math Biophys* 1952;14:1–18.