Partial Reconstruction of Wavelet Transforms: A Technique for Quantifying Amplitude Envelopes of Riser Response Modes

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Knowledge of the frequencies and amplitudes and single-mode versus multi-mode behavior of deepwater riser responses is important for accurate fatigue analysis. The extraction of this information from measured riser responses is a significant challenge given the complexity of the phenomena involved with riser vibrations. An existing technique for obtaining modal amplitudes, wavelet ridge extraction (WRE), has been shown to return a smoothed version of the actual amplitude envelope, resulting in attenuated peak values. In this paper, we introduce a new wavelet-based technique, partial reconstruction of wavelet transforms, for accurately extracting amplitude envelopes of the modes from multi-mode signals. The partial reconstruction technique is proposed to circumvent problems presented by WRE by making use of information available from the inverse wavelet transform. It is demonstrated with synthesized test signals that the new technique provides much more accurate modal envelopes than does WRE. Finally, to illustrate the technique for practical riser response signals, the partial reconstruction technique is applied to representative acceleration fluctuations measured from a drilling riser in the Schiehallion field. [DOI: 10.1115/1.1603310]

Introduction

As offshore oil production moves into depths of 1000–3000 m, the goal of understanding the dynamics and improving the design of marine risers becomes ever more important. One aspect of riser design is an accurate determination of fatigue life, which requires knowledge of the riser response amplitudes and frequencies (Allen [1]). Engineering models such as Shear7 are often used to determine riser response and the underlying amplitudes of the response frequencies. It is well known to the ocean-engineering community, however, that excitations from the top motion of the platform, wave action near the surface, and vortex induced vibrations from shear currents, as well as interactions with nearby risers make accurate prediction of riser motion a daunting challenge. Therefore, in order to have an accurate description of riser behavior in the ocean environment, riser responses are still being measured in full-scale experiments, underscored by recent efforts at monitoring the motion of risers attached to deepwater production platforms (Brower et al., [2], McCarthy et al., [3]).

The analysis of measured responses also presents its own set of challenges. Riser responses in shear flow have been observed to be nonstationary, with the response amplitudes and frequencies varying with time. For example, experimental studies by Larsen et al. [4] and Vandiver et al. [5] indicate that riser responses can be dominated by one mode or have many modes. Furthermore, the riser motion may intermittently switch between states of vibration even if the flow condition remains constant. Vandiver et al. [5] reported the following:

> It is important to note that, in sheared flow cases, stationary response is rarely observed. When single-mode dominance occurs, lock-in may shift from one mode to another due to temporary variation in flow, fluctuations in tension, or turbulence. Lock-in events are sometimes separated by a period of multiple-mode response. In cases such as those depicted in region II, where single-mode dominance is rare, there is considerable time-domain fluctuation of modal response energy between various modes.

Despite the fact that such observations have been reported in the literature, these time-dependent characteristics of riser response have not been quantified. A reason that the time-dependent characteristics have not been examined in detail is that the engineering tools for such an analysis are not well developed and the use of such tools often requires the expertise of the specialist.

The aim of the present paper is to demonstrate a new wavelet-based analysis technique that can be used to quantify the complex multiple-mode behavior observed in riser responses. The continuous wavelet transform, which was developed by Grossman and Morlet [6], allows one to quantify how modes having different frequencies start, stop, and develop over time. In this work, it is shown how the technique of partial reconstruction of wavelet transforms can be applied to quantify amplitude envelopes of intermittent response modes. The potential applications of this work include a straightforward analysis tool for extracting riser response amplitudes and frequencies and for quantifying the switching behavior between single-mode and multiple-mode responses. It should be noted that quantitative knowledge of the switching behavior between single-mode and multi-mode responses may have practical benefits for accurate fatigue life estimation. In the fatigue analysis performed for the SlenderEx drilling riser by Roveri and Vandiver [7], it was shown that the single-mode response produces more than eight times the fatigue damage than multi-mode response. It follows that if the riser response is switching between these states over time, then it is important to quantify both the amount of time spent in each state and the response amplitudes and frequencies within each of these states.

Wavelet ridge extraction is an existing technique for measuring the amplitude envelope and instantaneous frequency of multiple modes in a response and has been applied in a number of studies to examine time-dependent responses of multiple-degree-of-freedom systems. For example, Staszewski [8] and Ruzzene et al. [9] applied wavelet analysis to obtain the natural frequencies and decay rate of individual modes. Staszewski [10] used a WRE to

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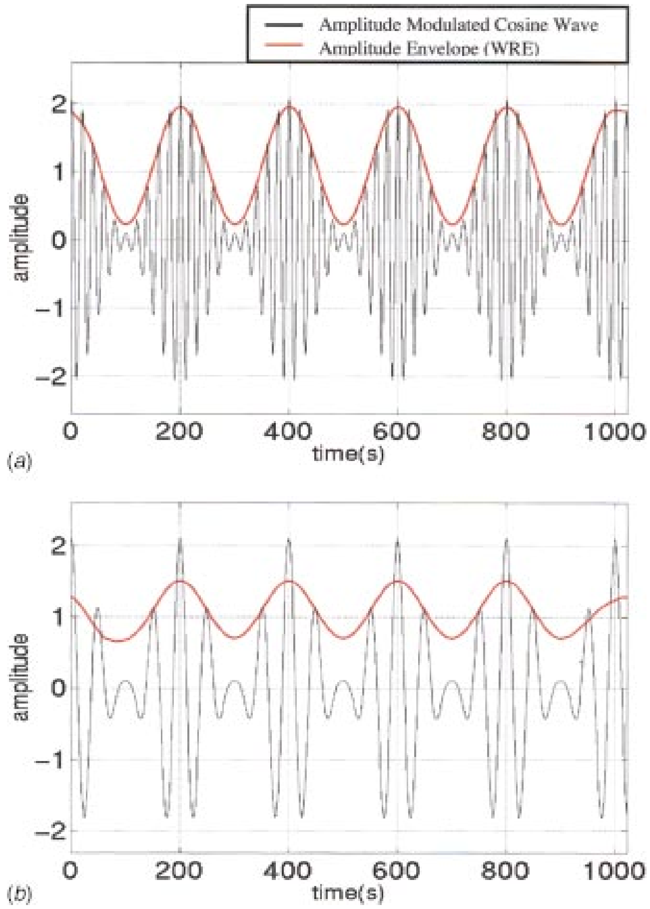

Fig. 1 Amplitude modulated cosine waves with amplitude envelopes measured from wavelet ridge extraction (*a*) $f_m/f_c=0.1$, (*b*) $f_m/f_c=0.25$

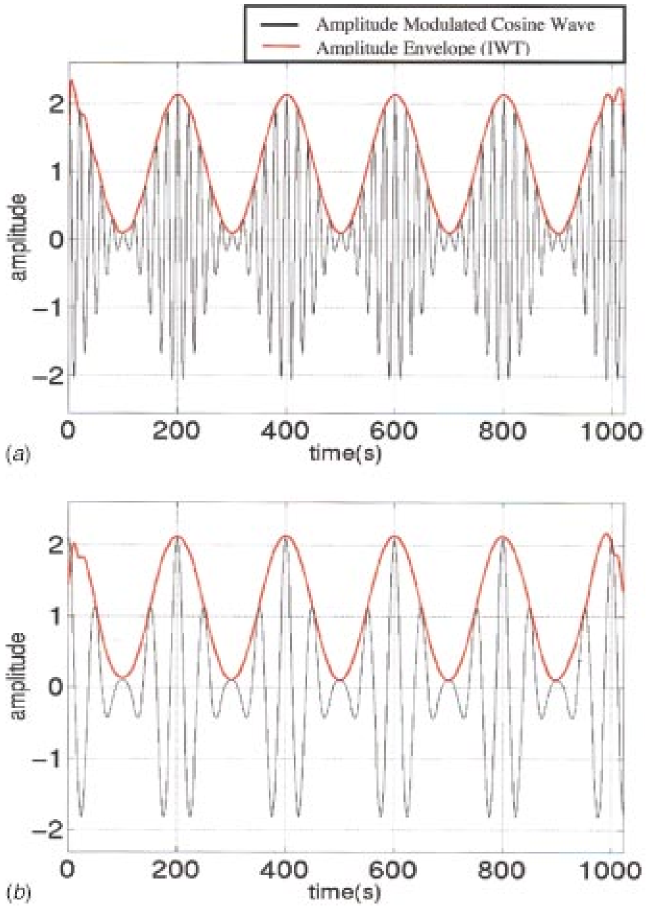

Fig. 2 Amplitude modulated cosine waves with amplitude envelopes measured from the inverse wavelet transform (*a*) $f_m/f_c=0.1$, (*b*) $f_m/f_c=0.25$

measure the instantaneous amplitude and frequency characteristics of nonlinear systems for the purpose of MDOF system identification. In the field of ocean engineering, Jordan et al. [11,12] quantified the transient response of the multiple-mode surge and pitch responses of a model spar platform tested in the OTRC wave basin. Weggel et al. [13,14] performed numerical simulations of a spar platform designed for the Gulf of Mexico and applied wavelet analysis to the computed response in order to examine nonlinear behavior of the natural response modes.

In the studies described above, it was demonstrated that the WRE technique is particularly well-suited for quantifying multiple-mode signals with *slowly* varying amplitudes and frequencies. However, as emphasized in the work by Jordan et al. [15], if the amplitude envelope or instantaneous frequency fluctuates too rapidly, then WRE measures smoothed versions of these instantaneous characteristics. Teisseire et al. [16], using a family of amplitude and frequency-modulated test signals, measured the frequency response function of wavelet ridges (with the Morlet wavelet) and showed that they act as low-pass filters. This result is consistent with the observation that wavelet ridges give smoothed amplitude fluctuations and attenuated localized extremes.

The attenuation of peaks in the amplitude envelope represents a loss of information that may be important for ocean-engineering applications such as fatigue assessment. A demonstration of this attenuation is given in Fig. 1, which shows two examples of amplitude modulated cosine waves:

$$AM(t)=[A+\Delta A\cos(2\pi f_m t)]\cos(2\pi f_c t) \tag{1}$$

where A is the carrier wave amplitude, f_m is the modulation frequency, f_c is the carrier frequency, and ΔA is the modulation amplitude. Both signals have the same properties, except for the carrier frequency, which is smaller in Fig. 1*b*, giving it a larger value of the parameter, f_m/f_c. The red line represents the amplitude envelope obtained by WRE. As the ratio f_m/f_c increases from the first signal to the second, the attenuation in the WRE amplitude envelope becomes more evident. In the first case, the maximum percent difference between the actual amplitude envelope peak and the wavelet ridge technique is 6.78 percent. However, in the second case, the maximum percent difference reaches 28.43 percent. These results are consistent with the results of Teisseire et al. [16], which demonstrate that the attenuation in the measured amplitude increases with the parameter f_m/f_c.

The purpose of this paper is to introduce a new wavelet-based analysis technique that gives accurate measurements of the fluctuating amplitudes of time-dependent and intermittent modes such as those encountered in marine riser responses. The scope of the work in this paper is as follows. A wavelet analysis technique based on information available from the inverse wavelet transform (IWT) is described and demonstrated on the amplitude modulated signal in Fig. 1, where WRE fails to obtain accurate envelopes. Next, in order to examine test signals more representative of riser responses, a test signal is constructed consisting of eight modes with intervals dominated by a single mode interspersed with intervals of multi-mode behavior. A technique termed partial reconstruction is applied by windowing the intermittent modes in the wavelet domain and applying the inverse wavelet transform to the windowed regions. The eight amplitude envelopes measured from the test signal are compared to the prescribed amplitudes and the accuracy of the technique is evaluated with global mean square error and peak resolution error measures. Finally, as an illustration

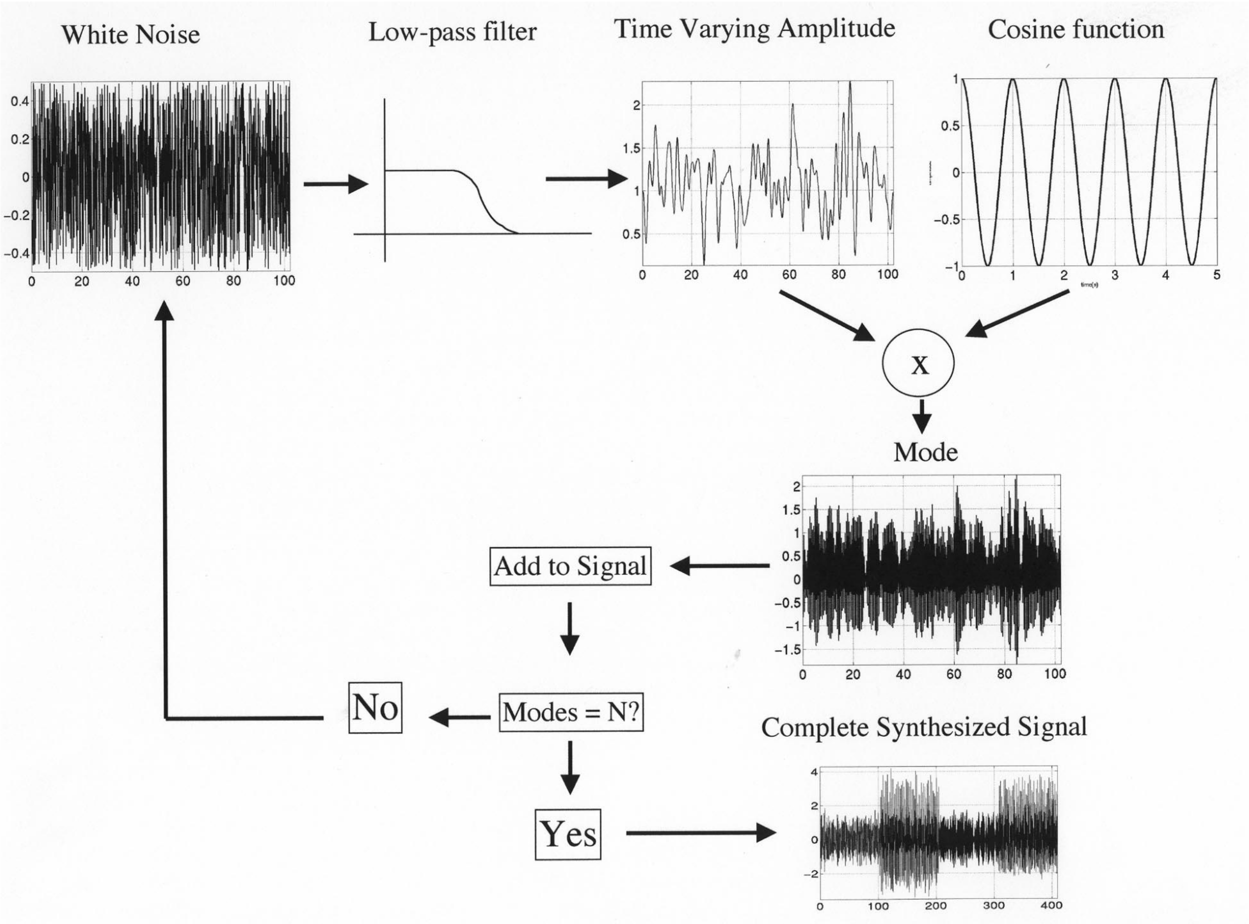

Fig. 3 Methodology for synthesis of a test signal

of the technique for practical application, partial reconstruction is applied to riser acceleration fluctuations measured from the Schiehallion field drilling riser.

Amplitude Envelopes From the Inverse Wavelet Transform

The wavelet transform of a function, $f(t)$, is given by:

$$W(a,\tau)=a^{-1/2}\int_{-\infty}^{\infty} f(t)\,\psi^{*}\left(\frac{t-\tau}{a}\right)dt \tag{2}$$

where a is the scale, τ is a time shift, and $\psi(t)$ is the analyzing wavelet. The wavelet transform projects a fluctuation onto time-localized wavelet functions, which adapt their window size to the frequency range to be examined. This work uses the complex-valued Morlet wavelet [6]:

$$\psi(t)=e^{i5.5t}e^{-t^{2}/2} \tag{3}$$

which is a Gaussian-windowed complex sinusoid.

The wavelet coefficients, $W(a,\tau)$, are defined on a two-dimensional time-scale domain and give a representation of the time evolution of frequency content in a signal. Wavelet theory also includes an inversion formula (IWT), which allows exact recovery of the function:

$$f(t)=\frac{1}{C_{\Psi}}\int_{-\infty}^{\infty}\int_{0}^{\infty} W(a,\tau)\,\frac{1}{\sqrt{a}}\,\psi\left(\frac{t-\tau}{a}\right)\frac{da}{a^{2}}\,d\tau. \tag{4}$$

In the above expression, C_{Ψ} is an admissibility constant whose value is finite if $\psi(t)$ is a wavelet, and is given by the equation:

$$C_{\psi}=2\pi\int_{-\infty}^{\infty}|\psi(\omega)|^{2}\frac{d\omega}{\omega} \tag{5}$$

where $\psi(\omega)$ is the Fourier transform of $\psi(t)$. To date, wavelet-based analysis techniques such as WRE measure signal characteristics from the wavelet coefficients $W(a,\tau)$ given by Eq. (2) and little emphasis has been given to the inverse wavelet transform. Note that the IWT integrates the wavelet coefficients over time and scale to perform an exact reconstruction of the original signal, $f(t)$. Since exact reconstruction is possible, it stands to reason that information regarding the amplitudes of the signal is encoded in the wavelet coefficients via the IWT. Note that the reconstructed function, $f(t)$, is complex since the Morlet wavelet is complex and the real part of the IWT gives $f(t)$. It follows that a simple approach for obtaining the amplitude is to measure the magnitude of the inverse wavelet transform, i.e.:

$$|f(t)|=\left|\frac{1}{C_{\Psi}}\int_{-\infty}^{\infty}\int_{0}^{\infty} W(a,\tau)\,\frac{1}{\sqrt{a}}\,\psi\left(\frac{t-\tau}{a}\right)\frac{da}{a^{2}}\,d\tau\right| \tag{6}$$

In order to give a demonstration of this approach, Fig. 2. shows the same amplitude-modulated signals shown in Fig. 1, with measured amplitude envelopes obtained from the magnitude of the IWT given by Eq. (6). The IWT exhibits virtually no attenuation in the reconstruction of these amplitude envelopes. Both of these envelopes created by the IWT reach maximum percent differences of less than 1.5 percent at the peaks, which is a significant improvement over the results obtained with WRE. The remaining error is most likely due to both discretization of scales and the approximation of continuous convolutions with discrete convolutions, and can be reduced with careful design of the reconstruction filters (Mallat, [17]).

Synthesized Signals Representative of Riser Responses

The results in the previous section serve to demonstrate that the magnitude of the IWT accurately measures the amplitude envelopes of the simple cases in Fig. 2 even where wavelet ridge extraction fails. Nevertheless, riser response signals measured in practice exhibit considerably more complexity than the signals in Fig. 2. Vandiver et al. [5] and Larsen et al. [4] observed from experimental measurements that marine riser response may be dominated by one frequency, i.e., lock-in, or may exhibit multiple frequencies and can intermittently switch frequencies and states of motion. Furthermore, risers have numerous natural frequencies that may be excited. In order to be representative of riser responses, synthesized test signals should include ''lock-in regions,'' characterized by a single dominant mode, intermittently interspersed with multiple modes, typical of nonlock-in behavior. Signals having these characteristics are generated and analyzed to examine performance of the IWT technique for more realistic signals. Wavelet transforms are used to identify individual modes and then a partial reconstruction is applied to IWT only those coefficients corresponding to the identified intermittent mode. We define the process of partial reconstruction in the following way:

1. Wavelet transform the full signal to visualize all modes
2. Window each mode in time and scale to ''isolate'' it from other modes
3. Perform the IWT over the selected windowed region of each mode to evaluate the amplitude expression in Eq. (6)

The accuracy of this approach is quantified by comparing the modal amplitudes obtained from the wavelet analysis with the prescribed amplitudes of the synthesized modes.

A diagram of the entire process for synthesizing test signals is given in Fig. 3. The total number of modes to include (eight in this work) is chosen beforehand and the objective is to generate a randomly varying amplitude for each mode. The process starts with a white noise signal obtained from a random number generator to create a signal of numbers ranging from -1 to 1. This signal is then amplified and filtered by a Butterworth low-pass filter to make the frequency content of the amplitude envelope well below the modal frequency. Finally, the random amplitude is multiplied by a cosine function with unit amplitude and frequency chosen to coincide with realistic natural frequencies of risers (Mekha et al. [18], McCarthy et al. [3]). This mode is then added to the signal and the process of generating and adding modes of different frequencies is repeated to build a ''simulated'' riser signal. The complete synthesized signal used in this study is presented in Fig. 4. The signal may be considered representative of a riser response in the sense that it possesses two periods of single-mode ''lock-in'' behavior, interspersed with two other periods of multi-mode behavior.

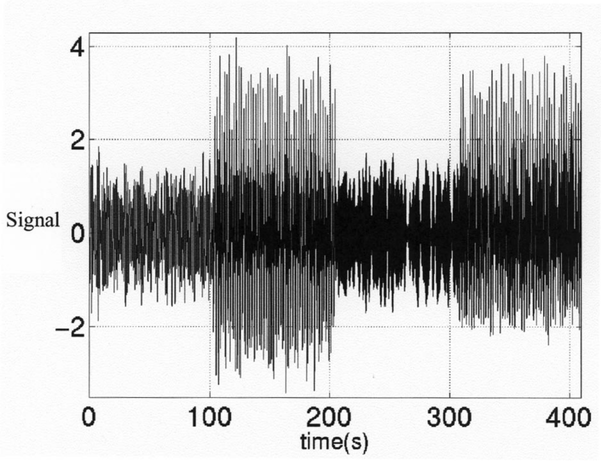

Fig. 4 Synthesized signal having characteristics representative of a riser response

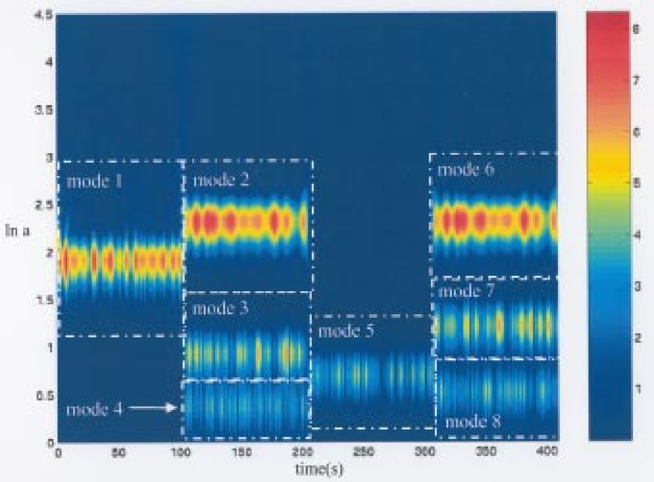

Fig. 5 Wavelet transform of synthesized signal in Fig. 4 depicting isolation of modes

Wavelet Analysis of Synthesized Riser Signals

Figure 5 shows the wavelet transform of the simulated riser signal presented in Fig. 4. Time is displayed along the horizontal axis and scale ($\ln a$) is displayed along the vertical axis with the color at each time-scale location representing the magnitude of the wavelet coefficients $W(a,\tau)$. Note that within the plot, the eight individual modes are visualized because of the time-scale separation that is inherent in the wavelet transform. Intervals 0–100 s and 200–300 s are dominated by a single mode and intervals 100–200 s and 300–400 s are characterized by multi-mode (three

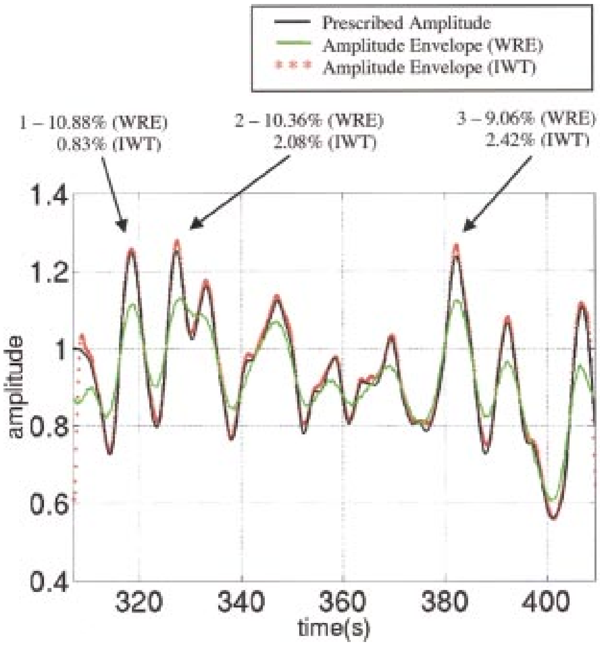

Fig. 6 Measured amplitude envelopes of mode six

modes) behavior. The dashed white boxes that appear on the plot represent the regions over which partial reconstruction with the IWT is performed.

As demonstrated in Fig. 5, once the wavelet transform has been performed upon a signal, the individual modes and their intervals can be visualized. The IWT can then be applied to the time-scale regions (in this case obtained with visual inspection) corresponding to independent modes. This in effect is a time-scale filtering of individual modes, which allows one to extract the modal amplitudes and the duration over which they are active within the signal.

The efficacy of the partial reconstruction technique is demonstrated through detailed examination of the amplitude obtained for mode six. In Fig. 6, the prescribed envelope is compared with the results of both WRE and partial reconstruction with the IWT. The result of WRE is a smoothed version of the prescribed amplitude, consistent with the results presented in Fig. 1 and underscoring the necessity for an alternative technique. The result from the partial reconstruction technique shows marked improvement over WRE in following the detail of the amplitude fluctuations. In terms of peak resolution, which may be of practical importance in design considerations, the three extreme peaks examined in Fig. 6 show an improvement from about 10 percent error for WRE to a range of 0.83 percent–2.42 percent error for IWT.

Figure 7 shows the amplitude envelopes of the full set of eight modes of the synthesized test signal. For each mode, the prescribed amplitude envelope (black) is plotted along with the measured amplitude envelope (red) obtained from partial reconstruction. In all cases, with the exception of mode four, the results from partial reconstruction are observed to accurately follow the random amplitude variation of the prescribed modes. In order to quantify the results, the global mean square error and the largest percent error in the highest peaks (considering at least the top five) are given for each mode in Table 1. The mean square error between the prescribed and measured envelopes ranges between 0.00014 and 0.00220. The maximum percent error in resolving the peaks ranges from 1.01 percent to about 6.14 percent giving excellent to reasonable agreement.

As observed in Table 1, the largest errors in the analysis occur with mode four. Figure 7 shows that there are regions in mode four, e.g., around 190 s, where there is considerable discrepancy between the measured and prescribed amplitude envelopes. A natural question arises as to the origin of the relatively large error for this mode. By examining mode four in the wavelet transform in Fig. 5, it is noted that it is in close proximity to mode three, such that the wavelet coefficients form continuous streaks between the two modes. Does this proximity introduce error?

In order to examine this question, additional calculations involving mode four were performed. Since there is complete control over the synthesized signal properties, a signal was used that involved mode four only. Table 2 presents the mean square error and peak resolution error obtained from the full signal with that of a signal composed only of mode four. The new calculation, using the same time-scale integration window size as before, results in a reduction of both the mean square error and the largest error in the extreme peaks, indicating that a large part of the error for mode four is due to interference with mode three. A further calculation addressed the question of whether using a much larger integration window would reduce error as well. It is also shown that by extending the scale range of the integration window from $\ln a = 0.0-0.61$ to $\ln a = 0.0-4.5$, something that is not normally possible when other modes are present, the error is reduced further. The reason for this improvement stems from the fact that time-localized signal features correspond to a large range of scales. For example, it is often demonstrated that a delta function generates nonzero wavelet coefficients at all scales. By extending the integration window in scale for the partial reconstruction technique, more nonzero wavelet coefficients (albeit small amplitude) corresponding to the mode are involved, resulting in a more accurate reconstruction.

Illustrative Example: Schiehallion Riser Responses

The purpose of this section is to demonstrate the application of wavelet analysis to riser response signals measured from a drilling riser in the Schiehallion field, west of Shetlands. The Schiehallion drilling riser was located in a water depth of 360 m and instrumented with X and Y accelerometers located at fractions $L/8$, $L/4$ and $L/2$ (measured from the bottom) of the riser length $L = 368$ m. The acceleration fluctuations were measured at a sampling frequency of 5 Hz for a duration of 27.3 min per record. A more thorough description of the experimental details can be found in Hassanein and Fairhurst [19], and Cornut and Vandiver [20].

Figure 8 presents two representative examples of acceleration signals, designated Case 1 and Case 2, both of which were measured on December 23, 1996. Case 1 was obtained from the X accelerometer at the $L/4$ location, while Case 2 was obtained from the Y accelerometer at the $L/2$ location. Note that these two signals were not measured simultaneously. The magnitudes of the Fourier transform and the continuous wavelet transform are plotted beneath each fluctuation.

In both cases, the acceleration fluctuations exhibit such complexity that it is difficult to determine the fluctuation characteristics directly from the time domain representation. Fourier transforms, however, readily measure the active *global* frequency content of the acceleration fluctuations. The labeled modes in each Fourier transform are based on the frequencies of the Schiehallion drilling riser modes estimated by Cornut and Vandiver [20], reproduced here in Table 3. The Fourier transforms of both Case 1 and Case 2 show a sharp mode one and broadband modes two and three that overlap one another. In addition, Case 2 exhibits a low frequency component that is much lower than any of the riser modal frequencies. In the wavelet transforms, the scales ($ln\, a$) that correspond to the riser frequencies are given in Table 3 and the modes corresponding to these scales are labeled. In comparing the wavelet transforms with the Fourier transforms, it is readily apparent that mode one is reasonably distinct, but in the wavelet transform modes two and three cannot be distinguished from one another. This is a consequence of the uncertainty principle, where the benefit of increased time resolution of the wavelet transform is traded for frequency resolution.

The continuous wavelet transform complements the Fourier transform by giving time-resolved information about the excited modes in the fluctuation. For example, in Case 1, the Fourier transform shows strong peaks for modes one and two, but no information about the time evolution of these modes. The wavelet transform clearly demonstrates that mode one stops fluctuating at about 1000 s, while modes two and three remain active for the entire signal duration. For Case 2, mode one is active throughout the entire duration but exhibits intermittency, most notably from the region between about 600 and 800 s, where the mode one oscillation disappears entirely.

Mode one in the wavelet transform of Case 1 is observed to be reasonably well-separated from the other modes and is therefore amenable to amplitude envelope measurement with partial reconstruction. Modes two and three cannot be individually windowed since the modes are not resolved in the wavelet transform. These modes can be windowed together and partial reconstruction of the combined modes yields the amplitude envelope of the fluctuation that results when adding the two modes together in the time domain (Pelstring [21]). For Case 2, mode one shows interference with modes two and three and therefore partial reconstruction would result in an amplitude envelope with error as demonstrated in the test signals in the previous section. The reason that more interference is observed for Case 2 than for Case 1 is because the frequency for mode one is slightly higher in Case 2, resulting

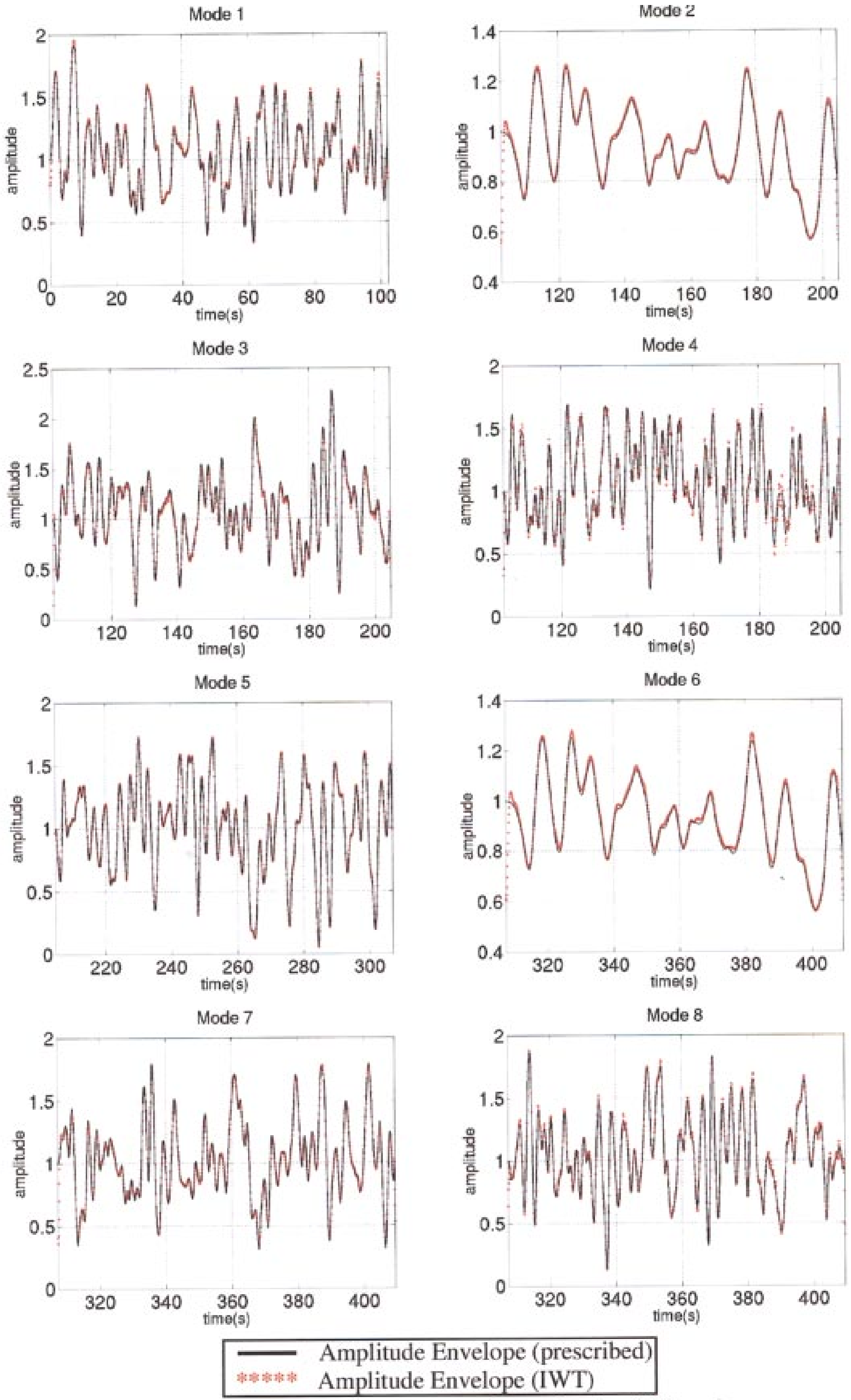

Fig. 7 Extracted modes of synthesized signal in Fig. 4

Table 1 Error calculations for IWT modal extraction

Mode	Frequency (Hz)	MSE	Max percent Difference in Peaks
One	0.75	0.00038	2.27
Two	0.5	0.00014	1.02
Three	2.0	0.00095	5.71
Four	3.5	0.00220	6.14
Five	2.5	0.00009	1.01
Six	0.5	0.00019	2.42
Seven	1.5	0.00079	6.01
Eight	3.0	0.00069	2.63

Table 2 Error analysis for mode four

	Scale Range	Frequency (Hz)	MSE	Max percent Diff in Peaks
Mode four within synthesized signal	0–0.61	3.5	0.00220	6.14
Mode four without interference	0–0.61	3.5	0.00024	2.77
Mode four without interference	0–4.5	3.5	0.00006	1.43

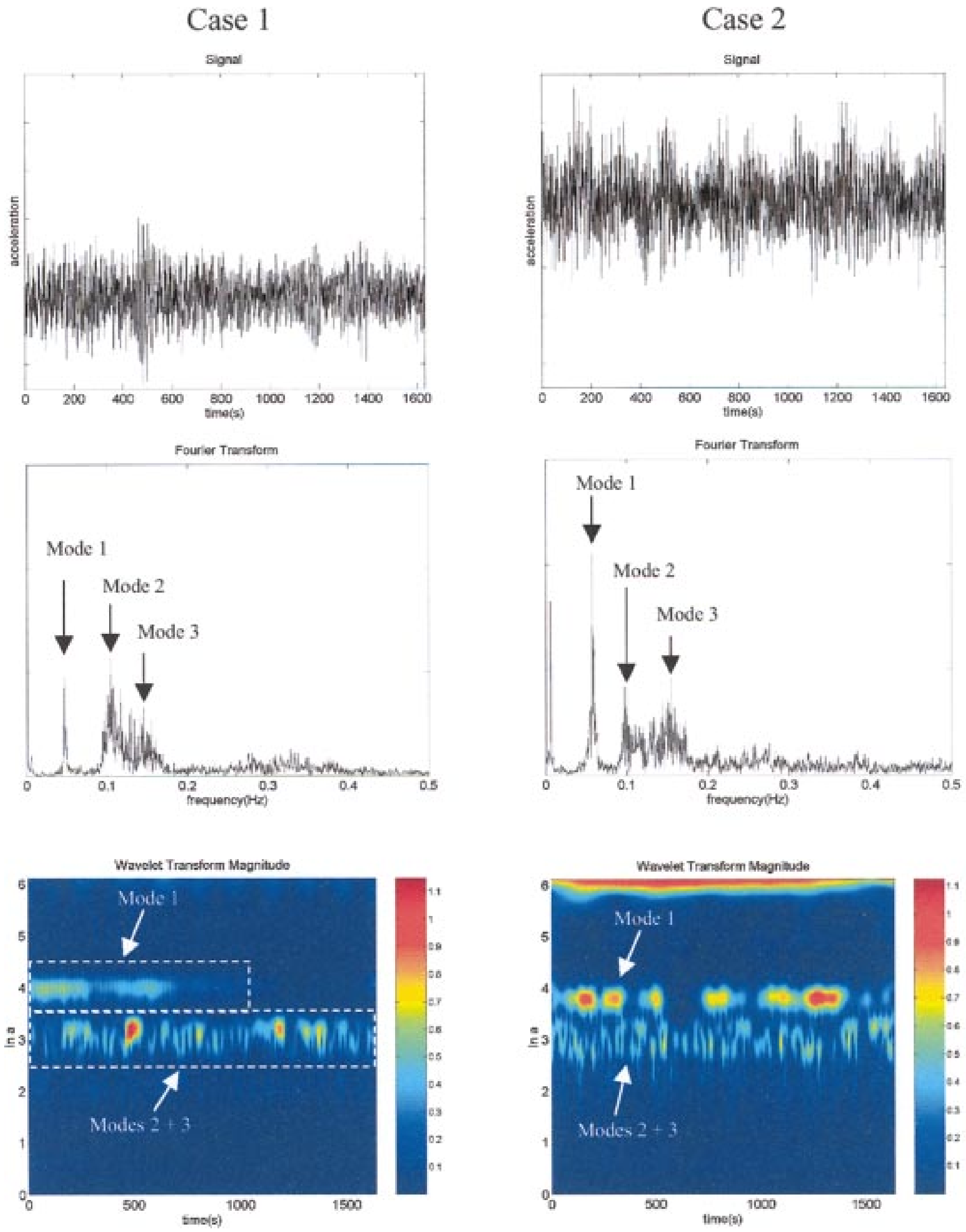

Fig. 8 Example acceleration fluctuations measured from the Schiehallion drilling riser with corresponding fourier and wavelet analysis. Case 1: *X* acceleration at *L*/4. Case 2: *Y* acceleration at *L*/2.

Table 3 Schiehallion drilling riser modal response frequencies (from Cornut and Vandiver, [20])

Riser Response Mode	Estimated Frequency (Hz)	Wavelet scale ($\ln a$)
One	0.057	3.78
Two	0.114	3.09
Three	0.156	2.77
Four	0.224	2.41

in more closely spaced modes one and two. Note that the closer spacing of modes for Case 2 is readily observed in the Fourier transforms.

Partial reconstruction is applied to the windowed regions in the wavelet transform of Case 1. The resulting amplitude envelopes for mode one and the combination of modes two and three are given in Fig. 9. Note that the envelopes are plotted at the same scale on the ordinate, but over different time intervals. Again, the purpose of applying the partial reconstruction technique is to quantify the amplitude variation that is visualized in the wavelet transform. For example, three noteworthy peaks (Peak A at about 500 s, Peak B at 1200 s, and Peak C at 1350 s) are labeled in the plot for modes two and three and it is clear that they correspond directly to wavelet coefficients visualized in the wavelet transform for Case 1 in Fig. 8.

The results obtained from the analysis of the synthetic test signals and the Schiehallion riser responses indicate that the wavelet analysis technique based on partial reconstruction has potential application to the analysis of riser responses, including fatigue assessment. Future work toward validating this technique should include examination of the effect of noise in the response signal and other phenomena observed in experimentally measured riser responses. One issue that needs to be carefully addressed is that of closely spaced modes and the resulting interference that occurs as shown in the results presented above. The time-frequency uncertainty principle does not allow for arbitrarily fine resolution of both time and frequency in a signal. Marine risers have numerous closely spaced natural frequencies that are intermittently excited and therefore the consequences of the uncertainty principle on analysis results in offshore applications need to be clearly understood.

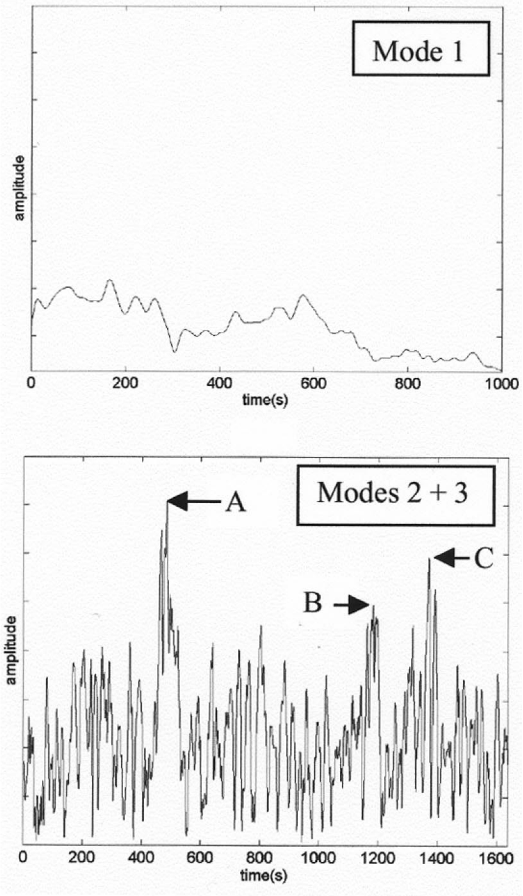

Fig. 9 Amplitude envelopes obtained from Case 1 using partial reconstruction of the windowed regions of the wavelet transform presented in Fig. 8

Conclusions

An analysis technique based on partial reconstruction of wavelet transforms was introduced and was demonstrated to measure amplitude envelopes of time-dependent modes more accurately than wavelet ridge extraction. A synthesized test signal was generated to include switching between lock-in (single-mode) and nonlock-in (multiple-mode) behavior, which is often observed in riser responses. This signal was analyzed to measure the amplitude envelopes of each mode and the accuracy of the partial reconstruction technique was quantified by comparing measured modal amplitudes with the prescribed amplitudes of the synthesized modes. The analysis technique was found to give accurate results, except where modes are spaced too closely in frequency. Finally, partial reconstruction was applied to measured riser responses from the Schiehallion drilling riser, demonstrating both the utility of visualizing time-dependent modes in the wavelet transform and the ability to quantify amplitude fluctuations of response modes. The results from this study indicate that partial reconstruction of wavelet transforms is a promising analysis technique for ocean-engineering applications, and especially for analysis of the complex time-dependent behavior of marine risers.

Acknowledgment

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