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Research Article

The Combination of Soft Sets and $\mathcal{N}$-Structures with Applications

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Using the notions of soft sets and $\mathcal{N}$-structures, $\mathcal{N}$-soft set theory is introduced. We apply it to both a decision making problem and a *BCK/BCI* algebra.

1. Introduction

To solve complicated problems in economics, engineering, and environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as the probability theory, the theory of (intuitionistic) fuzzy sets, the theory of vague sets, the theory of interval mathematics, and the theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [1]. Maji et al. [2] and Molodtsov [1] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov [1] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties which is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [2] described the application of soft set theory to a decision making problem. Maji et al. [3] also studied several operations on the theory of soft sets. Chen et al. [4] presented a new definition of soft set parametrization reduction and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh [5]. Roy and Maji [6] presented some results on an application of fuzzy soft sets in decision making problem. Aygünoglu and Aygün [7] introduced the notion of fuzzy soft group and studied its properties. Ali et al. [8] discussed new operations in soft set theory. Jun [9] applied the notion of soft set to BCK/BCI-algebras, and Jun et al. [10] considered applications of soft set theory in the ideals of d-algebras.

A (crisp) set A in a universe X can be defined in the form of its characteristic function $\mu_A : X \rightarrow \{0, 1\}$ yielding the value 1 for elements belonging to the set A and the value 0 for elements excluded from the set A. So far most of the generalization of the crisp set has been conducted on the unit interval $[0, 1]$, and they are consistent with the asymmetry observation. In other words, the generalization of the crisp set to fuzzy sets relied on spreading positive information that fit the crisp point $\{1\}$ into the interval $[0, 1]$. Because no negative meaning of information is suggested, we now feel a need to deal with negative information. To do so, we also feel a need to supply mathematical tool. To attain such object, Jun et al. [11] introduced a new function which is called negative-valued function and constructed $\mathcal{N}$-structures. They applied $\mathcal{N}$-structures to *BCK/BCI*-algebras, and discussed $\mathcal{N}$-subalgebras and $\mathcal{N}$-ideals in *BCK/BCI*-algebras. Jun et al. [12] considered closed ideals in BCH-algebras based on $\mathcal{N}$-structures.

In this paper we introduce the notion of $\mathcal{N}$-soft sets which are a soft set based on $\mathcal{N}$-structures by using the notions of soft sets and $\mathcal{N}$-structures, and then we apply it to both a decision making problem and a *BCK/BCI*-algebra.

2. Preliminaries

A *BCK/BCI*-algebra is an important class of logical algebras introduced by K. Iséki and was extensively investigated by several researchers.

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a *BCI-algebra* if it satisfies the following conditions:

(I) $(\forall x, y, z \in X)\,(((x * y) * (x * z)) * (z * y) = 0)$,

(II) $(\forall x, y \in X)\,((x * (x * y)) * y = 0)$,

(III) $(\forall x \in X)\,(x * x = 0)$,

(IV) $(\forall x, y \in X)\,(x * y = 0, y * x = 0 \Rightarrow x = y)$.

If a BCI-algebra X satisfies the following identity:

(V) $(\forall x \in X)\,(0 * x = 0)$,

then X is called a *BCK-algebra*. Any BCK-algebra X satisfies the following axioms:

(a1) $(\forall x \in X)\,(x * 0 = x)$,

(a2) $(\forall x, y, z \in X)\,(x \le y \Rightarrow x * z \le y * z, z * y \le z * x)$,

(a3) $(\forall x, y, z \in X)\,((x * y) * z = (x * z) * y)$,

(a4) $(\forall x, y, z \in X)\,((x * z) * (y * z) \le x * y)$,

where $x \le y$ if and only if $x * y = 0$.

Any BCI-algebra X satisfies the following axioms:

(a5) $(\forall x, y, z \in X)\,(0 * (0 * ((x * z) * (y * z))) = (0 * y) * (0 * x))$,

(a6) $(\forall x, y \in X)\,(0 * (0 * (x * y)) = (0 * y) * (0 * x))$.

A nonempty subset S of a *BCK/BCI*-algebra X is called a *BCK/BCI-subalgebra* of X if $x * y \in S$ for all $x, y \in S$.

For any family $\{a_i \mid i \in \Lambda\}$ of real numbers, we define

$$\bigvee \{a_i \mid i \in \Lambda\} := \begin{cases} \max\{a_i \mid i \in \Lambda\} & \text{if } \Lambda \text{ is finite,} \\ \sup\{a_i \mid i \in \Lambda\} & \text{otherwise,} \end{cases}$$
$$\bigwedge \{a_i \mid i \in \Lambda\} := \begin{cases} \min\{a_i \mid i \in \Lambda\} & \text{if } \Lambda \text{ is finite,} \\ \inf\{a_i \mid i \in \Lambda\} & \text{otherwise.} \end{cases} \quad (1)$$

Denote by $\mathcal{F}(S, [-1, 0])$ the collection of functions from a set S to $[-1, 0]$. We say that an element of $\mathcal{F}(S, [-1, 0])$ is a *negative-valued function* from S to $[-1, 0]$ (briefly, $\mathcal{N}$*-function* on S). By an $\mathcal{N}$*-structure* we mean an ordered pair (S, f) of S and an $\mathcal{N}$-function f on S.

3. $\mathcal{N}$-Soft Sets

Definition 1. Let X be an initial universe set and E a set of attributes. By an $\mathcal{N}$*-soft set* over X we mean a pair (f, A) where $A \subset E$ and f is a mapping from A to $\mathcal{F}(X, [-1, 0])$; that is; for each $a \in A$, $f(a) := f_a$ is an $\mathcal{N}$-function on X.

Denote by $\mathcal{N}(X, E)$ the collection of all $\mathcal{N}$-soft sets over X with attributes from E and we call it an $\mathcal{N}$-soft class.

We provide an example of an $\mathcal{N}$-soft set.

Example 2. As an initial universe set and a set of attributes, we consider $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ consists of six houses, and $E = \{e_1, e_2, e_3, e_4, e_5\}$, respectively, where

e_1 stands for the attribute "cheap,"

e_2 stands for the attribute "messy,"

e_3 stands for the attribute "brick,"

e_4 stands for the attribute "expensive,"

e_5 stands for the attribute "in the flooded area."

Let $f_{e_1}, f_{e_2}, f_{e_3}, f_{e_4}$, and f_{e_5} be $\mathcal{N}$-functions on X defined by

$$f_{e_1} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ -0.5 & 0 & -0.6 & 0 & -0.7 & -1 \end{pmatrix},$$
$$f_{e_2} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ 0 & -0.6 & 0 & -0.6 & -0.4 & -0.2 \end{pmatrix},$$
$$f_{e_3} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ -0.8 & -0.7 & 0 & 0 & 0 & -1 \end{pmatrix}, \quad (2)$$
$$f_{e_4} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ 0 & -1 & 0 & -0.8 & 0 & -0.8 \end{pmatrix},$$
$$f_{e_5} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ 0 & -0.9 & -0.5 & -0.7 & -0.8 & -0.7 \end{pmatrix}.$$

The $\mathcal{N}$-soft set (f, E) is an attributed family $\{f_{e_i} \mid i = 1, 2, \ldots, 5\}$ of all $\mathcal{N}$-functions on the set X and gives us a collection of approximate description of an object. The $\mathcal{N}$-function f_* here is "$*$ houses," where $*$ is to be filled up by an attribute $e \in E$. Therefore f_{e_1} means "cheap houses" whose functional value is represented by $\{x_1/-0.5, x_2/0, x_3/-0.6, x_4/0, x_5/-0.7, x_6/-1\}$, that is,

$$\text{cheap houses} = \left\{\frac{x_1}{-0.5}, \frac{x_2}{0}, \frac{x_3}{-0.6}, \frac{x_4}{0}, \frac{x_5}{-0.7}, \frac{x_6}{-1}\right\}. \quad (3)$$

Therefore, we can represent the $\mathcal{N}$-soft set (f, E) as follows:

$$(f, E) = \left\{\text{cheap houses} = \left\{\frac{x_1}{-0.5}, \frac{x_2}{0}, \frac{x_3}{-0.6}, \frac{x_4}{0}, \frac{x_5}{-0.7}, \frac{x_6}{-1}\right\},\right.$$
$$\text{messy houses} = \left\{\frac{x_1}{0}, \frac{x_2}{-0.6}, \frac{x_3}{0}, \frac{x_4}{-0.6}, \frac{x_5}{-0.4}, \frac{x_6}{-0.2}\right\},$$

$$\text{brick houses} = \left\{\frac{x_1}{-0.8}, \frac{x_2}{-0.7}, \frac{x_3}{0}, \frac{x_4}{0}, \frac{x_5}{0}, \frac{x_6}{-1}\right\},$$

$$\text{expensive houses} = \left\{\frac{x_1}{0}, \frac{x_2}{-1}, \frac{x_3}{0}, \frac{x_4}{-0.8}, \frac{x_5}{0}, \frac{x_6}{-0.8}\right\},$$

houses in the flooded area

$$= \left\{\frac{x_1}{0}, \frac{x_2}{-0.9}, \frac{x_3}{-0.5}, \frac{x_4}{-0.7}, \frac{x_5}{-0.8}, \frac{x_6}{-0.7}\right\}\Bigg\}, \quad (4)$$

where each approximation has two parts:

(i) a predicate p,

(ii) an approximate value $\mathcal{N}$-set v.

For example, for the approximation "cheap houses $= \{x_1/-0.5, x_2/0, x_3/-0.6, x_4/0, x_5/-0.7, x_6/-1\}$," we have

(i) the predicate name is "cheap houses,"

(ii) the approximate value $\mathcal{N}$-set is $\{x_1/-0.5, x_2/0, x_3/-0.6, x_4/0, x_5/-0.7, x_6/-1\}$.

Therefore, an $\mathcal{N}$-soft set (f, E) can be viewed as a collection of $\mathcal{N}$-approximations as follows:

$$(f, E) = \{p_1 = v_1, p_2 = v_2, \ldots, p_n = v_n\}. \quad (5)$$

For the purpose of storing an $\mathcal{N}$-soft set in a computer, we could represent an $\mathcal{N}$-soft set, which is described in the above, in the form of Table 1.

For convenience of explanation, we can represent the $\mathcal{N}$-soft set, which is described in the above, in matrix form as follows:

$$(f, E) = \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix} \overset{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{matrix}}{\begin{pmatrix} -0.50 & 0 & -0.6 & 0 & -0.7 & -1 \\ 0 & -0.6 & 0 & -0.6 & -0.4 & -0.2 \\ -0.8 & -0.7 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & -0.8 & 0 & -0.8 \\ 0 & -0.9 & -0.5 & -0.7 & -0.8 & -0.7 \end{pmatrix}}. \quad (6)$$

Definition 3. Let (f, A) and (g, B) be $\mathcal{N}$-soft sets in $\mathcal{N}(X, E)$. Then (f, A) is called an *$\mathcal{N}$-soft subset* of (g, B), denoted by $(f, A) \,\widetilde{\subseteq}\, (g, B)$, if it satisfies as following:

(i) $A \subseteq B$,

(ii) $(\forall e \in A)\,(f_e \subseteq g_e$, that is, $f_e(x) \leq g_e(x)$ for all $x \in X)$.

Example 4. Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and

$$\begin{aligned} E &= \{\text{cheap}(e_1), \text{beautiful}(e_2), \text{messy}(e_3), \text{brick}(e_4), \\ &\quad \text{beautiful}(e_5), \text{expensive}(e_6), \text{in the flooded area}(e_7), \\ &\quad \text{in the green surrounding}(e_8)\}. \end{aligned} \quad (7)$$

Let (f, A) and (g, B) be $\mathcal{N}$-soft sets in $\mathcal{N}(X, E)$ given by

$$A = \{\text{cheap}(e_1), \text{beautiful}(e_2), \text{messy}(e_3)\},$$

$$B = \{\text{cheap}(e_1), \text{beautiful}(e_2), \text{messy}(e_3), \text{expensive}(e_6), \text{in the flooded area}(e_7)\},$$

$$(f, A) = \begin{matrix} e_1 \\ e_2 \\ e_3 \end{matrix} \overset{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{matrix}}{\begin{pmatrix} -0.6 & -0.4 & -0.5 & -0.2 & -0.3 & 0 \\ 0 & -0.5 & -0.5 & 0 & -0.4 & -0.3 \\ -0.5 & -0.4 & -0.7 & -0.2 & -0.6 & -0.3 \end{pmatrix}}, \quad (8)$$

$$(g, B) = \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_6 \\ e_7 \end{matrix} \overset{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{matrix}}{\begin{pmatrix} -0.4 & -0.3 & -0.4 & -0.1 & -0.2 & 0 \\ 0 & -0.4 & -0.5 & 0 & -0.3 & 0 \\ -0.4 & -0.4 & -0.5 & 0 & -0.3 & 0 \\ -0.6 & -0.4 & -0.5 & -0.2 & -0.7 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{pmatrix}}.$$

Then (f, A) is an $\mathcal{N}$-soft subset of (g, B).

Let (f, A) be an $\mathcal{N}$-soft set in (X, E). The *complement* of (f, A), denoted by $(f, A)^c$, is defined to be an $\mathcal{N}$-soft set $(f^c, \rceil A)$, where $\rceil A$ is not the set of A, that is, $\rceil A = \{\neg a \mid a \in A\}$, and $f^c : \rceil A \to \mathcal{F}(X, [-1, 0])$ is an $\mathcal{N}$-function given by $f^c(\neg a)$ is $\mathcal{N}$-complement of $f(a)$ for all $\neg a \in \rceil A$, that is,

$$(\forall \neg a \in \rceil A) \quad (\forall x \in X) \quad (f^c(\neg a)(x) + f(a)(x) = -1). \quad (9)$$

Example 5. Consider the $\mathcal{N}$-soft set (f, E) in Example 2. Then the complement of (f, E) is represented as follows:

$$\begin{aligned} (f, E)^c = \Bigg\{ &\text{not cheap houses} \\ &= \left\{\frac{x_1}{-0.5}, \frac{x_2}{-1}, \frac{x_3}{-0.4}, \frac{x_4}{-1}, \frac{x_5}{-0.3}, \frac{x_6}{0}\right\}, \\ &\text{not messy houses} \\ &= \left\{\frac{x_1}{-1}, \frac{x_2}{-0.4}, \frac{x_3}{-1}, \frac{x_4}{-0.4}, \frac{x_5}{-0.6}, \frac{x_6}{-0.8}\right\}, \\ &\text{not brick houses} \\ &= \left\{\frac{x_1}{-0.2}, \frac{x_2}{-0.3}, \frac{x_3}{-1}, \frac{x_4}{-1}, \frac{x_5}{-1}, \frac{x_6}{0}\right\}, \\ &\text{not expensive houses} \\ &= \left\{\frac{x_1}{-1}, \frac{x_2}{0}, \frac{x_3}{-1}, \frac{x_4}{-0.2}, \frac{x_5}{-1}, \frac{x_6}{-0.2}\right\}, \\ &\text{not houses in the flooded area} \\ &= \left\{\frac{x_1}{-1}, \frac{x_2}{-0.1}, \frac{x_3}{-0.5}, \frac{x_4}{-0.3}, \frac{x_5}{-0.2}, \frac{x_6}{-0.3}\right\}\Bigg\}. \end{aligned} \quad (10)$$

For any $\mathcal{N}$-soft sets (f, A) and (g, B) in (X, E), we define

(i) "(f, A) AND (g, B)," denoted by $(f, A) \,\widetilde{\wedge}\, (g, B)$, to be an $\mathcal{N}$-soft set $(f, A) \,\widetilde{\wedge}\, (g, B) = (h, A \times B)$, where

$h_{(\alpha,\beta)} = \bigwedge\{f_\alpha, g_\beta\}$ for all $(\alpha,\beta) \in A \times B$; that is, $h_{(\alpha,\beta)}(x) = \bigwedge\{f_\alpha(x), g_\beta(x)\}$ for all $(\alpha,\beta) \in A \times B$ and $x \in X$.

(ii) "(f, A) OR (g, B)," denoted by $(f, A)\ \tilde{\vee}\ (g, B)$, to be an $\mathcal{N}$-soft set $(f, A)\ \tilde{\vee}\ (g, B) = (\lambda, A \times B)$, where $\lambda_{(\alpha,\beta)} = \bigvee\{f_\alpha, g_\beta\}$ for all $(\alpha,\beta) \in A \times B$; that is, $\lambda_{(\alpha,\beta)}(x) = \bigvee\{f_\alpha(x), g_\beta(x)\}$ for all $(\alpha,\beta) \in A \times B$ and $x \in X$.

Example 6. Consider two $\mathcal{N}$-soft sets (f, A) and (g, B) in (X, E) which describes the "cost of houses" and the "attractiveness of houses." Suppose that $X = \{x_1, x_2, \ldots, x_{10}\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$, where

e_1 stands for the attribute "cheap,"

e_2 stands for the attribute "costly,"

e_3 stands for the attribute "very costly,"

e_4 stands for the attribute "beautiful,"

e_5 stands for the attribute "in the green surroundings."

Take $A = \{e_1, e_2, e_3\}$ and $B = \{e_1, e_4, e_5\}$, and define

$$\begin{gathered}
(f, A) = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array} \begin{array}{c} \begin{array}{cccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} \end{array} \\ \left(\begin{array}{cccccccccc} -0.6 & -0.8 & -0.5 & -0.6 & -0.7 & 0 & -0.9 & -0.4 & -0.3 & -0.2 \\ -0.3 & 0 & -0.2 & 0 & -0.1 & -0.7 & 0 & 0 & -0.6 & -0.5 \\ 0 & 0 & -0.2 & 0 & -0.1 & -0.7 & 0 & 0 & -0.7 & -0.3 \end{array}\right) \end{array}, \\
(g, B) = \begin{array}{c} \\ e_1 \\ e_4 \\ e_5 \end{array} \begin{array}{c} \begin{array}{cccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} \end{array} \\ \left(\begin{array}{cccccccccc} -0.6 & -0.7 & -0.5 & -0.6 & -0.9 & 0 & -0.9 & -0.4 & -0.5 & -0.4 \\ -0.5 & -0.3 & -0.2 & -0.4 & 0 & 0 & -0.1 & 0 & -0.7 & -0.1 \\ -0.4 & 0 & 0 & -0.2 & -0.4 & -0.2 & 0 & -0.2 & -0.5 & -0.3 \end{array}\right) \end{array}.
\end{gathered} \tag{11}$$

Then $(f, A)\ \tilde{\wedge}\ (g, B)$ and $(f, A)\ \tilde{\vee}\ (g, B)$ are represented as follows:

$$(f, A)\ \tilde{\wedge}\ (g, B) = (h, A \times B) = \begin{array}{c} \\ (e_1, e_1) \\ (e_1, e_4) \\ (e_1, e_5) \\ (e_2, e_1) \\ (e_2, e_4) \\ (e_2, e_5) \\ (e_3, e_1) \\ (e_3, e_4) \\ (e_3, e_5) \end{array} \begin{array}{c} \begin{array}{cccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} \end{array} \\ \left(\begin{array}{cccccccccc} -0.6 & -0.8 & -0.5 & -0.6 & -0.9 & 0 & -0.9 & -0.4 & -0.5 & -0.4 \\ -0.6 & -0.8 & -0.5 & -0.6 & -0.7 & 0 & -0.9 & -0.4 & -0.7 & -0.2 \\ -0.6 & -0.8 & -0.5 & -0.6 & -0.7 & -0.2 & -0.9 & -0.4 & -0.5 & -0.3 \\ -0.6 & -0.7 & -0.5 & -0.6 & -0.9 & -0.7 & -0.9 & -0.4 & -0.6 & -0.5 \\ -0.5 & -0.3 & -0.2 & -0.4 & -0.1 & -0.7 & -0.1 & 0 & -0.7 & -0.5 \\ -0.4 & 0 & -0.2 & -0.2 & -0.4 & -0.7 & 0 & -0.2 & -0.6 & -0.5 \\ -0.6 & -0.7 & -0.5 & -0.6 & -0.9 & -0.7 & -0.9 & -0.4 & -0.7 & -0.4 \\ -0.5 & -0.3 & -0.2 & -0.4 & -0.1 & -0.7 & -0.1 & 0 & -0.7 & -0.3 \\ -0.4 & 0 & -0.2 & -0.2 & -0.4 & -0.7 & 0 & -0.2 & -0.7 & -0.3 \end{array}\right) \end{array}, \tag{12}$$

$$(f, A)\ \tilde{\vee}\ (g, B) = (\lambda, A \times B) = \begin{array}{c} \\ (e_1, e_1) \\ (e_1, e_4) \\ (e_1, e_5) \\ (e_2, e_1) \\ (e_2, e_4) \\ (e_2, e_5) \\ (e_3, e_1) \\ (e_3, e_4) \\ (e_3, e_5) \end{array} \begin{array}{c} \begin{array}{cccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} \end{array} \\ \left(\begin{array}{cccccccccc} -0.6 & -0.7 & -0.5 & -0.6 & -0.7 & 0 & -0.9 & -0.4 & -0.3 & -0.2 \\ -0.5 & -0.3 & -0.2 & -0.4 & 0 & 0 & -0.1 & 0 & -0.3 & -0.1 \\ -0.4 & 0 & 0 & -0.2 & -0.4 & 0 & 0 & -0.2 & -0.3 & -0.2 \\ -0.3 & 0 & -0.2 & 0 & -0.1 & 0 & 0 & 0 & -0.5 & -0.4 \\ -0.3 & 0 & -0.2 & 0 & 0 & 0 & 0 & 0 & -0.6 & -0.1 \\ -0.3 & 0 & 0 & 0 & -0.1 & -0.2 & 0 & 0 & -0.5 & -0.3 \\ 0 & 0 & -0.2 & 0 & -0.1 & 0 & 0 & 0 & -0.5 & -0.3 \\ 0 & 0 & -0.2 & 0 & 0 & 0 & 0 & 0 & -0.7 & -0.1 \\ 0 & 0 & 0 & 0 & -0.1 & -0.2 & 0 & 0 & -0.5 & -0.3 \end{array}\right) \end{array}. \tag{13}$$

4. Application in a Decision Making Problem

The problem in an $\mathcal{N}$-soft class is to choose an object from the initial universe set of given objects with respect to a set of choice attribute P. We present an algorithm for identification of an object based on multiobserves input data characterized by the color of roofs, size, and cost.

Algorithm 7. Consider the following.

(1) Input the $\mathcal{N}$-soft sets $(f, A), (g, B)$, and (h, C).

(2) Input the attribute set P as observed by the observe.

(3) Compute the corresponding resultant $\mathcal{N}$-soft set (ω, P) from the $\mathcal{N}$-soft sets $(f, A), (g, B)$, and (h, C) and place it in matrix form.

(4) Construct the comparison table of the $\mathcal{N}$-soft set (ω, P) where the comparison table is a square table in which

 (i) the number of rows and the number of columns are equal,

 (ii) rows and columns both are labelled by the object names $x_1, x_2, \ldots, x_n$ of the universe,

 (iii) the entries are c_{ij}, where c_{ij} is determined by the number of attributes for which the membership value of object x_i is less than or equal to the membership value of object x_j.

(5) Compute the row sum r_i for each x_i and column sum t_j for each x_j which are calculated by using the formula: $r_i = \sum_{j=1}^{n} c_{ij}$ and $t_j = \sum_{i=1}^{n} c_{ij}$.

(6) Compute the score S_i of each x_i, which is given as $S_i = r_i - t_i$.

(7) The decision is S_k if $S_k = \min_i S_i$.

(8) If k has more than one value, then any one of x_k may be chosen.

Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and $E = \{e_1, e_2, \ldots, e_{13}\}$ be a set of six houses and a set of attributes, respectively, where

e_1 stands for the attribute "black roof,"

e_2 stands for the attribute "brown roof,"

e_3 stands for the attribute "yellow roof,"

e_4 stands for the attribute "red roof,"

e_5 stands for the attribute "large size,"

e_6 stands for the attribute "small size,"

e_7 stands for the attribute "very small size,"

e_8 stands for the attribute "average size,"

e_9 stands for the attribute "very large size,"

e_{10} stands for the attribute "cheap,"

e_{11} stands for the attribute "expensive,"

e_{12} stands for the attribute "very cheap,"

e_{13} stands for the attribute "very expensive."

Consider three subsets A, B, and C of E as follows:

$A = \{e_1, e_2, e_3, e_4\}$ which represents the color of roof of the house,

$B = \{e_5, e_6, e_7, e_8, e_9\}$ which represents the size of the house,

$C = \{e_{10}, e_{11}, e_{12}, e_{13}\}$ which represents the cost of the house.

Let $(f, A), (g, B)$, and (h, C) be $\mathcal{N}$-soft sets in (X, E) defined by

$$
\begin{gathered}
(f, A) = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{array}
\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
\begin{pmatrix} -0.7 & -0.7 & -0.6 & -0.2 & -0.3 & -0.1 \\ -0.6 & -0.1 & -0.5 & -0.8 & -0.7 & -0.8 \\ -0.4 & -0.7 & -0.2 & -0.6 & -0.4 & -0.6 \\ -0.1 & -0.5 & -0.3 & -0.2 & -0.5 & -0.7 \end{pmatrix} \end{array}, \\
(g, B) = \begin{array}{c} \\ e_5 \\ e_6 \\ e_7 \\ e_8 \\ e_9 \end{array}
\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
\begin{pmatrix} -0.6 & -0.2 & -0.4 & -0.1 & -0.8 & -0.7 \\ -0.8 & -0.4 & -0.6 & -0.2 & -0.9 & -0.8 \\ -0.2 & -0.7 & -0.6 & -0.8 & -0.1 & -0.2 \\ -0.4 & -0.9 & -0.9 & -0.9 & -0.2 & -0.4 \\ -0.5 & -0.3 & -0.3 & -0.6 & -0.3 & -0.5 \end{pmatrix} \end{array}, \\
(h, C) = \begin{array}{c} \\ e_{10} \\ e_{11} \\ e_{12} \\ e_{13} \end{array}
\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
\begin{pmatrix} -0.7 & -0.4 & -0.5 & -0.3 & -0.4 & -0.2 \\ -0.6 & -0.5 & -0.4 & -0.4 & -0.4 & -0.3 \\ -0.9 & -0.6 & -0.7 & -0.4 & -0.5 & -0.3 \\ -0.1 & -0.5 & -0.4 & -0.7 & -0.6 & -0.1 \end{pmatrix} \end{array},
\end{gathered} \tag{14}
$$

respectively. We perform "(f, A) AND (g, B)" and it is represented as follows:

$$
(f, A) \barwedge (g, B) = \begin{array}{c} \\ (e_1, e_5) \\ (e_1, e_6) \\ (e_1, e_7) \\ (e_1, e_8) \\ (e_1, e_9) \\ (e_2, e_5) \\ (e_2, e_6) \\ (e_2, e_7) \\ (e_2, e_8) \\ (e_2, e_9) \\ (e_3, e_5) \\ (e_3, e_6) \\ (e_3, e_7) \\ (e_3, e_8) \\ (e_3, e_9) \\ (e_4, e_5) \\ (e_4, e_6) \\ (e_4, e_7) \\ (e_4, e_8) \\ (e_4, e_9) \end{array}
\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
\begin{pmatrix}
-0.7 & -0.7 & -0.6 & -0.2 & -0.8 & -0.7 \\
-0.8 & -0.7 & -0.6 & -0.2 & -0.9 & -0.8 \\
-0.7 & -0.7 & -0.6 & -0.8 & -0.3 & -0.2 \\
-0.7 & -0.9 & -0.9 & -0.9 & -0.3 & -0.4 \\
-0.7 & -0.7 & -0.6 & -0.6 & -0.3 & -0.5 \\
-0.6 & -0.2 & -0.5 & -0.8 & -0.8 & -0.8 \\
-0.8 & -0.4 & -0.6 & -0.8 & -0.9 & -0.8 \\
-0.6 & -0.7 & -0.6 & -0.8 & -0.7 & -0.8 \\
-0.6 & -0.9 & -0.9 & -0.9 & -0.7 & -0.8 \\
-0.6 & -0.3 & -0.5 & -0.8 & -0.7 & -0.8 \\
-0.6 & -0.7 & -0.4 & -0.6 & -0.8 & -0.7 \\
-0.8 & -0.7 & -0.6 & -0.6 & -0.9 & -0.8 \\
-0.4 & -0.7 & -0.6 & -0.8 & -0.4 & -0.6 \\
-0.4 & -0.9 & -0.9 & -0.9 & -0.4 & -0.6 \\
-0.5 & -0.7 & -0.3 & -0.6 & -0.4 & -0.6 \\
-0.6 & -0.5 & -0.4 & -0.2 & -0.8 & -0.7 \\
-0.8 & -0.5 & -0.6 & -0.2 & -0.9 & -0.8 \\
-0.2 & -0.7 & -0.6 & -0.8 & -0.5 & -0.7 \\
-0.4 & -0.9 & -0.9 & -0.9 & -0.5 & -0.7 \\
-0.5 & -0.5 & -0.3 & -0.6 & -0.5 & -0.7
\end{pmatrix} \end{array}. \tag{15}
$$

Table 1

X	Cheap	Messy	Brick	Expensive	In the flooded area
x_1	−0.5	0	−0.8	0	0
x_2	0	−0.6	−0.7	−1	−0.9
x_3	−0.6	0	0	0	−0.5
x_4	0	−0.6	0	−0.8	−0.7
x_5	−0.7	−0.4	0	0	−0.8
x_6	−1	−0.2	−1	−0.8	−0.7

If we require the $\mathcal{N}$-soft set for the attributes $D = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7\}$, where

$$\begin{gathered} d_1 = (e_1, e_5), \qquad d_2 = (e_1, e_9), \\ d_3 = (e_2, e_5), \qquad d_4 = (e_2, e_8), \\ d_5 = (e_3, e_7), \qquad d_6 = (e_4, e_8), \qquad d_7 = (e_4, e_9), \end{gathered} \tag{16}$$

then the resultant $\mathcal{N}$-soft set for the $\mathcal{N}$-soft sets (f, A) and (g, B) will be (k, D), say, which is represented as follows:

$$(k, D) = \begin{matrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_7 \end{matrix} \overset{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{matrix}}{\begin{pmatrix} -0.7 & -0.7 & -0.6 & -0.2 & -0.8 & -0.7 \\ -0.7 & -0.7 & -0.6 & -0.6 & -0.3 & -0.5 \\ -0.6 & -0.2 & -0.5 & -0.8 & -0.8 & -0.8 \\ -0.6 & -0.9 & -0.9 & -0.9 & -0.7 & -0.8 \\ -0.4 & -0.7 & -0.6 & -0.8 & -0.4 & -0.6 \\ -0.4 & -0.9 & -0.9 & -0.9 & -0.5 & -0.7 \\ -0.5 & -0.5 & -0.3 & -0.6 & -0.5 & -0.7 \end{pmatrix}}. \tag{17}$$

If we perform "(k, D) AND (h, C)," then we will have $7 \times 4 = 28$ attributes. Let

$$\begin{aligned} P = \{&(d_1, e_{10}), (d_2, e_{12}), (d_3, e_{11}), \\ &(d_4, e_{13}), (d_5, e_{12}), (d_6, e_{12}), (d_7, e_{12})\} \end{aligned} \tag{18}$$

be the set of choice attributes of an observer, where $(d_i, e_{jk}) = \max\{d_i, e_{jk}\}$. Then the resultant $\mathcal{N}$-soft set (ω, P) is represented as follows:

$$(\omega, P) = \begin{matrix} (d_1, e_{10}) \\ (d_2, e_{12}) \\ (d_3, e_{11}) \\ (d_4, e_{13}) \\ (d_5, e_{12}) \\ (d_6, e_{12}) \\ (d_7, e_{12}) \end{matrix} \overset{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{matrix}}{\begin{pmatrix} -0.7 & -0.4 & -0.5 & -0.2 & -0.4 & -0.2 \\ -0.7 & -0.6 & -0.6 & -0.4 & -0.3 & -0.3 \\ -0.6 & -0.2 & -0.4 & -0.4 & -0.4 & -0.3 \\ -0.1 & -0.5 & -0.4 & -0.7 & -0.6 & -0.1 \\ -0.4 & -0.6 & -0.6 & -0.4 & -0.4 & -0.3 \\ -0.4 & -0.6 & -0.7 & -0.4 & -0.5 & -0.3 \\ -0.5 & -0.5 & -0.3 & -0.4 & -0.5 & -0.3 \end{pmatrix}}. \tag{19}$$

The comparison table for the $\mathcal{N}$-soft set (ω, P) is given by Table 2.

We now compute the raw sum, column sum, and the score for each x_i, and it is given by Table 3.

Table 2: Comparison table.

	x_1	x_2	x_3	x_4	x_5	x_6
x_1	7	4	4	6	5	7
x_2	4	7	4	5	5	6
x_3	3	5	7	5	5	7
x_4	3	2	3	7	4	7
x_5	4	4	3	5	7	7
x_6	1	1	1	1	1	7

From the score table (Table 3), it is clear that the minimum score is −29, scored by x_6, and the decision is in favor of selecting x_6.

5. Application in *BCK/BCI*-Algebras

In what follows let E denote a set of attributes unless otherwise specified. We will use the terminology "soft machine" which means that it produces a *BCK/BCI*-algebra.

Definition 8 (see [11]). By a *subalgebra* of a BCK/BCI-algebra X based on $\mathcal{N}$-function f (briefly, $\mathcal{N}$-*subalgebra* of X), we mean an $\mathcal{N}$-structure (X, f) in which f satisfies the following assertion:

$$(\forall x, y \in X) \quad \left(f(x * y) \leq \bigvee \{f(x), f(y)\}\right). \tag{20}$$

Definition 9. Let (f, A) be an $\mathcal{N}$-soft set over a BCK/BCI-algebra X, where A is a subset of E. If there exists an attribute $u \in A$ for which the $\mathcal{N}$-structure (X, f_u) is an $\mathcal{N}$-subalgebra of X, then we say that (f, A) is an $\mathcal{N}$*-soft* BCK/BCI*-algebra* related to the attribute u (briefly, $\mathcal{N}_u$*-soft* BCK/BCI*-algebra*). If (f, A) is an $\mathcal{N}_u$-soft BCK/BCI-algebra for all $u \in A$, we say that (f, A) is an $\mathcal{N}$*-soft* BCK/BCI*-algebra*.

Example 10. Suppose there are five colors in the universe U, that is,

$$U := \{\text{white, blackish, reddish, green, yellow}\} \tag{21}$$

and E := {beautiful, fine, moderate, delicate, elegant, smart, chaste} be a set of attributes. Let $\heartsuit$ be a soft machine to mix two colors according to order in such a way that we have the following results:

$$\begin{aligned} &\text{white} \heartsuit x = \text{white } \forall x \in U, \\ &\text{blackish} \heartsuit y \\ &\quad = \begin{cases} \text{white} & \text{if } y \in \{\text{blackish, green, yellow}\}, \\ \text{blackish} & \text{if } y \in \{\text{white, reddish}\}, \end{cases} \\ &\text{reddish} \heartsuit z = \begin{cases} \text{white} & \text{if } z \in \{\text{reddish, yellow}\}, \\ \text{reddish} & \text{if } z \in \{\text{white, blackish, green}\}, \end{cases} \end{aligned}$$

Table 3: Raw-sum, column-sum, and the score.

	Row-sum (r_i)	Column-sum (t_i)	Score (S_i)
x_1	33	22	11
x_2	31	23	8
x_3	32	22	10
x_4	26	29	−3
x_5	30	27	3
x_6	12	41	−29

$$\text{green} \heartsuit u = \begin{cases} \text{white} & \text{if } u \in \{\text{green}, \text{yellow}\}, \\ \text{green} & \text{if } u \in \{\text{white}, \text{blackish}, \text{reddish}\}, \end{cases}$$

$$\text{yellow} \heartsuit v = \begin{cases} \text{white} & \text{if } v = \text{yellow}, \\ \text{reddish} & \text{if } v = \text{green}, \\ \text{green} & \text{if } v = \text{reddish}, \\ \text{yellow} & \text{if } v \in \{\text{white}, \text{blackish}\}. \end{cases} \quad (22)$$

Then $(U, \heartsuit, \text{white})$ is a BCK-algebra. Consider a set of attributes

$$A := \{\text{beautiful}, \text{fine}, \text{moderate}\} \subseteq E \quad (23)$$

and define an $\mathcal{N}$-soft set (f, A) over the BCK-algebra $(U, \heartsuit, \text{white})$ as follows:

$$\begin{aligned}(f, A) = \{ & f_{\text{beautiful}}(\text{white}) = -0.7, f_{\text{beautiful}}(\text{blackish}) = -0.7, \\ & f_{\text{beautiful}}(\text{reddish}) = -0.7, f_{\text{beautiful}}(\text{green}) = -0.4, \\ & f_{\text{beautiful}}(\text{yellow}) = -0.4, f_{\text{fine}}(\text{white}) = -0.9, \\ & f_{\text{fine}}(\text{blackish}) = -0.7, f_{\text{fine}}(\text{reddish}) = -0.3, \\ & f_{\text{fine}}(\text{green}) = -0.6, f_{\text{fine}}(\text{yellow}) = -0.3, \\ & f_{\text{moderate}}(\text{white}) = -0.8, f_{\text{moderate}}(\text{blackish}) = -0.2, \\ & f_{\text{moderate}}(\text{reddish}) = -0.6, f_{\text{moderate}}(\text{green}) = -0.2, \\ & f_{\text{moderate}}(\text{yellow}) = -0.2\}. \end{aligned} \quad (24)$$

The tabular representation of (f, A) is given by Table 4.

Then (f, A) is an $\mathcal{N}$-soft BCK-algebra over the BCK-algebra $(U, \heartsuit, \text{white})$.

Now let (g, A) be an $\mathcal{N}$-soft set over the BCK-algebra $(U, \heartsuit, \text{white})$ with the tabular representation which is given by Table 5.

Then (g, A) is not an $\mathcal{N}_{\text{fine}}$-soft BCK-algebra over $(U, \heartsuit, \text{white})$ since

$$\begin{aligned} g_{\text{fine}}(\text{blackish} \heartsuit \text{green}) & = g_{\text{fine}}(\text{white}) = -0.2 > -0.4 \\ & = \bigvee \{g_{\text{fine}}(\text{blackish}), g_{\text{fine}}(\text{green})\}. \end{aligned} \quad (25)$$

Table 4: Tabular representation of (f, A).

(f, A)	White	Blackish	Reddish	Green	Yellow
Beautiful	−0.7	−0.7	−0.7	−0.4	−0.4
Fine	−0.9	−0.7	−0.3	−0.6	−0.3
Moderate	−0.8	−0.2	−0.6	−0.2	−0.2

Hence (g, A) is not an $\mathcal{N}$-soft BCK-algebra over $(U, \heartsuit, \text{white})$. But we can verify that (g, A) is both an $\mathcal{N}_{\text{beautiful}}$-soft BCK-algebra and $\mathcal{N}_{\text{moderate}}$-soft BCK-algebra over $(U, \heartsuit, \text{white})$.

Proposition 11. *Every $\mathcal{N}$-soft BCK/BCI-algebra (f, A) over a BCK/BCI-algebra X satisfies the following inequality:*

$$(\forall x \in X) \quad (\forall u \in A) \quad (f_u(0) \leq f_u(x)). \quad (26)$$

Proof. For any $x \in X$ and $u \in A$, we have

$$f_u(0) = f_u(x * x) \leq \bigvee \{f_u(x), f_u(x)\} = f_u(x). \quad (27)$$

This completes the proof. □

The problem we now discuss is as follows.

If (g, B) is an $\mathcal{N}$-soft BCK/BCI-algebra over a BCK/BCI-algebra X, then is every $\mathcal{N}$-soft subset of (g, B) an $\mathcal{N}$-soft BCK/BCI-algebra over X?

Unfortunately this is not true as seen in the following example.

Example 12. Consider the universe:

$$U := \{\text{white}, \text{blackish}, \text{reddish}, \text{green}, \text{yellow}\} \quad (28)$$

which is considered in Example 10. Consider a soft machine \$ which produces the following products:

$$\text{white} \ \$ \ x = \begin{cases} \text{white} & \text{if } x \in \{\text{white}, \text{blackish}, \text{reddish}\}, \\ \text{green} & \text{if } x \in \{\text{green}, \text{yellow}\}, \end{cases} \quad (29)$$

$$\text{blackish} \ \$ \ y = \begin{cases} \text{white} & \text{if } y = \text{blackish}, \\ \text{blackish} & \text{if } y \in \{\text{white}, \text{reddish}\}, \\ \text{green} & \text{if } y = \text{yellow}, \\ \text{yellow} & \text{if } y = \text{green}, \end{cases} \quad (30)$$

$$\text{reddish} \ \$ \ z = \begin{cases} \text{white} & \text{if } z = \text{reddish}, \\ \text{reddish} & \text{if } z \in \{\text{white}, \text{blackish}\}, \\ \text{green} & \text{if } z \in \{\text{green}, \text{yellow}\}, \end{cases} \quad (31)$$

$$\text{green} \ \$ \ u = \begin{cases} \text{white} & \text{if } u \in \{\text{green}, \text{yellow}\}, \\ \text{green} & \text{if } u \in \{\text{white}, \text{blackish}, \text{reddish}\}, \end{cases} \quad (32)$$

$$\text{yellow} \ \$ \ v = \begin{cases} \text{white} & \text{if } v = \text{yellow}, \\ \text{blackish} & \text{if } v = \text{green}, \\ \text{green} & \text{if } v = \text{blackish}, \\ \text{yellow} & \text{if } v \in \{\text{white}, \text{reddish}\}. \end{cases} \quad (33)$$

Table 5: Tabular representation of (g, A).

(g, A)	White	Blackish	Reddish	Green	Yellow
Beautiful	−0.6	−0.6	−0.6	−0.06	−0.06
Fine	−0.2	−0.5	−0.4	−0.4	−0.3
Moderate	−0.7	−0.2	−0.6	−0.2	−0.2

Table 6: Tabular representation of (g, B).

(g, B)	White	Blackish	Reddish	Green	Yellow
Beautiful	−0.8	−0.5	−0.3	−0.2	−0.2
Fine	−0.5	−0.3	−0.5	−0.4	−0.3
Moderate	−0.6	−0.2	−0.4	−0.3	−0.2
Elegant	−0.7	−0.7	−0.6	−0.3	−0.3
Smart	−0.8	−0.2	−0.7	−0.5	−0.2

Then $(U, \$, \text{white})$ is a BCI-algebra. Take

$$B = \{\text{beautiful, fine, moderate, elegant, smart}\} \tag{34}$$

and let (g, B) be an $\mathcal{N}$-soft set over the BCI-algebra $(U, \$, \text{white})$ with the tabular representation which is given by Table 6.

Then (g, B) is an $\mathcal{N}$-soft BCI-algebra over $(U, \$, \text{white})$. Now let (f, A) be an $\mathcal{N}$-soft subset of (g, B), where

$$A = \{\text{fine, elegant, smart}\} \subset B, \tag{35}$$

and the tabular representation of (f, A) is given by Table 7.

Then

$$\begin{aligned} & f_{\text{elegent}}(\text{blackish } \$ \text{ green}) \\ &\quad = f_{\text{elegent}}(\text{yellow}) = -0.4 \\ &\quad > -0.7 = \bigvee \{f_{\text{elegent}}(\text{blackish}), f_{\text{elegent}}(\text{green})\}, \end{aligned} \tag{36}$$

and so (f, A) is not an $\mathcal{N}_{\text{elegent}}$-soft BCI-algebra over $(U, \$, \text{white})$. Hence (f, A) is not an $\mathcal{N}$-soft BCI-algebra over $(U, \$, \text{white})$.

But, we have the following theorem.

Theorem 13. *For any subset A of E, let (f, A) be an $\mathcal{N}$-soft BCK/BCI-algebra over a BCK/BCI-algebra X. If B is a subset of A, then $(f|_B, B)$ is an $\mathcal{N}$-soft BCK/BCI-algebra over X.*

Proof. Straightforward. □

The following example shows that there exists an $\mathcal{N}$-soft set (f, A) over a BCK/BCI-algebra X such that

(i) (f, A) is not an $\mathcal{N}$-soft BCK/BCI-algebra over a BCK/BCI-algebra X,

(ii) there exists a subset B of A such that $(f|_B, B)$ is an $\mathcal{N}$-soft BCK/BCI-algebra over a BCK/BCI-algebra X.

Example 14. Let $(U, \heartsuit, \text{white})$ be a BCK-algebra as in Example 10. Consider a set of attributes $A := \{$beautiful, fine, moderate, smart, chaste$\} \subseteq E$. Let (f, A) be an $\mathcal{N}$-soft set over $(U, \heartsuit, \text{white})$ with the tabular representation which is given by Table 8.

Table 7: Tabular representation of (f, A).

(f, A)	White	Blackish	Reddish	Green	Yellow
Fine	−0.6	−0.3	−0.6	−0.5	−0.3
Elegant	−0.9	−0.8	−0.7	−0.7	−0.4
Smart	−0.8	−0.1	−0.5	−0.3	−0.1

Then (f, A) is neither an $\mathcal{N}_{\text{smart}}$-soft BCK-algebra nor an $\mathcal{N}_{\text{chaste}}$-soft BCK-algebra over $(U, \heartsuit, \text{white})$. Hence (f, A) is not an $\mathcal{N}$-soft BCK-algebra over $(U, \heartsuit, \text{white})$. But if we take

$$B := \{\text{beautiful, fine, moderate}\} \subseteq A, \tag{37}$$

then $(f|_B, B)$ is an $\mathcal{N}$-soft BCK-algebra over $(U, \heartsuit, \text{white})$.

Definition 15. Let (f, A) and (g, B) be two $\mathcal{N}$-soft sets in (X, E). The *union* of (f, A) and (g, B) is defined to be the $\mathcal{N}$-soft set (h, C) in (X, E) satisfying the following conditions:

(i) $C = A \cup B$,

(ii) for all $x \in C$,

$$h_x = \begin{cases} f_x & \text{if } x \in A \setminus B, \\ g_x & \text{if } x \in B \setminus A, \\ f_x \cap g_x & \text{if } x \in A \cap B. \end{cases} \tag{38}$$

In this case, we write $(f, A) \tilde{\cup} (g, B) = (h, C)$.

Lemma 16 (see [11]). *If (X, f) and (X, g) are $\mathcal{N}$-subalgebras of a BCK/BCI-algebra X, then the union $(X, f \cup g)$ of (X, f) and (X, g) is an $\mathcal{N}$-subalgebra of X.*

Theorem 17. *If (f, A) and (g, B) are $\mathcal{N}$-soft BCK/BCI-algebras over a BCK/BCI-algebra X, then the union of (f, A) and (g, B) is an $\mathcal{N}$-soft BCK/BCI-algebra over X.*

Proof. Let $(f, A) \tilde{\cup} (g, B) = (h, C)$ be the union of (f, A) and (g, B). Then $C = A \cup B$. For any $x \in C$, if $x \in A \setminus B$ (resp. $x \in B \setminus A$), then $(X, h_x) = (X, f_x)$ (resp. $(X, h_x) = (X, g_x)$) is an $\mathcal{N}$-subalgebra of X. If $A \cap B \neq \emptyset$, then $(X, h_x) = (X, f_x \cup g_x)$ is an $\mathcal{N}$-subalgebra of X for all $x \in A \cap B$ by Lemma 16. Therefore (h, C) is an $\mathcal{N}$-soft BCK/BCI-algebra over a BCK/BCI-algebra X. □

Definition 18. Let (f, A) and (g, B) be two $\mathcal{N}$-soft sets in (X, E). The *intersection* of (f, A) and (g, B) is the $\mathcal{N}$-soft set (h, C) in (X, E) where $C = A \cup B$ and, for every $x \in C$,

$$h_x = \begin{cases} f_x & \text{if } x \in A \setminus B, \\ g_x & \text{if } x \in B \setminus A, \\ f_x \cap g_x & \text{if } x \in A \cap B. \end{cases} \tag{39}$$

In this case, we write $(f, A) \tilde{\cap} (g, B) = (h, C)$.

Theorem 19. *Let (f, A) and (g, B) be $\mathcal{N}$-soft BCK/BCI-algebras over a BCK/BCI-algebra X. If A and B are disjoint, then the intersection of (f, A) and (g, B) is an $\mathcal{N}$-soft BCK/BCI-algebra over X.*

Proof. Let $(f, A) \tilde{\sqcap} (g, B) = (h, C)$ be the intersection of (f, A) and (g, B). Then $C = A \cup B$. Since $A \cap B = \emptyset$, if $x \in C$, then either $x \in A \setminus B$ or $x \in B \setminus A$. If $x \in A \setminus B$, then $(X, h_x) = (X, f_x)$ is an $\mathcal{N}$-subalgebra of X. If $x \in B \setminus A$, then $(X, h_x) = (X, g_x)$ is an $\mathcal{N}$-subalgebra of X. Hence (h, C) is an $\mathcal{N}$-soft BCK/BCI-algebra over a BCK/BCI-algebra X. □

The following example shows that Theorem 19 is not valid if A and B are not disjoint.

Example 20. Suppose there are five colors in the universe U, that is,

$$U := \{\text{white}, \text{blackish}, \text{reddish}, \text{green}, \text{yellow}\}. \tag{40}$$

Let Ξ be a soft machine to mix two colors according to order in such a way that we have the following results:

$$\begin{gathered}
x \,\Xi\, \text{white} = x \quad \forall x \in U, \\
x \,\Xi\, \text{blackish} = \begin{cases} \text{white} & \text{if } x \in \{\text{white}, \text{blackish}\}, \\ x & \text{if } x \in \{\text{reddish}, \text{green}, \text{yellow}\}, \end{cases} \\
x \,\Xi\, \text{reddish} = \begin{cases} \text{reddish} & \text{if } x \in \{\text{white}, \text{blackish}\}, \\ \text{white} & \text{if } x = \text{reddish}, \\ \text{yellow} & \text{if } x = \text{green}, \\ \text{green} & \text{if } x = \text{yellow}, \end{cases} \\
x \,\Xi\, \text{green} = \begin{cases} \text{green} & \text{if } x \in \{\text{white}, \text{blackish}\}, \\ \text{yellow} & \text{if } x = \text{reddish}, \\ \text{white} & \text{if } x = \text{green}, \\ \text{reddish} & \text{if } x = \text{yellow}, \end{cases} \\
x \,\Xi\, \text{yellow} = \begin{cases} \text{yellow} & \text{if } x \in \{\text{white}, \text{blackish}\}, \\ \text{green} & \text{if } x = \text{reddish}, \\ \text{reddish} & \text{if } x = \text{green}, \\ \text{white} & \text{if } x = \text{yellow}. \end{cases}
\end{gathered} \tag{41}$$

Then (U, Ξ, white) is a BCI-algebra. Consider sets of attributes:

$$\begin{gathered}
A = \{\text{beautiful}, \text{fine}, \text{elegant}, \text{smart}\}, \\
B = \{\text{elegant}, \text{smart}, \text{chaste}\}.
\end{gathered} \tag{42}$$

Then A and B are not disjoint. Let (f, A) and (g, B) be $\mathcal{N}$-soft sets over (U, Ξ, white) having the tabular representations which are given in Tables 9 and 10, respectively.

Then (f, A) and (g, B) are $\mathcal{N}$-soft BCI-algebras of (U, Ξ, white). But the intersection $(f, A) \tilde{\sqcap} (g, B) = (h, C)$ of (f, A) and (g, B) is not an $\mathcal{N}$-soft BCI-algebra of (U, Ξ, white) since

$$\begin{aligned}
&\left(f_{\text{elegant}} \cap g_{\text{elegant}}\right)(\text{green} \,\Xi\, \text{reddish}) \\
&\quad = \left(f_{\text{elegant}} \cap g_{\text{elegant}}\right)(\text{yellow}) \\
&\quad = -0.1 > -0.3 = \bigvee \{\left(f_{\text{elegant}} \cap g_{\text{elegant}}\right)(\text{green}), \\
&\qquad \left(f_{\text{elegant}} \cap g_{\text{elegant}}\right)(\text{reddish})\},
\end{aligned} \tag{43}$$

that is, $(U, f_{\text{elegant}} \cap g_{\text{elegant}})$ is not an $\mathcal{N}$-subalgebra of (U, Ξ, white).

Table 8: Tabular representation of (f, A).

(f, A)	White	Blackish	Reddish	Green	Yellow
Beautiful	−0.6	−0.6	−0.6	−0.3	−0.3
Fine	−0.8	−0.7	−0.2	−0.5	−0.2
Moderate	−0.7	−0.1	−0.5	−0.1	−0.1
Smart	−0.1	−0.2	−0.3	−0.4	−0.5
Chaste	−0.3	−0.2	−0.6	−0.7	−0.2

Table 9: Tabular representation of (f, A).

(f, A)	White	Blackish	Reddish	Green	Yellow
Beautiful	−0.7	−0.6	−0.3	−0.3	−0.3
Fine	−0.6	−0.5	−0.4	−0.2	−0.2
Elegant	−0.8	−0.5	−0.1	−0.3	−0.1
Smart	−0.5	−0.5	−0.2	−0.2	−0.4

Table 10: Tabular representation of (g, B).

(g, B)	White	Blackish	Reddish	Green	Yellow
Elegant	−0.8	−0.6	−0.3	−0.1	−0.1
Smart	−0.7	−0.6	−0.3	−0.3	−0.5
Chaste	−0.9	−0.5	−0.2	−0.4	−0.2

Theorem 21. *If (f, A) and (g, B) are two $\mathcal{N}$-soft BCK/BCI-algebras over a BCK/BCI-algebra X, then $(f, A) \tilde{\vee} (g, B)$ is a fuzzy soft BCK/BCI-algebra over X.*

Proof. We note that

$$(f, A) \tilde{\vee} (g, B) = (h, A \times B), \tag{44}$$

where $h_{(u,v)} = \bigvee \{f_u, g_v\}$ for all $(u, v) \in A \times B$. For any $x, y \in X$, we have

$$\begin{aligned}
h_{(u,v)}(x * y) &= \bigvee \{f_u(x * y), g_v(x * y)\} \\
&\le \bigvee \{\bigvee \{f_u(x), f_u(y)\}, \bigvee \{g_v(x), g_v(y)\}\} \\
&= \bigvee \{\bigvee \{f_u(x), g_v(x)\}, \bigvee \{f_u(y), g_v(y)\}\} \\
&= \bigvee \{h_{(u,v)}(x), h_{(u,v)}(y)\}.
\end{aligned} \tag{45}$$

Hence $(h, A \times B) = (f, A) \tilde{\vee} (g, B)$ is an $\mathcal{N}$-soft BCK/BCI-algebra based on (u, v). Since (u, v) is arbitrary, we know that $(h, A \times B) = (f, A) \tilde{\vee} (g, B)$ is an $\mathcal{N}$-soft BCK/BCI-algebra over X. □

6. Conclusions

Using the notions of soft sets and $\mathcal{N}$-structures, we have introduced the concept of $\mathcal{N}$-soft sets and considered its application in both a decision making problem and a BCK/BCI-algebra. In an imprecise environment, the importance of the problem of decision making has been emphasized in recent years. We have presented an $\mathcal{N}$-soft set theoretic approach towards solution of the decision making problem. We have taken the algorithm that involves Construction of Comparison Table from the resultant $\mathcal{N}$-soft set and the final decision based on the minimum score computed from the Comparison Table (see Tables 2 and 3). Through the application in a BCK/BCI-algebra, we have introduced the notion of $\mathcal{N}$-soft BCK/BCI-algebras and have investigated related properties.

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