Does the Use of Diagrams as Communication Tools Result in their Internalization as Personal Tools for Problem Solving?

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Abstract

Although diagrams are considered as effective personal tools for solving problems, applied research in education has identified a widespread problem: that students lack spontaneity in diagram use. One way to address this problem was reported by Uesaka and Manalo (2007): their findings indicate the effectiveness of using peer instruction to enhance students' spontaneous use of diagrams. However, it was not clear from their study whether actual interaction is necessary, and whether formulation of explanations in using diagrams to solve problems would in itself be sufficient. The present study sought to clarify the role of communication in enhancing the spontaneous use of diagrams, and involved 5 days of experimental classes for 59 participants in the 8th grade. Two conditions were used: one where participants really interacted with each other in peer instruction sessions (the experimental condition), and another where the participants formulated explanations but were not involved in peer instruction interactions. At post test, both quantity and quality of diagrams spontaneously produced by participants in the experimental condition were higher than those in the control condition, suggesting that the communication process involved in actual interactions with peers is a critical factor. This result supports the notion that using diagrams as communication tools results in their internalization as personal tools for problem solving.

Keywords: Diagram Use, Math Word Problem Solving, Communication Tools, Problems Solving Tools.

Introduction

1.1 Students' Lack of Spontaneity in Diagram Use

Diagram use is considered as one of the most effective strategies for problem solving and many studies have also empirically demonstrated that diagrams effectively promote the efficient performance of many types of tasks (e.g., Ainsworth & Th Loizou, 2003; Larkin & Simon, 1987; Mayer, 2003; Schoenfeld, 1985). After PISA (Program for International Student Assessment, OECD, 2004) specified the ability to efficiently use diagrams in problem solving as being an important aspect of literacy for living, the educational concern for this ability has also been mounting.

It appears, however, that school students do not appreciate the efficiency that diagram use brings to problem solving as much as teachers and researcher do. Research into actual day-to-day educational practices suggests that students fail to spontaneously use diagrams despite plenty of exposure to appropriate use of diagrams. In a tutoring case study, for example, Ichikawa (1993) described an 8th-grade girl who did not spontaneously use diagrams in a test situation and failed to solve the problem given – despite previously being taught how to solve similar problems with the use of diagrams. Ichikawa (2000) described the extent to which this problem is found among students. Dufour-Janiver, Bednarz, and Belanger (1987) also noted the same lack of spontaneity and observed that this problem occurs even though math teachers use a lot of diagrams in class. Uesaka, Manalo, and Ichikawa (2007) confirmed this tendency particularly among Japanese students.

It is important for students to be able to construct and use diagrams by themselves when diagrams are required and/or appropriate in problem solving situations they encounter. Not being able to do so can only be considered a serious disadvantage especially as, in most educational and 'real world' contexts, suitable diagrams are neither provided nor suggested with the problems given.

Up until recently, however, there has been a tendency for diagrams research to focus only on diagram effects and functions (e.g., Cheng, 2002; Larkin & Simon, 1987; Mayer, 2003). Few studies have empirically examined possible teaching methods to address the identified problem of lack of spontaneity in diagram use. This study therefore aimed to develop a practical instruction method to effectively promote spontaneous diagram use, particularly where math word problem solving is concerned.

1.2 Promoting Strategy Use via Collaboration

Uesaka (2003) is one of only a few studies that have proposed teaching methods to enhance students' spontaneous use of diagrams when attempting to solve math word problems. She found evidence that diagram use can be promoted by addressing two factors: students' perception of the efficacy of diagram use, and their diagram construction skills.

The idea of addressing both students' perception about and skills in using a strategy is familiar in the research area of academic strategy use. For example, Brown (1983) proposed the term 'informed training' to suggest the importance of providing skills training in the target strategy in conjunction with providing information about the efficacy of the strategy.

Although many interventions for enhancing spontaneous use of a desired strategy, including that described in Uesaka

(2003), have employed teacher-provided instruction and/or encouragement, some studies have included peer instruction as part of the intervention. One such study was that reported by Palincsar and Brown (1984). They proposed 'reciprocal teaching' for enhancing strategy use and reading performance, which included activities whereby a student explained the content of selected text and other students raised questions stemming from the use of reading strategies. The important point of this study was that students shared strategies used externally in a collaborative learning situation, and this was found to promote subsequent generalization and spontaneous use of the reading strategies in non-collaborative situations.

1.3 Diagrams as Communication Tools

The approach of enhancing strategy use through incorporating collaborative learning situations in the intervention may also prove useful in the case of diagram use because diagrams are not only tools for problem solving but also tools for communication, though this latter aspect has only recently been explored in diagrams research (e.g., Heiser & Tversky, 2006; Tversky, Lozano, Heiser, Lee, & Daniel, 2005). The effectiveness of diagrams as tools for communication has been demonstrated in some earlier studies. For example, Lyon (1995) reported that diagrams facilitated communication with adults who, because of aphasia, found communication through verbal means difficult (see also review by Sacchett, 2002).

This suggests the possibility that giving students the opportunity to use diagrams in collaborative situations as tools for communication could promote subsequent spontaneous use of diagrams as personal tools for problem solving. Their efforts at explaining how to solve problems with diagrams could promote subsequent spontaneous use of diagrams – without the need for teachers to provide encouragement.

An investigation of the hypothesis, that facilitating learning situations where students explain how to solve problems to each other with the use of diagrams subsequently promotes spontaneous diagram use, was carried out and reported by Uesaka and Manalo (2007). They compared two conditions: the first group incorporated peer instruction whereby opportunities were provided for participants to explain to each other within their small groups by applying the jigsaw method (Aronson, 1978); the second group was without peer instruction, but otherwise equivalent to the first group in the instructional procedures used. The results revealed that the spontaneous construction, and appropriateness, of diagrams employed in the post test was higher in the group which incorporated peer instruction. This finding suggests a possible relationship between these two aspects: that using diagrams as communication tools may lead to internalizing them as personal tools for problem solving.

This study also suggested the possible mechanisms involved in the results observed. Firstly, understanding the ways in which diagrams are used in problem solving is enhanced when participants get a turn at explaining as well as when they listen and get opportunities to ask questions. With increased understanding comes greater appreciation of the efficacy that diagram use brings to problem solving. Secondly, the peer instruction sessions made it inevitable for participants to use diagrams to represent their thinking (as words alone often proved inadequate), and hence get practice in constructing diagrams and further appreciate the benefits that come with their use. Thus, perception of the efficiencies that come with diagram use, as well as construction skills, are enhanced: Uesaka (2003) suggested that both these factors are necessary for promoting the spontaneous use of diagrams.

1.4 Purpose of this Study

Although Uesaka and Manalo's (2007) findings were important in that they demonstrated the beneficial effects of incorporating peer instruction on spontaneity of strategy (diagram) use, the exact role of "communication" was not clarified. More specifically, it was not entirely clear whether the actual interaction between the students that was involved in peer instruction was the crucial component, or if it was the provision of opportunities to explain how the problem given was solved with diagrams. If Uesaka and Manalo's (2007) results can be obtained simply through engaging in virtual explanations (without the accompanying interaction with others), it would suggest that real interaction is not necessary (interactive communication is not the important factor in this process). On the other hand, if peer instruction is found to have advantages over virtual explanations, it would suggest that the actual communicative interaction between students is the effective ingredient in promoting its subsequent spontaneous use.

Therefore, this study examined whether actual explanation among students has additional effects compared to simply engaging in the provision of virtual explanations.

Method

### 2.1 Participants and Experimental Design

The participants were 59 8th-grade students from public junior high schools in two wards of Tokyo and a junior high school affiliated with the University of Tokyo. They participated voluntarily in the study.

The participants were assigned to one of two conditions: the actual explanation condition (the experimental condition) or the virtual explanation condition (the control condition). The process of assignment controlled for the participants' school achievement through the use of randomized block design. Information about the participants' school achievements was gathered through the use of a questionnaire which was sent to the participants' parents by mail prior to the start of the experimental classes.

2.2 Materials

Math Word Problems Used in Instruction Sessions. The 5 days of experimental classes were held at the University of Tokyo and comprised of a pre-test session (held on the first day), instruction sessions (held on days 2-4), and a post test session (held on the last day). During the instruction classes, two different types of problems were used, referred to as 'arrangement problems' and 'mobile phone problems'. Two math word problems were prepared under each type. Problems belonging to the same category shared a similar story context and required a similar type of diagram to efficiently arrive at the correct solution. Two of the 'mobile phone problems' are shown as examples in Appendix 1.

For the 'mobile phone problems', graphs were deemed as helpful for arriving at the correct answer. On the other hand, for the 'arrangement problems', using tables and pictures representing the situation were considered to be effective for solving them. Examples of diagrams constructed by participants during the instruction sessions are shown in Figure 1.
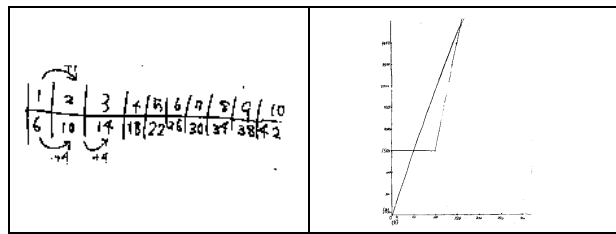
(A) Arrangement Problem   (B) Mobile Phone Problem

Fig. 1. Typical Diagrams Participants Produced During the Instruction Sessions. (A) is for one of the arrangement problems, and (B) for one of the mobile phone problems.

Math Word Problem Solving Assessment. This assessment was administered on the last day of the experimental classes to evaluate the extent to which participants spontaneously used diagrams (without the provision of any encouragement or hints to use diagrams). The appropriateness of the diagrams that participants constructed was also evaluated.

This assessment consisted of four math word problems, for which the participants were allowed 8 minutes for each. These problems were relatively more difficult than problems used during the instruction sessions to make it more likely that diagrams would be needed for their efficient solution (i.e., easier problems would not have assessed the participants predisposition to use diagrams as many would have likely been able to quickly solve them without diagrams). For two of the problems – the 'tile' and the 'pentagon' problems – using tables was considered the most appropriate for arriving at the correct solutions. For the other two problems – the 'water' and the 'travel' problems – the use of graphs was considered most appropriate.

Examples of problems used in this assessment are shown in Appendix 2. The judgments made about the most effective types of diagrams for each of the problems given were based on raters decisions reported in Uesaka and Manalo (2007), where the same problems were used.

Basic Skills Assessment. The basic skills assessment was administered on the first day of the experimental classes as a pre-test to determine the equivalence (or otherwise) of the diagram construction skills of the participants assigned to the two conditions. Establishing this at the start of the study was important to avoid the potential criticism that subsequent differences between the two groups could have arisen from pre-existing discrepancies in their diagram construction abilities.

The assessment consisted of two questions, in which participants were asked to construct a table and a graph from details contained in sentences they were given. This assessment explicitly asked participant to draw diagrams; thus it evaluated their ability to correctly construct diagrams when given instructions to do so.

2.3 Procedure

Each of the daily sessions lasted about 50 minutes. The following is an outline of the instruction sessions provided.

In both experimental and control conditions, the two problems belonging to the same category were provided to the participants simultaneously. The participants worked in small groups of 4-5 members. These group members divided into two subgroups (with 2-3 members) so that one subgroup could work on one of the problems given, and the other subgroup on the other problem given.

After the problem they were to work on had been decided, participants started to work on and construct a solution for the problem they had. Participants in the experimental condition were asked to write a solution with the intention of later providing an explanation to members of the other subgroup who worked on the other problem. The participants in the control condition, on the other hand, were asked to write an explanation with the intention of providing an explanation to hypothetical other people (e.g., for the mobile phone problem, they were asked to imagine that they were working in a phone shop and had to draft a pamphlet for explaining to customers). Participants in the control condition were further told that some of them would be asked to present their solution in front of the class.

The instructions provided to the participants also included the teachers' explicit advice about the efficacy of diagram use, and opportunities to construct diagrams for solving the problems given. Just before starting to solve the problems, participants in both conditions received explicit instructions from the teacher about efficiencies that diagram use brings to problem solving as well as to providing explanations. In addition, the participants received hints from the teacher while they were attempting to solve the problems: these were given as required, and included suggestions for steps to take in solving the problems and using diagrams – as appropriate.

After preparing the material for explanation, participants in the experimental group were asked to actually explain to other students (in small groups) how to solve the problem they worked on. Two minutes were allowed for each participant's explanation, and students who were recipients of the explanation were encouraged to ask questions. After this, one participant from the class was asked to present his or her explanation to the entire class, and the teacher provided comments and/or additional information as necessary. Thus, the participants in the experimental groups, usually listened to a total of three explanations about how to solve the problems given: two in small group, interactive peer instruction situations, and one presentation in front of the class.

In contrast, for the participants in the control groups, three participants were selected and asked to present how they solved their problem in front of the class. As in the experimental condition, the teacher also provided comments and information as necessary. Thus, the only real difference was that participants in the control condition never experienced the peer instruction interaction in small group settings, even though they were exposed to the same number of participant explanations (i.e., three).

In the pre- and post tests, participants were asked to solve the problems given on their own: any interaction between participants was prohibited. In the math word problem solving assessment, which was administered as the post test, the use of an eraser was also prohibited as it would have limited the worksheet information available to the researchers for carefully analyzing the processes involved in the participants' problem solving (which includes initial errors they made, changes they decided on, etc.). The participants were asked to solve the problems one by one, with the teacher signaling when they could move on to the next problem according to the time allotment.

Results

In the analyses, only participants who attended all sessions were included. The number of participants included was 42 (for the experimental condition, n = 20; for the control condition, n = 22).

3.1 Math Word Problem Solving Assessment Findings

The students' spontaneous use of diagrams in the math word problem solving assessment was analyzed. Before conducting the analyses, the participants' responses to each problem were scored. Responses, in which at least one diagram was used, were scored as 'used diagrams (1)'; otherwise they were scored as 'no diagram (0)'. For the purpose of this study, the definition of a diagram used was 'any representations of the problem other than words, sentences, or numerical formulas', and the definition of a table (counted as a diagram) was 'a depiction of at least a pair of values arrayed to represent two related variables'.

The average numbers of problems for which participants constructed diagrams were compared for the two conditions

by using a t-test. The average number of problems for which the participants in the experimental condition spontaneously use diagrams was significantly higher compared to the control condition ($t_{(40)} = 2.66$, $p < .05$; see Figure 2).

An analysis of the quality of diagrams was also undertaken. If participants produced a diagram similar to those deemed as effective, the diagram was scored as 'appropriate (1)', and the rest was scored 'inappropriate (0)' (examples of diagrams produced are shown in Figure 3). A t-test was carried out to compare the average number of problems in which the appropriate types of diagrams were produced by participants in the two conditions (also shown in Figure 2). The average number of appropriate types of diagrams was higher in the experimental condition compared to the control condition, but the difference proved to be only marginally significant ($t_{(40)} = 1.31$, $p < .10$).

Together, these findings suggest that adding the opportunity for students to interact in peer instruction sessions, and employ diagrams in actually explaining and communicating with each other how to solve the problems given, made a positive difference to their subsequent spontaneous use of diagrams as well as to the quality of diagrams they produced. This suggests that the actual interaction between students is an important component in enhancing their spontaneous use of diagrams.

3.2 Basic Skills Assessment Findings

In order to confirm the equivalence of the experimental and control conditions at the beginning of the experiment, participants' responses in the basic skills assessment were analyzed. If the participants produced a perfectly correct response, full credit (2 points) was given. A response that was mostly correct but included a small mistake was given partial credit (1 point). For the rest, no credit was given (0 point). The total scores for the two questions were compared for the two conditions using a t-test. This revealed no significant difference ($t_{(40)} = 1.10$, n.s.).

This result suggests that the participants assigned to the two conditions were equivalent in their ability to use diagrams at least at the beginning of the study. The possible criticism about pre-existing differences in the participants' abilities in using diagrams, noted earlier, has therefore been negated by this finding.
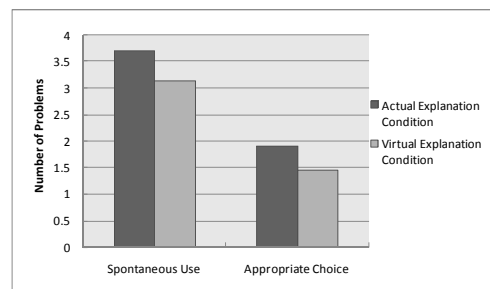

Fig. 2. The Mean Number of Problems in which Diagrams Were Spontaneously Used (left), and in which Appropriate Diagrams Were Constructed (right) in the Math Word Problem Solving Assessment.
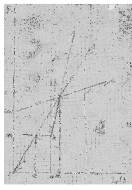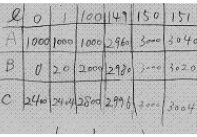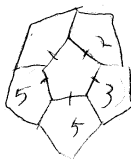
	Water Problem	Pentagon Problem
Diagrams Deemed as Appropriate		
Diagrams Deemed as Inappropriate		

Fig. 3. Examples of Diagrams Produced During the Post Test Assessment.

3.3 Example of Protocols Used During the Classes

To better understand what occurred during the peer instruction situations in the experimental condition, analyses were carried out of typical protocols observed during those situations. An example is shown in Table 1. The problem that S1 choose was the first of the mobile phone problems (see Appendix 1). She explained by constructing a graph, shown as an example in Figure 4. The graph she made was appropriate and the explanation she initially wrote was also comparatively sophisticated – which was quite similar to good solutions written by participants in the control condition. However, the explanation included errors: she neglected the intersection of the two variables, and treated calling time as a discrete quantity instead of a continuous quantity. S2 raised a question about the intersection on the graph. By answering S2's question, she was led to question her own understanding of the graph.

As shown in this case, participants in the experimental condition received questions (usually several each time) and were confronted with situations in which they had to answer questions generated by their peers. It provided them with opportunities to explain more. After experiencing these kinds of activities and interactions, S1 wrote on the review sheet administered on the final day of the experiment: "Previously I did not construct graphs and tables because I felt they were troublesome, but I found I can more easily understand when I use these, so I have decided to draw more of them from now on". This comment suggests that she also came to appreciate the efficiency that diagram use brings to problem solving (something she obviously did not appreciate previously).

This example suggests that real interactions between students in peer tutoring situations instigates "prompts" for providing more explanations and for reflection on what one is doing. Through experiences of trying to answer questions raised, greater understanding of the content included in diagrams often follows – together with perceiving the efficiencies they bring. This likely leads to the increased spontaneity in subsequent diagram use.

Table 1. Example of Protocols Observed During an Instruction Session in the Experimental Condition.

> S1: I selected the first problem. In this graph, this line shows Plan B, and it usually costs 4500 yen. This line shows Plan A, with constantly increasing cost … Oh, opposite! Sorry – this line is Plan A, and this is Plan B.
> ….
> This cross point shows 150 minutes, thus, hmm … explaining is difficult! Anyway, when people speak under 149 minutes, Plan B is cheaper. When people speak over 151 minutes, Plan A is cheaper.
> S2: I got a question, how about this point?
> S1: The cost is similar at this point. Huh? Is that correct?

Discussion

The main finding of this study is that participants in the experimental condition, where they were given opportunities to really explain how to solve the problems given in interactive peer instruction sessions, subsequently evidenced greater spontaneous use of diagrams in problem solving. This is in comparison to participants in the control condition, where they were given opportunities to construct explanations for hypothetical people, but never experienced the real interactive provision of explanations to their peers. In addition, the quality of the diagrams used in the post test was higher in the experimental group. These results suggest that real interactive communication with the use of diagrams is the critical component when peer instruction enhances subsequent spontaneity in diagram use (as earlier observed in Uesaka & Manalo, 2007).

The differences found between the two conditions in their diagram use at post test were deemed not to originate from any pre-existing discrepancy in the participants' diagram use abilities. This is because the basic skills assessment administered during the pre-test session revealed no differences between the two groups in their diagram construction skills. Thus, the differences observed at post test can confidently be attributed to differences in the instructional interventions used.

Although the present study used a "virtual explanation" condition in the control group, it does not intend to question the benefits of explaining for promoting students' spontaneous use of diagrams. Rather, this investigation aimed to identify the most important contributing components in providing explanations: to find out whether thinking through and devising ways to explain is adequate,
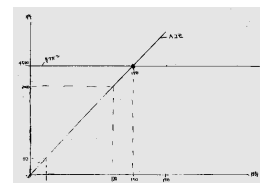

Fig. 4. The Graph that S1 Constructed and Used.

or if actually explaining in an interactive manner is additionally required. The findings suggest that incorporating the latter is more effective. Analysis of the protocols used suggest that in providing interactive explanations to peers, students get urged to explain more, to question, and to reflect on the processes involved in solving the problems – including the benefits associated with diagram use.

The present study contributes to both research in diagrams and in math education. It demonstrates that using diagrams as communication tools results in its internalization as personal tools for problem solving. This is a new perspective because these two aspects of diagram use had only been examined independently in earlier diagrams research. In addition, the findings of this study clearly indicate that teachers' demonstrating the use of diagrams in class is insufficient for students to use diagrams spontaneously as their own tools for problem solving. Teachers need to additionally provide opportunities for students to use diagrams in interactive situations – to communicate their uses in problem solving to their peers.

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Appendix 1: Examples of Math Problems Used in the Instruction Sessions

Mobile Phone Problem (1)

You are a clerk in a mobile phone shop. In this shop, two types of mobile phone plans are sold. When a customer who wants to buy a mobile phone asks your advice, which mobile would you recommend as being cheaper depending on calling time?

Plan A: A basic fee is 4500 yen, and no extra cost is charged.

Plan B: There is no basic fee, and no free calling time. The cost of calls is 30 yen per minute.

Mobile Phone Problem (2)

In another shop, different two types of mobile phone plans are sold. When a customer who wants to buy a mobile phone asks your advice, which mobile would you recommend as being cheaper depending on calling time?

Plan B: There is no basic fee, and no free calling time. The cost of calls is 30 yen per minute.

Plan C: A basic fee of 1500 yen including 100 minutes of free calling time. After 100 minutes, 80 yen per minute is charged.

Appendix 2: Examples of Math Problems Used at Post Test

Water Problem

The head of a company asked Taro to find out which of three countries – A, B, or C – would be best for establishing a factory. The factory uses water and water charges differed between the three countries. The different charging methods are described below. Please imagine you are Taro, and come up with an explanation that he could provide to the head of the company.

Country A: 1000 yen is charged as a basic fee, but you can use water without additional charge up to 100 litres. After 100 litres, 40 yen/litre is charged.

Country B: There is no basic fee. Water cost is 20 yen/litre.

Country C: In addition to 2400 yen as a basic fee, there is a charge of 4 yen/litre of water used.

Pentagon Problem

There are many sheets of paper in the shape of a regular pentagon, with each side being 1 cm. These sheets are arranged one by one with the rule that a new sheet shares only one side with already arranged sheets. Find the circumference when arranging 1, 5, 10 and 20 sheets.