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## TOOL RUNOUT ESTIMATION USING FEED FORCE SPECTRAL COMPONENTS

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### ABSTRACT

Cutter Runout is the eccentricity caused by a combination of imperfections in the cutting tool, tool holder, and spindle. While magnitudes of cutter runout are generally small (e.g. 10  $\mu\text{m}$ ), the effect is noticeable in high tolerance finishing operations, and can cause significant variations in the force experienced on a multi tooth cutter. The goal of this research is to design and evaluate an indirect runout estimation method, using feed force components at the spindle and tooth passing frequency. The accuracy of the method is evaluated for a range of cutting conditions for 4 and 6 tooth helical flat end mills. The standard error of the runout estimation method for 27 test cases was determined to be 7.1  $\mu\text{m}$ . The error was skewed by several data points from a 6 tooth cutter. Additional testing is needed with 2, 3 and 5 tooth cutters.

### INTRODUCTION

The effect of tool runout (Figure 1) on cutting forces is well documented. Kline and DeVor [1] presented methods to incorporate runout magnitude and orientation into mechanistic cutting force models. Hekman and Liang [2] developed a method to calculate runout magnitude and orientation recursively given the prior part material properties, cutting tool geometry, cutting parameters, and machining characteristics. This study found that runout is affected by machining process dynamics during the cutting process. Seethaler and Yellowley [3] used Fourier series representation to generate real time identification of cutting tool runout for an array of radial immersion values for peripheral cutting. Their results allow for the tracking of runout changes during end milling operations, as well as cutter chipping and breaking. Research by Yalcin et al [4] approximated cutting tool runout magnitude and

orientation by systematic iteration of both values to match simulated x and y force profiles to experimental profiles.

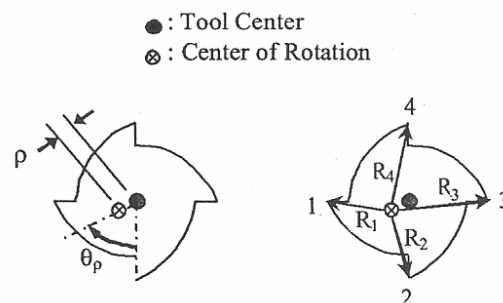


Figure 1: Runout  $\rho$ , runout locating angle  $\theta$ , and effective tooth radii  $R_i$  on a four tooth cutter.

Previous methods to find cutter runout require expensive sensors, such as a 3-axis dynamometer [2,3,4]. This research focuses on an indirect runout estimation method using a single axis load cell. Direct methods are available, e.g. laser and dial indicator, however they are either very expensive or inconvenient. We want to develop an indirect method so that runout can be estimated while cutting a sacrificial test block. This method should be more convenient and may also be a better indicator of the effective runout since the tool is cutting.

The indirect method uses a non-dimensional ratio of forces ( $R_f$ ) to estimate the runout. The ratio is determined theoretically (closed form), using mechanistic force models, and from experimental cutting tests, using the magnitude of feed force harmonics. Equating the two ratios allows the calculation of the cutter runout. The ratio is defined such that the cutting model material parameters are not required to

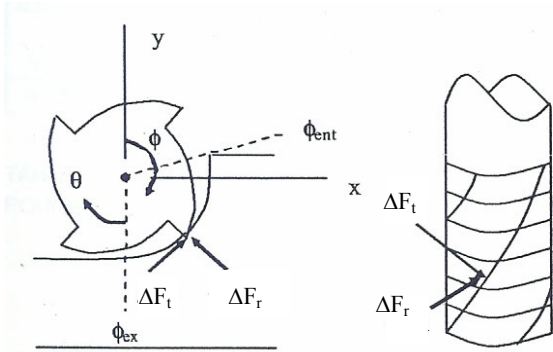
calculate the theoretical  $R_f$ . The ratio is also selected so that the force magnitude of the FFT components at the spindle and tooth passing frequency can be directly used in calculating the experimental ratio. A potential drawback to this method is the effect of machine and tool vibration on the tooth and spindle frequency components of cutting force. Light cuts are taken to reduce the effects of vibration.

This paper discusses the methodology and results obtained using the feed force components at the tooth passing and spindle frequencies to estimate runout. Section two discusses the mechanistic force model and deflection model used in this research. Section three describes the effect of runout on the feed force spectral harmonic magnitudes. Section four focuses on the derivation of a closed form ratio which is used to estimate runout magnitude. Results using this method are evaluated for a range of cutting conditions.

## FORCE AND DEFLECTION MODELS

### Discrete Force Model

The discrete mechanistic force model used in this research is based on the individual tangential and radial force components of a cutting tooth segment engaged in a workpiece (Figure 2). The individual force components of each axial slice are summed to yield the total cutting forces of the end mill in a workpiece.



**Figure 2: Tangential and radial forces utilized in the discrete mechanistic force model.**

The tangential and radial components of cutting force on a tooth segment can be further separated into a shearing force and an edge force:

$$\Delta F_t(\phi) = K_{ic} \cdot h(\phi) \cdot \Delta a + K_{ie} \cdot \Delta a \quad (1)$$

$$\Delta F_r(\phi) = K_{rc} \cdot h(\phi) \cdot \Delta a + K_{re} \cdot \Delta a \quad (2)$$

where  $h$  is the chip thickness,  $\Delta a$  is the cutting edge length in an axial slice of the tool,  $K_{ic}$  and  $K_{rc}$  represent the cutting energies ( $N/mm^2$ ) associated with shearing, and  $K_{ie}$  and  $K_{re}$  are the parasitic friction component of force ( $N/mm$ ) associated with edge forces. The location of each cutting tooth is denoted by an angular displacement for each cutting tooth,  $\phi$ . Equations 1 and 2 are in a form proposed by several researchers [5,6,7].

The total forces acting on the tool can be expressed as the sum of the forces acting on each cutting tooth for each axial cross section engaged in the workpiece:

$$F_x(\phi) = \sum_{i=1}^{N_z} \sum_{j=1}^{N_t} [-\Delta F_t \cdot \cos(\phi) - \Delta F_r \cdot \sin(\phi)] \quad (3)$$

$$F_y(\phi) = \sum_{i=1}^{N_z} \sum_{j=1}^{N_t} [+ \Delta F_t \cdot \sin(\phi) - \Delta F_r \cdot \cos(\phi)] \quad (4)$$

where  $N_t$  and  $N_z$  are number of teeth and number of axial disks of the tool respectively.

### Static Deflection of a Cutting Tool

Static deflections for each cutting edge are considered when developing the ratio  $R_f$  to estimate runout magnitude. The method to determine individual static tooth deflection was originally investigated by Doherty et. Al [8]. Static deflection for a given cutting edge is defined as the following:

$$\delta = \frac{F_r L^3}{3EI} \quad (5)$$

where  $F_r$  is the radial force component acting on a cutting edge, and  $L$ ,  $E$  and  $I$  are the tool length, elastic modulus and moment of inertia respectively. An expression for the radial force can be substituted into Equation (5), and an expression for the deflection of each tooth of a cutter can be determined. As an example, the deflection for tooth 1 of a 4 flute cutter is found to be:

$$\delta_1 = \frac{f_t \cdot \sin(\phi_{ent}) + (R_4 - R_1) + \delta_4}{\Gamma + 1} \quad (6)$$

$$\Gamma = 3EI / (K_R K_T L^3 \Delta a) \quad (7)$$

where  $\Gamma$  is a dimensionless ratio relating tool stiffness to the total cutting force on the tool,  $K_T$  and  $K_R$  are experimentally determined cutting coefficients (see [8] for determination of  $K_T$  and  $K_R$ , coefficients that are dependent on average chip thickness),  $\phi_{ent}$  is the entrance angle of the cutter into the workpiece, and  $R_4$  and  $R_1$  are the radii of the 4<sup>th</sup> and 1<sup>st</sup> teeth including the effect of runout.

Deflection equations for each tooth can be derived in a similar fashion. From these equations, a closed form expression can be determined for each tooth [8]. For example, for a 4 flute cutter, the expression for the deflection of tooth 1 is found to be:

$$\delta_1 = \frac{(\Gamma+1)^3 (f'_t + R_{14}) + (\Gamma+1)^2 (f'_t + R_{43}) + (\Gamma+1) (f'_t + R_{32}) + f'_t + R_{21}}{(\Gamma+1)^4 - 1} \quad (8)$$

where  $f'_t$  is equal to  $f_t \cdot \sin(\phi_{ent})$ , and  $R_{14}$  represents the difference between  $R_1$  and  $R_4$ . Deflections of the remaining 3 teeth are all determined in the same fashion. Expressions for two, three, and six tooth cutters can be derived in a similar manner.

**SPECTRAL FEED FORCE ANALYSIS**  
**Spectral Feed Force Components**

A time domain force signal can be decomposed into individual spectral force components using a Fast Fourier Transform (FFT) [9]. Force signatures are comprised primarily of force magnitudes at the spindle and tooth passing frequencies. The effect of runout has been shown in prior work [1,2,10] to generate force variations at the spindle frequency.

The FFT force components at the spindle and tooth passing frequency for light peripheral cuts can be estimated from the time domain force profiles. The amplitude of the force at the spindle frequency is estimated as:

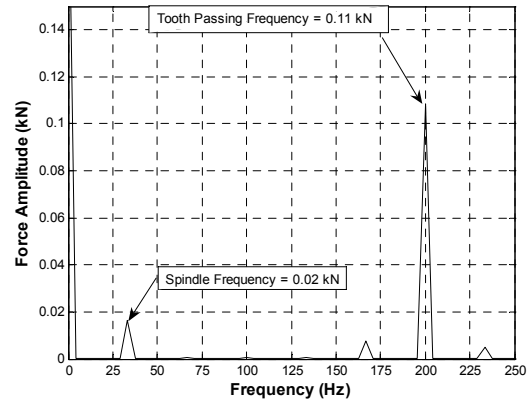
$$F_{xs} = \frac{1}{2} [F_{x,max} - F_{x,min}] \tag{9}$$

where  $F_{x,max}$  and  $F_{x,min}$  are the maximum and minimum peak forces as shown in Figure 3. A case with zero runout would result in no difference of magnitude between maximum and minimum peak forces. The force amplitude at the tooth passing frequency is estimated as:

$$F_{xt} = \frac{1}{2} [F_{x,max} + F_{x,min}] \tag{10}$$

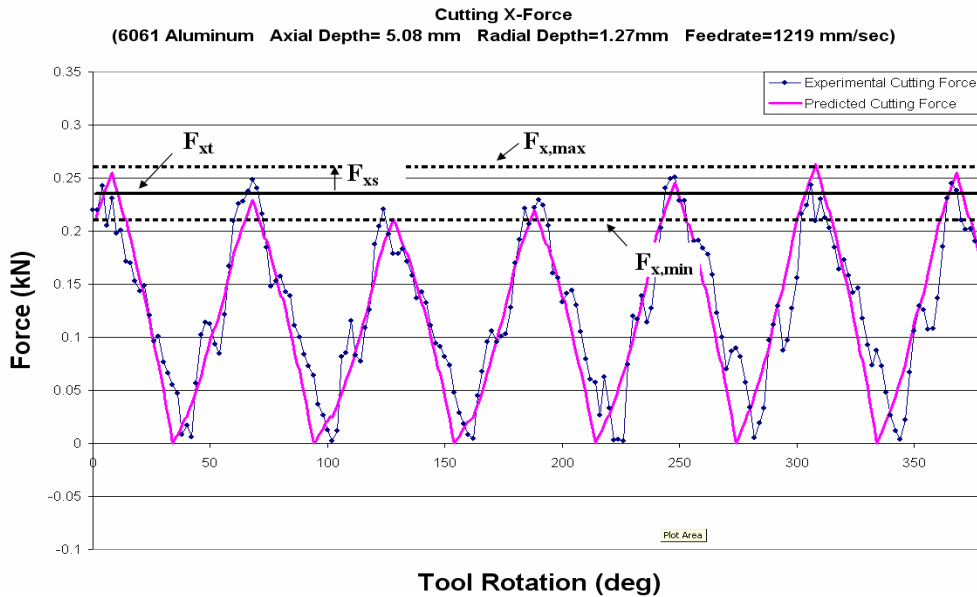
This estimation is reasonable for light peripheral cuts, where the force drops to zero after each tooth engagement. Simulation and experimental results of light peripheral cuts are used to verify that time domain peak forces can be used in estimating the spindle and tooth passing frequency force magnitudes. Experimental forces are measured with a 3-axis Kistler dynamometer, model 9257B. The dynamometer is

aligned with the workpiece so that the feed force is in the x-direction of the cut as shown in Figure 2. The experimental test cut in Figure 3 has magnitudes at the spindle and tooth passing frequencies of 0.02 kN and 0.11 kN respectively (Figure 4). The amplitude of  $F_{xs}$  in the time domain profile is equal to 0.025 kN. This value is roughly equal to the magnitude of the FFT at the spindle frequency. The amplitude of  $F_{xt}$  in the time domain profile is 0.12 kN, which is approximately equal to the magnitude of the FFT at the tooth passing frequency. Discrete force signals are processed such



**Figure 4: Fourier analysis of the experimental 6 flute peripheral cut shown in Figure 3**

that the magnitude of the FFT signal is the actual force magnitude [9].



**Figure 3: Experimental and estimated cutting force for 6 flute down milling peripheral cut (6061 aluminum, feed rate 1219 mm/sec 2000 rpm)**

**Table 1: Force harmonic FFT magnitudes are compared to estimated values using simulated time domain peak force values in Equations 5 and 6.**

Runout	Axial Immersion	Feed Per tooth	Force at spindle freq. $\omega_{sp}$ (FFT)	Force at tooth freq. $\omega_{Tp}$ (FFT)	Estimated force at spindle freq. (Equation 9)	Estimated force at tooth freq. (Equation 10)
$\mu\text{m}$	mm	$\mu\text{m}/\text{tooth}$	kN	kN	kN (error %)	kN (error %)
16.2	2.5	76.2	0.03	0.062	0.035 (14)	0.064 (3.5)
32.4	2.5	127	0.04	0.06	0.060 (33)	0.066 (09)
6.7	5.1	101.6	0.01	0.07	0.010 (00)	0.071 (02)
32.4	5.1	101.6	0.035	0.07	0.055 (36)	0.071 (02)
6.7	7.6	76.2	0.026	0.08	0.027 (04)	0.087 (08)

Further verification is provided by simulation. Five cuts are simulated with the conditions and results given in Table 1. The locating angle is assumed to be zero in all tests. The results in Table 1 indicate that the estimation of force magnitude at the tooth passing frequency is accurate, but that the estimation at the spindle frequency is not that accurate for large runouts, e.g. 32.4  $\mu\text{m}$ . For small runouts, the method is accurate.

### RUNOUT ESTIMATION FROM A NON- DIMENSIONAL RATIO OF CUTTING FORCES

Effective cutter runout can be estimated by comparing an experimentally calculated ratio  $R_{fm}$  to a force model ratio  $R_f$ . Experimental FFT components of the time domain feed force are used to calculate the ratio. A closed form expressions for  $R_f$  is developed for a cutting tool and compared to the experimental ratio. The runout is then estimated from this comparison. The assumption of a perfectly rigid tool allows for the cancellation of cutting parameters  $K_{tc}$  and  $K_{te}$  in equations. A more general solution requires consideration of tool flexibility in the chip thickness equation. We incorporate this effect in the calculation of the ratio  $R_f$  as discussed in the following section.

### Non-Dimensional Force Ratio

A ratio proportional to runout can be derived based upon the difference between peak forces and the average peak forces:

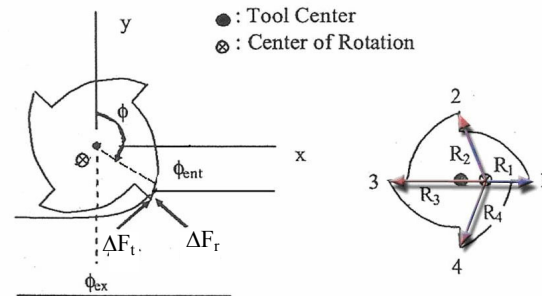
$$R_f = \frac{F_{ts}}{F_{tt,2} - F_{tt,1}} \quad (11)$$

where  $F_{ts}$  and  $F_{tt}$  are defined as:

$$F_{ts} = \frac{1}{2} [F_{t,\max} - F_{t,\min}] \quad (12)$$

$$F_{tt} = \frac{\frac{1}{2} [F_{t,\max} + F_{t,\min}]}{2} \quad (13)$$

The expressions for  $F_{ts}$  and  $F_{tt}$  are the same as given in Equations (9) and (10), with the feed force component  $F_x$  replaced by the tangential force component  $F_t$ . For light peripheral cuts, this introduces an acceptable error in the estimation of the spectral components of the feed force. The error worsens with larger radial immersion cuts. Equation (12) and (13) approximate the amplitudes of the feed force at the spindle frequency and the tooth frequency. The difference of forces in the denominator of Equation (11) eliminates the edge forces thereby eliminating the need for  $K_{te}$  (see Equation (1)). The subscripts of  $F_{tt}$  refer to the two feedrates used in calculating  $F_{tt,1}$  and  $F_{tt,2}$ . The ratio of forces in Equation (11) eliminates the need for the force model coefficient  $K_{tc}$ .



**Figure 5: Tangential and radial components for low radial and axial immersion cuts, and effective tool radius definitions for 0 degree locating angle**

A final expression for Equation (11) is developed for a four tooth cutter. Expressions for other cutters, e.g. 6 tooth, can be developed in a similar manner. The maximum and minimum chip thicknesses for all the teeth engaged in a light peripheral cut with cutter runout and deflection (Figure 5) can be expressed as:

$$h_{\max} = f_t \cdot \sin(\phi_{ent}) - (R_2 - R_1) + \delta_2 - \delta_1 \quad (14)$$

$$h_{\min} = f_t \cdot \sin(\phi_{ent}) - (R_2 - R_3) + \delta_4 - \delta_3 \quad (15)$$

where  $\phi_{ent}$  is the tooth entrance angle,  $f_t$  is the cutter feed-per-tooth,  $R_i$  is the effective radius of the  $i^{th}$  tooth, and  $\delta_i$  is the radial deflection of the  $i^{th}$  tooth. As seen in Figure 5, the runout locating angle is selected as 0 degrees, in line with tooth 3. Since the methodology described in this paper cannot predict the locating angle, a worse case condition is selected for analysis.

Substituting Equations (14) and (15) into the tangential force equation (equation (1)), and assuming a small axial depth of cut, yields expressions for the maximum and minimum peak tangential forces:

$$F_{t,max} = K_{tc} [f_t \sin(\phi_{ent}) - (R_2 - R_1) + \delta_2 - \delta_1] \Delta a + K_{te} \Delta a \quad (16)$$

$$F_{t,min} = K_{tc} [f_t \sin(\phi_{ent}) - (R_4 - R_3) + \delta_4 - \delta_3] \Delta a + K_{te} \Delta a \quad (17)$$

where  $\Delta a$ , the tooth length, is equal to the axial depth of cut.

Substituting the maximum and minimum peak forces into ratio  $R_f$  along with the deflection relationships (Equation (10)) results in the following expression for a four tooth cutter:

$$R_f(\rho) = \frac{2 \cdot \rho}{\sin(\phi_{ent}) \cdot (f_{t2} - f_{t1})} \cdot \Psi \quad (18)$$

where

$$\Psi = \frac{1}{1 + \frac{2}{\Gamma} + \frac{2}{\Gamma^2}} \quad (19)$$

The effect of deflection is contained in the  $\Psi$  term. Assuming a perfectly rigid tool with an infinitely high elastic modulus,  $\Gamma$  becomes infinity, and  $\Psi$  reduces to 1. For most light cuts, tool deflection is not significant; however, for long slender tools  $\Psi$  can be significant when using Equation (18) to solve for runout. Unfortunately, incorporating deflection in  $R_f$  requires the use of mechanistic cutting parameters. All runout estimations in this research utilize the deflection term  $\Psi$  in order to evaluate the general expression given in Equation (18).

As a further justification for equating the tangential force to the feed force, consider a light peripheral cut with the cutter entrance angle equal to 160 degrees. Using Equation (3), and some typical model constants, it can be shown that the tangential force is approximately 85 percent of the total x force. Fortunately, when the forces are substituted into the ratio expression, the numerator and denominator tend to cancel this effect, resulting in a smaller error.

### Runout Estimation using Ratio $R_f$

Runout for a four tooth cutter can be estimated by comparing an experimentally determined ratio to the closed form ratio expression given in Equation (18). An experimentally measured ratio ( $R_{fm}$ ) is determined by dividing the spindle frequency FFT amplitude by the difference in the tooth passing frequency FFT components at two feedrates. The assumption in this analysis is that the spindle frequency

FFT amplitude is equal to  $F_{ts}$  and the tooth passing component is equal to  $F_{tt}$ . Once the experimental ratio  $R_{fm}$  is determined, it can be substituted into the ratio from Equation (18), and runout magnitude can be calculated. A ratio expression has also been developed for a six tooth cutter.

Runout estimations for both 4 and 6 flute cutters are compared to measured runout to determine the accuracy of the method. See Table 2, 3 and 4 for the various test process conditions, feed per tooth conditions used in the estimations, and estimations results.

**Table 2: Runout estimation results for a 4 flute helical end mill, axial depth = 4.3mm, radial depth = 1.27mm**

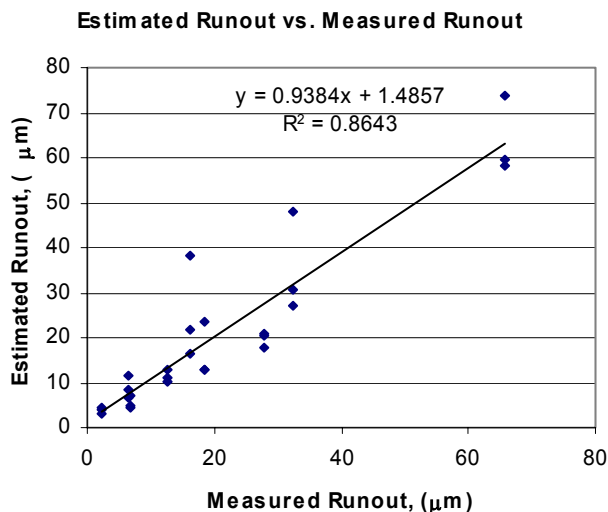
Test	Measured Runout ( $\mu\text{m}$ )	Estimated Runout ( $\mu\text{m}$ )	Feed per tooth 1 $f_{t1}$ ( $\mu\text{m}$ )	Feed per tooth 2 $f_{t2}$ ( $\mu\text{m}$ )
5	2.4	4.3	25.4	38.1
6	6.35	11.4	25.4	38.1
7	12.7	11.3	25.4	38.1
8	18.3	13.0	25.4	38.1
9	27.9	21.0	25.4	38.1
5	2.4	3.3	25.4	50.8
6	6.35	6.9	25.4	50.8
7	12.7	10.2	25.4	50.8
8	18.3	12.7	25.4	50.8
9	27.9	17.7	25.4	50.8
5	2.40	4.1	38.1	50.8
6	6.35	8.63	38.1	50.8
7	12.7	12.9	38.1	50.8
8	18.3	23.6	38.1	50.8
9	27.9	20.3	38.1	50.8

**Table 3: Runout estimation results for a 6 flute helical end mill, axial depth = 2.5mm, radial depth = 1.27mm**

Test	Measured Runout ( $\mu\text{m}$ )	Estimated Runout ( $\mu\text{m}$ )	Feed per tooth 1 $f_{t1}$ ( $\mu\text{m}$ )	Feed per tooth 2 $f_{t2}$ ( $\mu\text{m}$ )
1	6.7	7.1	76.2	101
2	16.2	21.8	76.2	101
3	32.4	48.2	76.2	101
4	65.8	73.6	76.2	101
1	6.7	5.0	76.2	127
2	16.2	16.6	76.2	127
3	32.4	26.9	76.2	127
4	65.8	58.4	76.2	127
1	6.7	4.3	76.2	154
2	16.2	38.1	76.2	154
3	32.4	30.5	76.2	154
4	65.8	59.7	76.2	154

**Table 4: Standard error of estimated runout using Equation 18. The overall standard error of the estimation method is 7.1  $\mu\text{m}$**

Test	Measured Runout ( $\mu\text{m}$ )	Estimated Runout ( $\mu\text{m}$ )	Standard Error ( $\mu\text{m}$ )
1	6.7	7.1, 5.0, 4.3	1.5
2	16.2	21.8, 16.6, 38.1	11.3
3	32.4	48.2, 26.9, 30.5	8.4
4	65.8	73.6, 58.4, 59.7	6.2
5	2.4	4.3, 3.3, 4.1	1.4
6	6.35	11.4, 6.9, 8.6	2.8
7	12.7	11.3, 10.2, 12.9	1.4
8	18.3	13.0, 12.7, 23.6	4.7
9	27.9	21.0, 17.7, 20.3	7.2



**Figure 6: Evaluation of the indirect ratio runout estimation method. All 4 and 6 tooth cutting tests are included**

The results of all the runout estimations are given in the evaluation plot shown in Figure 6. For perfect estimation, all the estimated runout values should fall on a line with a slope of one and a zero intercept. In this case, the best straight line fit gives a slope of 0.94, with a small intercept value. The standard error of the estimated runout from measured runout is 7.1  $\mu\text{m}$ . Assuming the error can be represented by a Gaussian distribution, a 95% probability of bracketing the actual runout is  $\pm$  two standard deviations, or  $\pm$  14.2  $\mu\text{m}$ . Thus, any runout measurement made using this indirect method would have an error band of  $\pm$  14.2  $\mu\text{m}$ . This error is rather large in terms of finding a precise value for runout, however it can be used to determine if the runout of a tool is outside an allowable

limit. Further investigation of Figure 6 shows that three of the data points, all from the 6 flute tests, are far outside the straight line fit. If these points are eliminated from the data set, the standard error drops to 4.5 mm. More testing is needed to see if these points are anomalies and to better define the error statistics of this method. Additional testing is needed with 2, 3 and 5 tooth cutters.

Several observations can be made about the cutting tests used in obtaining the time domain and spectral components of the force. Any chatter or excessive tool vibrations make it difficult to obtain the peak forces and the force harmonics. In general, it is better to take light cuts that reduce the chance for excessive tool deflection and the possibility that one tooth becomes disengaged from the workpiece. If tool runout is small, it does not take much tool deflection for it to become significant in the chip thickness calculation. A check on the significance of deflection can be made by estimating  $\delta$ , before cutting, using a deflection equation similar to that given for the 4 flute cutter.

## CONCLUSIONS

An indirect runout estimation method that uses the spectral feed force components has been investigated. An experimental ratio is developed using the cutting force FFT magnitudes at the tooth passing frequency and the spindle frequency. A theoretical ratio is derived from force models and cut geometry and equated to the experimental ratio to estimate runout. Cutting tests with 4 and 6 tooth cutters resulted in estimations with a standard deviation of 7.1  $\mu\text{m}$ . The results for the 4 tooth cutter were more accurate than those for the 6 tooth. Additional testing is necessary on other cutters, e.g. 2, 3 and 4 tooth, to determine the accuracy and repeatability of this indirect measurement method.

## ACKNOWLEDGMENTS

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