Balaji, C., and Venkateshan, S. P., 1994, "Combined Surface Radiation and Free Convection in Cavities," *AIAA Journal of Thermo Physics and Heat Transfer*, Vol. 8, pp. 373–376.

Gosman, A. D., Pun, W. M., Runchal, A. K., Spalding, D. B., and Wolfshtein, M., 1969, *Heat and Mass Transfer in Recirculating Flows*, Academic Press, London.

Nishimura, T., Shiraishi, M., Nagasawa, F., and Kawamura, Y., 1988 "Natural Convection Heat Transfer in Enclosures With Multiple Vertical Partitions," *Int. J. Heat Mass Transfer*, Vol. 31, No. 8, pp. 1679–1686.

Sri Jayaram, K., 1996, "Interaction of Surface Radiation and Free Convection in Partitioned Enclosures," M.S. thesis, Indian Institute of Technology, Madras, India.

Reexamination of the Transmittance Formulae of a Lamina

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Expressions for the optical properties of laminated layers have drawn much attention from researchers in recent years because of emerging optoelectronic applications (Chen and Tien, 1992; Cunsolo et al., 1992; Zhang and Flik, 1993, Engelbrecht, 1994; Grossman and McDonald, 1995; Anderson and Bayazitoglu, 1996). Different equations must be applied in particular situations (Chen and Tien, 1992; Zhang, 1994). After carefully examining the transmittance formulae of a lamina, this work shows that the geometric-optics formula may result in a significant error for a highly absorbing medium even in the incoherent limit (when interference effects are negligible).

Introduction

Consider the transmission of electromagnetic radiation through a lamina with smooth and parallel surfaces. In the incoherent limit when radiation coherence length is much smaller than the thickness of the lamina, the transmittance (or reflectance) may be obtained either by tracing the multiply reflected radiant power fluxes (ray-tracing method) or by separating the power flux at each interface into an outgoing component and an incoming component (net-radiation method), viz. (Siegel and Howell, 1992)

$$T = \frac{(1-\rho)^2\tau}{1-\rho^2\tau^2} \tag{1}$$

where ρ is the reflectance at the interface and τ is the internal transmittance. This formula is also called the geometric-optics formula since it is obtained without considering interference effects. For a plane wave, ρ equals the square of the absolute value of the complex Fresnel reflection coefficient (i.e., the ratio of the reflected electric field to the incident electric field at the interface). The Fresnel reflection coefficient is (Heavens, 1965)

$$r_{12} = \begin{cases} \dfrac{N_1 \cos\theta_2 - N_2\cos\theta_1}{N_1\cos\theta_2 + N_2\cos\theta_1}, & \text{for } p - \text{polarization} \\[2ex] \dfrac{N_1\cos\theta_1 - N_2\cos\theta_2}{N_1\cos\theta_1 + N_2\cos\theta_2}, & \text{for } s - \text{polarization} \end{cases} \tag{2}$$

where θ_1 is the angle of incidence, $N_1 = 1$ is the refractive index of air or vacuum, $N_2 = n + i\kappa$ is the complex refractive index of the lamina material, and θ_2 is the (complex) angle of refraction, which is related to θ_1 by Snell's law: $N_1 \sin\theta_1 = N_2 \sin\theta_2$. Since $r_{21} = -r_{12}$, the reflectance at both interfaces is equal to $\rho = r_{12}r_{12}^*$, where * denotes the complex conjugate. The internal transmittance τ is related to the (complex) phase change δ by $\tau = \exp[-2\,\mathrm{Im}(\delta)]$, where Im denotes the imaginary part. The phase change inside the lamina is

$$\delta = \frac{2\pi d}{\lambda} N_2 \cos\theta_2 \tag{3}$$

where d is the lamina thickness and λ is the wavelength in vacuum.

In the coherent limit, the transmittance of a lamina may be obtained from thin-film optics (i.e., wave optics) either by tracing the reflected and transmitted waves (Airy's method) or by separating the electric fields into a forward-propagating component (forward wave) and a backward-propagating component (backward wave), viz. (Heavens, 1965; Born and Wolf, 1980; Yeh, 1988)

$$T = \frac{[1+\rho^2 - 2\,\mathrm{Re}(r_{12}^2)]\tau}{1+\rho^2\tau^2 - 2\tau\,\mathrm{Re}(r_{12}^2 e^{i2\phi})} \tag{4}$$

where $\phi = \mathrm{Re}(\delta)$ is the real part of the phase change. The resulting spectral transmittance oscillates because of interference effects.

As pointed out by Cunsolo et al. (1992), integrating Eq. (4) over a period of oscillation yields

$$T = \frac{[1+\rho^2 - 2\,\mathrm{Re}(r_{12}^2)]\tau}{1-\rho^2\tau^2} = \frac{(1-\rho)^2\tau}{1-\rho^2\tau^2} + \frac{4[\mathrm{Im}(r_{12})]^2\tau}{1-\rho^2\tau^2}. \tag{5}$$

Although the second term at the right is often very small, the above equation is different from Eq. (1). It is worthwhile to investigate the physical origin of this discrepancy and to discuss practical situations where Eq. (5) should be used instead of Eq. (1).

Analysis and Discussion

The power transmittance at the interface between the air (or vacuum) and the medium (lamina) is (Yeh, 1988)

$$T_{12} = \frac{\mathrm{Re}(N_2\cos\theta_2)}{\mathrm{Re}(N_1\cos\theta_1)}(1+r_{12})(1+r_{12}^*) \tag{6}$$

where $(1 + r_{12})$ is the Fresnel transmission coefficient. The power transmittance at the second interface between the medium and the air can be obtained by exchanging the subscripts 1 and 2 in Eq. (6). At normal incidence, $r_{12} = (1 - n - i\kappa)/(1 + n + i\kappa)$ and $\rho = [(n-1)^2 + \kappa^2]/[(n+1)^2 + \kappa^2]$. Therefore, $T_{12} = 4n/[(n+1)^2 + \kappa^2] = 1 - \rho$ and,

$$T_{21} = \frac{1}{n}\frac{4(n^2+\kappa^2)}{(n+1)^2+\kappa^2} = 1 - \rho + \frac{2\kappa\,\mathrm{Im}[r_{21}]}{n}. \tag{7}$$

If both κ and $\mathrm{Im}(r_{21})$ are nonzero, $T_{21} \neq 1 - \rho$. As discussed by Salzberg (1948) and Knittl (1976), this inequality is caused by the interference effect between the reflected wave and the incident wave at the interface for radiation incident from an absorbing medium. No incident power flux and reflected power flux can be defined in an absorbing medium because of the coupling between the incident and the reflected waves, although it is always possible to separate the electric field into forward and backward components. In addition to the assumption that the interference effects are negligible, geometric optics implies that the power flux in the medium equals the sum of the power fluxes of the forward wave and the backward wave, i.e., the coupling term is negligible.

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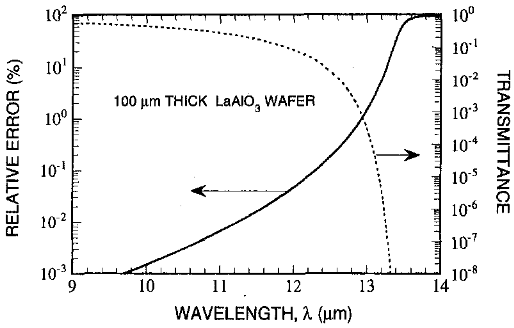

Fig. 1 Relative error of Eq. (1) and the transmittance of a $LaAlO_3$ lamina calculated using Eq. (8) at normal incidence

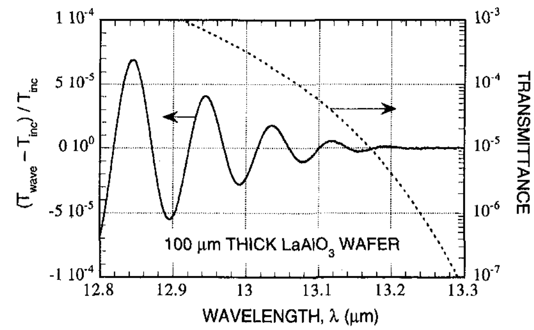

Fig. 2 Comparison between the wave-optics formula, Eq. (4) for T_{wave}, and the incoherence formula, Eq. (8) for T_{inc}. The transmittance value is also calculated from Eq. (8)

Zhang (1994) derived an equation for the transmittance in the incoherent limit using partial-coherence theory:

$$T = \frac{T_{12}T_{21}\tau}{1 - \rho^2\tau^2}. \tag{8}$$

The above equation is identical to Eq. (5). However, it is not a simple replacement of $(1 - \rho)^2$ in Eq. (1) with $T_{12}T_{21}$, because the ray-tracing method and the net-radiation method inherently assume that $T_{12} = T_{21} = 1 - \rho$ in order to satisfy the first law of thermodynamics. For an absorbing medium, T_{21} cannot be interpreted as the power transmittance at the interface because no incident power flux can be defined in the medium. In fact, T_{21} could be greater than 1. Hence, Eq. (8) is distinct from the geometric-optics formulation. All the above equations, except Eq. (7), are applicable to both normal and oblique incidence. The partial-coherence formulation was verified by Anderson and Bayazitoglu (1996) for arbitrary angles of incidence and polarization states. Taking Eq. (5) or (8) as the exact expression for the incoherent limit, the relative error of Eq. (1) can be evaluated. At normal incidence, the relative error equals $\kappa^2/(n^2 + \kappa^2)$. Since the internal transmittance is $\tau = \exp(-4\pi\kappa d/\lambda)$, the requirements of $\lambda < d$ and transparency of the lamina often exclude large values of κ.

As an example, suppose the lamina is a $LaAlO_3$ wafer of thickness $d = 100$ μm. The optical constants are calculated from the Lorentian dielectric function determined by Zhang et al. (1994). At 1 $\mu m < \lambda < 11$ μm, $\kappa < 0.01$ and $n > 1$. The relative error is less than 0.01 percent, which is smaller than the uncertainty of most experiments. The relative error of Eq. (1) and the transmittance for a $LaAlO_3$ lamina at wavelengths from 9 to 14 μm at normal incidence are shown in Fig. 1. As the wavelength increases, the relative error increases, but the transmittance decreases because of a decreasing n and an increasing κ. The relative error of Eq. (1) is less than 1 percent for $\lambda < 13$ μm, where the transmittance is greater than 0.001. The error of Eq. (1) becomes substantial for very low transmittance.

The difference between the wave-optics formula and the incoherent formula is shown in Fig. 2, where T_{wave} and T_{inc} are calculated using Eqs. (4) and (8), respectively. Because of an increasing κ, the strength of oscillation decreases as the wavelength increases. Notice that $N_2 = (0.8 + i\ 0.06)$ at 12.8 μm and $(0.44 + i\ 0.17)$ at 13.3 μm for $LaAlO_3$. The agreement between the incoherent formula and the wave-optics formula in the case with strong absorption further confirms the applicability of Eq. (5) or (8). Similar trends can be shown for other dielectric materials and/or in different spectral regions.

For a highly absorbing lamina (i.e., $\tau \ll 1$), multiple reflections may be neglected. The transmittance obtained from Eq. (1), when multiple reflections are negligible, is $(1 - \rho)^2\tau$. The transmittance calculated from Eq. (8) for $\tau \ll 1$ is

$$T = T_{12}T_{21}\tau = \frac{16(n^2 + \kappa^2)\tau}{[(n + 1)^2 + \kappa^2]^2} \tag{9}$$

where the last expression is for normal incidence only. Eq. (9) agrees with the wave-optics equation for $\tau \ll 1$ (Born and Wolf, 1980). The error of using $(1 - \rho)^2\tau$, instead of Eq. (9), can be substantial for a metallic film since κ is on the same order of n. Take a 50 nm thick free-standing gold film at 2 μm as an example. Using $n = 0.47$ and $\kappa = 12.5$, from Siegel and Howell (1992), the normal transmittance calculated from Eq. (9) is $\approx 2 \times 10^{-3}$, whereas $(1 - \rho)^2\tau \approx 2.8 \times 10^{-6}$. The nearly three orders of magnitude discrepancy is caused by the difference between $T_{21} = 8.4$ and $1 - \rho = 0.012$.

Concluding Remarks

By inspecting the energy balance at the second interface, this work reveals an implicit assumption associated with Eq. (1), that is, the power flux equals the sum of the power fluxes of the forward wave and the backward wave. Using the expressions given by Eqs. (5) and (8), the relative error of Eq. (1) is evaluated. Corrections are rarely needed since $\kappa \ll n$ for most engineering applications when interference effects may be neglected. For a highly absorbing lamina however, the power transmittance and reflectance at the second interface cannot be defined. Hence, for applications that involve very low transmittance laminae, Eq. (1) is inappropriate even though interference effects are negligible.

Certain important applications require the determination of transmittance below 10^{-4}. Examples are in the characterization of attenuation filters, bandpass filters, and materials with strong absorption bands (Frenkel and Zhang, 1994; Zhang et al., 1995a, b). Infrared transmittance as low as 10^{-10} can be measured using modern spectrometric and laser techniques as reviewed by Gentile et al. (1995).

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References

Anderson, C. F., and Bayazitoglu, Y., 1996, "Radiative Properties of Films Using Partial Coherence Theory," *Journal of Thermophysics and Heat Transfer*, Vol. 10, pp. 26–32.

Born, M., and Wolf, E., 1980, *Principles of Optics*, 6th ed., Pergamon Press, Oxford, UK, chap. 1, pp. 36–66, and chap. 13, pp. 627–633.

Chen, G., and Tien, C. L., 1992, "Partial Coherence Theory of Thin Film Radiative Properties," ASME Journal of Heat Transfer, Vol. 114, pp. 636–643.

Cunsolo, S., Dore, S., and Varsamis, C. P., 1992, "Refractive Index of Crystals from Transmission and Reflection Measurements: MgO in the Far-Infrared Region," *Applied Optics*, Vol. 31, pp. 4554–4558.

Engelbrecht, J. A. A., 1994, "Modelling of the Reflectance of Silicon," *Infrared Physics and Technology,* Vol. 35, pp. 701–708.
Frenkel, A., and Zhang, Z. M., 1994, "Broadband High Optical Density Filters in the Infrared," *Optics Letters,* Vol. 19, pp. 1495–1497.
Gentile, T. R., Frenkel, A., Migdall, A. L., and Zhang, Z. M., 1995, "Neutral Density Filter Measurements at the National Institute of Standards and Technology," Spectrophotometry, Luminescence and Colour; Science and Compliance, C. Burgess and D. G. Jones, eds., Elsevier, Amsterdam, The Netherlands, pp. 129–139.
Grossman, E. N., and McDonald, D. G., 1995, "Partially Coherent Transmittance of Dielectric Lamellae," *Optical Engineering,* Vol. 34, pp. 1289–1295.
Heavens, O. S., 1965, *Optical Properties of Thin Solid Films,* Dover Publications, Inc., New York, chap. 4, pp. 46–95.
Knittl, Z., 1976, *Optical of Thin Films,* John Wiley & Sons, Inc., NY, pp. 203–204.
Salzberg, B., 1948, "A Note on the Significance of Power Reflection," *American Journal of Physics,* Vol. 16, pp. 444–446.
Siegel, R., and Howell, J. R., 1992, *Thermal Radiation Heat Transfer,* 3rd ed., Hemisphere Publishing Corp., Washington D.C., chap. 4, p. 120, and chap. 18, pp. 928–930.
Yeh, P., 1988, *Optical Waves in Layered Media,* John Wiley & Sons, Inc., New York, chap. 4, pp. 83–101.
Zhang, Z. M., 1994, "Optical Properties of Layered Structures for Partially Coherent Radiation," *Heat Transfer 1994—Proceedings of the Tenth International Heat Transfer Conference,* Vol. 2, G. F. Hewitt, ed., Institution of Chemical Engineers, Rugby, UK, pp. 177–182.
Zhang, Z. M., and Flik, M. I., 1993, "Predicted Absorptance of $YBa_2Cu_3O_7$/YSZ/Si Multilayer Structures for Infrared Detectors," *IEEE Transactions on Applied Superconductivity,* Vol. 3, pp. 1604–1607.
Zhang, Z. M., Choi, B. I., Flik, M. I., and Anderson, A. C., 1994, "Infrared Refractive Indices of $LaAlO_3$, $LaGaO_3$, and $NdGaO_3$," *Journal of the Optical Society of America B,* Vol. 11, pp. 2252–2257.
Zhang, Z. M., Datla, R. U., and Hanssen, L. M., 1995a, "Development of Neutral-Density Infrared Filters Using Metallic Thin Films," *MRS Symp. Proc.,* Vol. 374, pp. 117–122.
Zhang, Z. M., Hanssen, L. M., and Datla, R. U., 1995b, "High-Optical-Density Out-of-Band Spectral Transmittance Measurements of Bandpass Filters," *Optics Letters,* Vol. 20, pp. 1077–1079.

Thermal Convection in an Infinite Porous Medium Induced by a Heated Sphere

R. Ganapathy [1]

This paper investigates the transient behavior of the free convection motion and heat transfer induced by a heated sphere with prescribed wall temperature embedded instantaneously in an infinite porous medium. Solutions for the velocity and temperature fields have been obtained in the form of series expansions in Rayleigh number which is based on the medium permeability and the temperature of the sphere. All discussions are based on the assumption that the flow is governed by Darcy's law and the thermal Rayleigh number is small.

Nomenclature

a = radius of the sphere (L)
c_p = specific heat of fluid at constant pressure $(L^2T^2\Theta^{-1})$
f = function of η (Eq. 13)
G = function of η (Eq. 18)
g = gravitational acceleration (LT^{-2})
K = medium permeability (L^2)
L = length
Q = prescribed wall temperature on the sphere (θ)
R = nondimensional radial coordinate
R_o = nondimensional radius of the sphere $(a/\sqrt{K})$
Ra = thermal Rayleigh number, $(\beta g K\sqrt{K}/\alpha\nu)Q$
Re = Reynolds number, (UL/ν)
r = radial coordinate (L)
T = temperature (θ)
t = time (T)
U = nondimensional radial velocity
u = radial velocity (LT^{-1})
V = nondimensional transverse velocity
v = transverse velocity (LT^{-1})

Greek Symbols

α = effective thermal diffusivity of the porous medium $[L^2T^{-1}]$
β = coefficient of thermal expansion $[\theta^{-1}]$
Θ = azimuthal angle
ϕ = meridian angle
η = similarity variable, $R/2\sqrt{t}$
η_o = similarity variable $R_o/2\sqrt{t}$
ν = kinematic viscosity (L^2T^{-1})
σ = heat capacity ratio (eq. 2)
ψ = stream function $[L^3T^{-1}]$

Subscripts

0 = zeroth-order solution
1 = first-order solution
∞ = reference state

1 Introduction

Since the work of Yamamoto (1974), there has been a spate of research papers on thermal convection due to the presence of heated spheres in saturated porous media. Most of them, with the possible exception of the work of Ganapathy and Purushothaman (1990), were primarily concerned with steady-state solutions only. However, as the analysis of such flows is essential for the solution of many engineering problems such as the hydrodynamics of weak thermal explosions, cooling of the components of electrical and electronic equipment, and the management of nuclear waste, a knowledge of the transient behavior of the flow and heat transfer becomes necessary, especially when the heated sphere is buried instantaneously. It is towards this end that we propose to present a solution to this problem of transient convection due to the presence of a heated sphere embedded instantaneously in an unbounded porous medium and investigate the ensuing flow field and heat transfer in the context of thermal flows in porous media.

2 Mathematical Formulation

We consider the natural convection around a heated sphere of radius a and of constant temperature Q (in excess of the reference temperature), buried instantaneously in an unbounded fluid-saturated porous medium of low permeability. The medium is assumed to be rigid, homogeneous, and isotropic, and the fluid saturating the medium is assumed to be Boussinesq incompressible.

A spherical-polar coordinate system (r, ϕ, Θ) is chosen (Fig. 1), with the origin of the system at the centre of the sphere and the axis $\phi = 0$ vertically upwards. Taking advantage of the continuity equation, we define a stream function ψ such that

$$u = (r^2 \sin\phi)^{-1}\partial\psi/\partial\phi, \quad v = (r\sin\phi)^{-1}\partial\psi/\partial r \qquad (1)$$

where u and v are the radial and transverse components of velocity and introduce the nondimensional quantities (see nomenclature)

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