

The fog of fraud – how to reduce insurance fraud

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Abstract

Most insurance companies publish few data on the occurrence and detection of insurance fraud, although the credible announcement of thoroughly auditing claim reports might act as a powerful deterrence. We show that uncertainty about fraud detection may be an effective strategy to deter ambiguity averse agents from reporting false insurance claims. As committing to a fraud detection strategy eliminates this uncertainty, it might be optimal not to commit and abstain from publishing data on fraud detection.

JEL classifications: D8, K4

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1 Introduction

Fraudulent claims on insurance policies are an important issue for insurers. The extent of insurance fraud varies widely from small overstatements in the claims made to deliberate pretending damages that never occurred or that were especially arranged. Due to the nature of fraud estimating the losses for the insurance industry is not an easy task. Nevertheless the Insurance Information Institute for example estimates that in both 2004 and 2005 insurance fraud amounted to \$30 billion in the US property and casualty insurance market.¹

Therefore the strategies of insurers to deter insurance fraud do matter. However in the mass market and with small claims it is too costly to audit each claim that is made. That's why usually claim reports are scanned for known patterns of fraud and only a certain fraction of these reports is verified in detail. Previous literature, like Picard (1996) or Mookherjee and Png (1989), suggests that it would be in the insurer's interest to announce the level of auditing to deter insurance fraud. Given the expected level of auditing the insured indeed report only few fraudulent claims. However in this case it is optimal for the insurers to audit only few claims which makes their announcement not credible in the first place. The situation changes, if the insurer can credibly commit to a certain

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¹ See Rees and Wambach (2008), p. 135.

level of auditing. Therefore the equilibrium without commitment implies a welfare loss.² Yet few insurers publish their level of auditing and try to overcome the credibility issue by having an industry association scrutinise their level of auditing or using another third-party verification.³ In addition, there is to our knowledge few subsidizing of auditing costs in the sense of a common agency, i.e. the mechanism proposed by Picard (1996), to lower the variable costs of auditing claims in order to solve the credibility problem. Therefore we suggest that there is an additional issue here.

As the insured do not know the exact costs of auditing a claim by the insurers, there is even under rational expectations some uncertainty regarding the level of auditing. This uncertainty makes ambiguity averse agents less inclined to commit insurance fraud. If the insurer commits to a certain level of auditing, this uncertainty however is lost and the level of insurance fraud might increase. Consequently the insurer may want to keep the insured uninformed about the fraud detection strategy.⁴

The rest of the article is organized as follows. Section 2 sets up a stylised model to give an intuition how the uncertainty about the level of auditing might decrease insurance fraud and explains the decision process of the ambiguity averse insured. In addition, we show there that our results are robust to the way ambiguity aversion is modelled. Section 3 explains the main model and its results. Thus it is actually in the insurers' interest to abstain from committing, even if they had the possibility to do so. In section 4 an extension of the model shows that the average level of auditing needed to completely deter insurance fraud is due to the ambiguity lower without commitment. Consequently in addition to the individual incentives not to commit the insurance industry may gain from the uncertainty generated in this way. Section 5 contains the concluding remarks.

2 How uncertainty works

We begin with a stylised model to make clear where the effects we are describing come from. A risk and ambiguity averse agent takes out an insurance with a premium P and coverage q against a possible loss $L > 0$. Without loss of generality we normalise the outside wealth of the agent to 0. The agent is represented by an increasing and concave utility function u . A loss L occurs with probability δ and no loss with probability $1 - \delta$. Given this loss distribution an insured who reports a loss different from L is immediately recognized as a fraudster. However if no loss occurs, the insured can nevertheless claim a loss of L , as the loss is private information of the insured. As we chose a costly state verification model, there is no direct cost or disutility faced by the insured for this behaviour.⁵

The insurer cannot observe the loss directly. She just receives the report of the insured. If she pays out the claim, the insured gets q and therefore in case of fraud ends up with a final wealth w of $q - P$.

However the insurer has a technology to audit a fraction p of all reports for their truth. If the

² This problem also appears in different settings as Border and Sobel (1987) or Melumad and Mookherjee (1989) have shown.

³ For such mechanisms see Dewatripont (1988).

⁴ We were encouraged in this view, when one insurance executive told us that besides being bad publicity communicating detailed data on fighting insurance fraud, like the level of auditing, might induce more insured to give it a try.

⁵ Townsend (1979) already considers such a model, but with deterministic auditing. New approaches are Krasa and Villamil (2000) or Dionne et al. (2008). However in Dionne et al. (2008) the insured faces moral costs for filing a fraudulent claim and the insurer can specify the level of auditing on the observable characteristics of the insured.

insurers audit a report, they know for sure, whether it is true or not. In the case the insurer detects a fraud, she pays no indemnity and the insured has to pay a fine M . The technology is private knowledge to the insurer. The insured only know that some reports will be verified. However the insurer may choose to disclose this fraction p to the insured. In the first case there is uncertainty about the level of auditing. This uncertainty about probabilities due to the lack of relevant information is denoted by ambiguity.⁶

We distinguish therefore ambiguity and risk. With risk there is a known probability distribution, while under ambiguity the exact probabilities are unknown. There are various reasons for this distinction. From a rational point of view it seems appropriate to take into account the amount of information a decision is based on. This point was first made by Ellsberg (1961). In addition, there are empirical observations, like Kunreuther et al. (1995) or Cabantous (2007), that support the hypothesis that the subjective expected utility approach neglects the distinction between risk and ambiguity. Third, there is evidence from neuroeconomics, like Hsu et al. (2005) or Rustichini et al. (2005), that ambiguity and risk are perceived differently. Finally the effects of diversification are limited under ambiguity compared to risk as Marinacci (1999) proves.

To model ambiguity averse agents, Schmeidler (1989) introduced an axiomatisation of the Choquet expected utility, that allows for probability intervals. These intervals are formalised by capacities. A capacity is a non-additive probability measure. Thus the insured judges the level of auditing here to be in the range of $[(1 - \mathcal{A})p, (1 - \mathcal{A})p + \mathcal{A}]$ with a parameter $\mathcal{A} \in [0; 1]$, given their estimates of the auditing costs. The parameter \mathcal{A} describes the amount of ambiguity and the degree of ambiguity aversion of the insured. With $\mathcal{A} = 0$ there is no ambiguity or the insured is ambiguity neutral. Consequently the probability is exactly p . Thus the insured is confident to know the probability of an audit, because for example the insurer committed to a certain level of auditing. The higher \mathcal{A} , the more the insured doubts her assessment about the level of auditing. If $\mathcal{A} > 0$, the agent is ambiguity averse and the probability interval has a strictly positive length. This is formalized by a capacity v of

$$\text{a false claim is audited: } v(w = -P - M) = (1 - \mathcal{A})p \quad (1)$$

$$\text{a false claim is not audited: } v(w = -P + q) = (1 - \mathcal{A})(1 - p) \quad (2)$$

An ambiguity averse agent will consider the probability from the interval, that is the worst for her. An agent who does not care about ambiguity and is therefore ambiguity neutral assigns each event a unique probability and acts as an expected utility maximiser.

2.1 The case with announcement

If the level of auditing is disclosed, there is no ambiguity, $\mathcal{A} = 0$, as the probabilities are known and the expected utility EV of the insured without a loss L has the following form:

$$\int u(w)dp = (1 - p)u(-P + q) + pu(-P - M), \text{ if fraud is committed or } u(-P) \text{ otherwise}$$

⁶ Unfortunately the literature uses various notions. Sometimes ambiguity is called (Knightian) uncertainty or imprecision.

Therefore the insured overstates the loss, if the probability of an audit p is smaller than p' given by $\frac{u(-P+q)-u(-P)}{u(-P+q)-u(-P-M)}$. If $p \geq p'$, all insured will only report true losses. To make the problem interesting we will assume $p' < 1$ in this article.

2.2 The case without announcement

In this case the insured lacks relevant information. The difference to the first case depends on the ambiguity aversion and the amount of ambiguity perceived by the insured. An ambiguity neutral insured, i.e. with $\mathcal{A} = 0$, evaluates her options as before. Otherwise the insured judges according to the capacity v , described above, and attempts to maximise her Choquet expected utility CEV . Consequently she assumes the worst possible distribution that is compatible with her beliefs.⁷ So the probability of detection increases to $(1 - \mathcal{A})p + \mathcal{A}$. Therefore we get:

$$\int u(w)dv = (1 - \mathcal{A})(1 - p)u(-P + q) + ((1 - \mathcal{A})p + \mathcal{A})u(-P - M), \text{ if fraud is committed or} \\ u(-P) \text{ otherwise}$$

This time the threshold is p^* , such that $p^* = \frac{(1-\mathcal{A})u(-P+q)+\mathcal{A}u(-P-M)-u(-P)}{(1-\mathcal{A})[u(-P+q)-u(-P-M)]}$. However it holds $CEV < EV$ in the case of fraud for any given auditing probability. Additionally, CEV is decreasing in p . Therefore we can conclude that $p^* \leq p'$. Thus for lower levels of auditing the insured will abstain from committing fraud. This means that keeping the insured in the dark about the level of auditing may reduce their inclination to commit insurance fraud.

To show that this result is robust to the way ambiguity aversion is modelled we repeat this results with a recent representation of preferences by Klibanoff et al. (2005) allowing for 'smooth' ambiguity aversion. Here ambiguity is captured by second order probabilities. The intuition is that the insured have some theories or models of the world. The trust in each model is denoted by its second order probability. The models themselves assign probabilities to the states of the world. An ambiguity neutral insured simply takes the expectation and derives simple probabilities for each state of the world. If the insured is ambiguity averse, first the expected utility for each model is calculated separately and transformed by a function ϕ . Finally using the second order probabilities the ambiguity expected utility is computed with these values. Thus this representation requires a transformation function ϕ that is increasing and concave. The concavity of this function represents the ambiguity aversion of the insured. So an ambiguity neutral insured is represented by a linear function ϕ , which gives the common expected utility representation. The amount of ambiguity or number of the different models is represented by a set Π of possible values for the first order probability \hat{p} of a claim being audited. In addition, some second order probabilities $\mu(\hat{p})$ for the models are available. The value of p used so far should correspond with the expected value now, i.e. $p = \int_{\Pi} \hat{p}d\mu(\hat{p})$ and it should hold that $p \in \Pi$. Then the ambiguity expected utility AEV of the insured is:

$$\int_{\Pi} \phi [(1 - \hat{p})u(-P + q) + \hat{p}u(-P - M)] d\mu(\hat{p}), \text{ if fraud is committed or } \phi(u(-P)) \text{ otherwise}$$

⁷ This is caused by the convexity of the capacity. For a proof see Schmeidler (1986).

We want to show that the insured abstain from committing fraud, whenever this is the case under risk.

Proof: By Jensen's inequality we get:

$$\begin{aligned} & \int_{\Pi} \phi [(1 - \hat{p})u(-P + q) + \hat{p}u(-P - M)] d\mu(\hat{p}) \leq \\ & \leq \phi \left(\int_{\Pi} (1 - \hat{p})u(-P + q) + \hat{p}u(-P - M) d\mu(\hat{p}) \right) = \\ & = \phi [(1 - p)u(-P + q) + pu(-P - M)] = \phi (EV) \end{aligned}$$

Hence if the average $p = \int_{\Pi} \hat{p} d\mu(\hat{p})$ is at least p' , no insurance fraud is committed. If the second order distribution is non-degenerated, this holds even for lower averages. \square

This confirms our earlier result and shows that it is robust to the way the ambiguity is modelled. Thus additional uncertainty can decrease the inclination of the insured to commit insurance fraud.

Proposition 1: In the case the insurer does not announce the level of auditing and the ambiguity averse insured has not all the relevant information to determine the level of auditing exactly, less insurance fraud is committed than in the case of a (credible) announcement.

The next section sets up the main model in the framework of Picard (1996).

3 The main model

In the main model we have the following timing. First, at $t = 0$ an insurance contract is signed and the insurer decides whether to commit to some auditing level. At $t = 1$ a loss might occur, which is observed only by the insured. Thus at $t = 2$ the insured can make an insurance claim. After that at $t = 3$ the insurer decides whether and to what extent to audit the filed claims. Finally the insurer pays the insurance claim or gets part of the fine if an audited claim was fabricated.

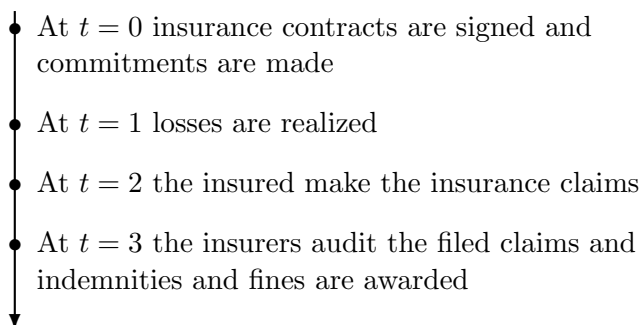


Figure 1: The timing of the model

The model works as follows. The loss distribution is the same as in the last section. A loss L occurs with probability δ . There is a continuum of mass 1 of agents. Each agent can insure her possible loss on a competitive market with free entry for a premium P and a coverage of q . Independent of the realisation of a loss, which is private information, the insured can make any insurance claims. The auditing technology works as before and to audit a claim costs the insurer an amount $k > 0$, that is the

same for all firms.⁸ The exact costs k are private knowledge of the insurer. The insured only knows that $k \in \{k_L, k_H\}$ with $k_L < k_H$ and both strictly positive. An insured who does not care about the ambiguity, expects to face each type of insurer with probability $1/2$ due to the principle of insufficient reason. As she does not know anything, she treats both states of nature symmetrically. An insured who is ambiguity averse also treats both states of nature symmetrically, but does not distribute the whole probability mass in order to allow for errors. This safety margin is denoted by \mathcal{A} as in section 2. We restrict the analysis here to the case of the Choquet expected utility and use the following capacity:

$$v(k_i) = (1 - \mathcal{A})\frac{1}{2}, \quad i \in \{L, H\}$$

Due to the symmetry the capacity assigns both types of insurers the same value. Another approach for this is the one proposed by Gajdos et al. (2008), in which they propose an axiomatic foundation for such a contraction representation. Consequently the insured expects to face rather the low cost insurer depending on her degree of ambiguity aversion.

If a false claim is audited, the insured has to pay a fine M , of which $m \leq M$ is awarded to the insurer that detected the fraud. As in Picard (1996), M and m are exogenous in the model. First we consider the case in which the insurer commits herself to a certain level of auditing in period 0.

3.1 The case with commitment

We solve the model backwards. In period 3 the insurer decides which claims to audit. As the insurer committed to a certain level of auditing p , the audits will be conducted accordingly. In the next step we analyse the decision of the insured in period 2 to make a claim in the case that no loss occurred. The level of auditing has been announced, so the insured does not care about the auditing costs of the insurer. As before insurance fraud is committed, if $(1 - p)u(-P + q) + pu(-P - M) \geq u(-P)$. Consequently the critical value for the level of auditing p is $p' = \frac{u(-P+q)-u(-P)}{u(-P+q)-u(-P-M)}$. If more claims are audited no fraud is committed. For lower levels of p every insured without a loss reports a claim. At the first stage the insurers choose p , depending on the costs of auditing k , to maximise their profits.

An equilibrium⁹ in this game is the same as the one described in Proposition 1 of Picard (1996)¹⁰ and depends on the costs of auditing k . If the costs are high, i.e. $k > k' = \frac{(1-\delta)q}{\delta p'}$, the insurer does not audit any claims and all the insured without a loss claim a loss. In a competitive market the premium P would be set equal to the coverage q . In this case the agents are indifferent between entering an insurance contract or not. If the costs of auditing are low, a fraction p' of all claims is audited and no insurance fraud is committed. If the market is competitive, the premium P would be such that it covers the valid claims, δq , and the expected costs of auditing, $\delta p' k$.

We now turn to the case, if the insurer decides not to commit.

⁸ See Jost (1988) for a model with heterogeneous costs.

⁹ As most effects in the model are linear, there are multiple equilibria, because the insurer or the insured is indifferent at the critical values.

¹⁰ Picard (1996) assumes an exogenously given fraction θ of opportunistic policyholders in an otherwise honest population. σ is the fraction of opportunistic policyholders of a certain insurer. Setting $\theta = \sigma = 1$ resembles our model with credible announcement. However with commitment the insured are homogeneous in our model, such that there is no adverse selection in contrast to Picard (1996).

3.2 The case without commitment

Once again we solve the model backwards. As no commitment was made, the insurer will choose the level of auditing p to maximise her profits given the fraction α of insured without a loss that reported a false claim.¹¹ Consequently, the insurers' expected profit is the premium income P minus reimbursements q for claims that are true or not audited minus the auditing costs k plus the part m of the fines the insurer gets from false claims that were audited:

$$\text{Expected profits} = P - q[\delta + \alpha(1 - \delta)(1 - p)] - k(\delta + \alpha(1 - \delta))p + m\alpha p(1 - \delta)$$

The insurer acts after the insured reported their claims. Thus the level of fraud α is taken as given. Differentiating the expected profits with respect to p we get a critical value $k^*(\alpha)$, such that for $k = k^*(\alpha)$ the insurer is indifferent between auditing or not.

$$k^*(\alpha) = \frac{\alpha(1 - \delta)(q + m)}{\delta + \alpha(1 - \delta)} \quad \text{with} \quad \frac{\partial k^*(\alpha)}{\partial \alpha} > 0 \quad (3)$$

We can rewrite $k^*(\alpha)$ by noting that the insurer's beliefs π in period 2 about a claim to be false is:

$$\pi(\alpha) = \frac{\alpha(1 - \delta)}{\alpha(1 - \delta) + \delta}$$

and thus $k^*(\alpha) = (q + m)\pi(\alpha)$. This means that the costs of auditing are equal to the expected benefits of auditing, i.e. the claims that need not to be paid and the fines awarded to the insurer. If the costs of auditing are lower all claims are audited or none if the costs are higher than $k^*(\alpha)$.

Turning one step back we analyse the decision of the insured in period 2 to make a claim in the case that no loss occurred. Without commitment there is uncertainty due to the lack of information about the costs of auditing faced by the insurers. The insured expects the following probability of auditing conditional on the fraction α of insured willing to report a false claim:

$$E[p|\alpha] = \begin{cases} 1 & \text{if } k_H < k^*(\alpha) \\ p \in [\frac{1}{2}(1 + \mathcal{A}), 1] & \text{if } k_H = k^*(\alpha) \\ \frac{1}{2}(1 + \mathcal{A}) & \text{if } k_L < k^*(\alpha) < k_H \\ p \in [0, \frac{1}{2}(1 + \mathcal{A})] & \text{if } k_L = k^*(\alpha) \\ 0 & \text{if } k_L > k^*(\alpha) \end{cases}$$

We see that the expectation depends on the degree of ambiguity aversion, \mathcal{A} , of the insured. If the insured is ambiguity neutral, i.e. $\mathcal{A} = 0$, she expects to face both types of insurer with equal probability. However the more ambiguity averse she gets, the more probability she puts on being confronted with a low cost insurer. As in section 2 it holds that the insured reports truthfully, if:

$$(1 - E[p|\alpha])u(-P + q) + E[p|\alpha]u(-P - M) \leq u(-P) \quad (4)$$

¹¹ The insurer can derive this fraction α from the number of claims made. Assuming the insurer receives claims from a fraction β of the insured, α equals $(\beta - \delta)/(1 - \delta)$.

We assume a population of agents with different degrees of ambiguity aversion. The degree of ambiguity aversion \mathcal{A} is uniformly distributed on $[0, \bar{A}]$ with $\bar{A} > 0$. The insurers, who know this distribution, cannot observe the degree of ambiguity aversion of an insured. We assume that every insured correctly anticipates the behavior of the other insured. With these assumptions we can describe a Perfect Bayesian equilibrium of the game so far given premiums P and reimbursements q .

Proposition 2: Without commitment an equilibrium has the following form:

- Complete fraud, $\alpha = 1$, and no audits, $p_L = p_H = 0$, if the costs are high, $k_L > k^*(1) = (1 - \delta)(q + m)$
- A level of fraud of $\alpha = \frac{\delta k_L}{(1 - \delta)(q + m - k_L)}$, and a low level of audits $p_H = 0$ and $p_L = \frac{2}{1 + \alpha \bar{A}} p'$, if fraud is not too attractive, i.e. $\frac{1}{2}(1 + \alpha \bar{A}) \geq p' = \frac{u(-P + q) - u(-P)}{u(-P + q) - u(-P - M)}$ and the costs are intermediate, $k_L \leq k^*(1)$
- Complete fraud, $\alpha = 1$, and audits of $p_H = 0$ and $p_L = 1$, if fraud is very attractive $p' \geq \frac{1}{2}(1 + \bar{A})$ and the costs are intermediate, $k_L \leq k^*(1) \leq k_H$
- A level of fraud, $\alpha = \frac{1}{\bar{A}}(2p' - 1) < 1$ and audits of $p_H = 0$ and $p_L = 1$, if fraud is attractive $\frac{1}{2} < p' < \frac{1}{2}(1 + \bar{A})$ and the costs are intermediate, $k_L \leq k^*(\alpha) \leq k_H$
- A level of fraud of, $\alpha = \frac{\delta k_H}{(1 - \delta)(q + m - k_H)} < 1$, and a high level of audits $p_H = \frac{1}{1 - \alpha \bar{A}}(2p' - 1 - \alpha \bar{A})$ and $p_L = 1$, if fraud is very attractive $\frac{1}{2}(1 + \alpha \bar{A}) < p'$ and the costs are low, $k_H < k^*(1)$

Proof: Solving the equilibrium backwards, we consider the insurer setting the level of auditing. Thus we distinguish the following five cases of average auditing $p = (p_L + p_H)/2$: no auditing $p = 0$, low partial auditing $0 < p < 1/2$, partial auditing $p = 1/2$, high partial auditing $1/2 < p < 1$ and complete auditing $p = 1$. If there is in expectation no auditing of claim reports, $p = 0$, every insured will report a claim. Ex post however it will still be optimal to abstain from auditing for the insurer, if the costs of auditing k for both types of insurer are higher than the expected benefit of detecting a fraudster, $(1 - \delta)(q + m)$. This is the first case of the proposition. If the costs are lower, this is not an equilibrium as the insurer does some auditing and the expectations of the insured are not satisfied.

If the level of auditing is low, i.e. $0 < p < 1/2$, the low cost insurer is exactly indifferent between auditing claim reports or not. Thus the high cost insurer will abstain from auditing any claims. Thus we can solve the equilibrium backwards and calculate $\alpha = \frac{\delta k_L}{(1 - \delta)(q + m - k_L)}$ by equation (3) to make the low cost insurer indifferent. The intuition for this result is that we have the ratio of the cost of falsely targeted audits of valid claims and the gains of auditing the fabricated claims, which need to be identical to make the insurer indifferent. This level of fraud determines the necessary level of auditing. Equation (4) gives us the exact level of auditing in equilibrium. This leads to $p_L = \frac{2}{1 + \alpha \bar{A}} \frac{u(-P + q) - u(-P)}{u(-P + q) - u(-P - M)}$ or $\frac{2}{1 + \alpha \bar{A}} p'$. If fraud were too rewarding, i.e. $p' > 1/2(1 + \alpha \bar{A})$, the low cost insurer could not on her own deter enough insured from filing false reports. This holds, because p' is the level of auditing necessary to deter ambiguity neutral insured from filing false claims as we have seen before. Thus if her costs were close to $k^*(1)$, this wouldn't be an equilibrium. If her costs

are higher than $k^*(1)$, it would not be worthwhile to audit any claims for her. This gives us the second part of the proposition.

In the next step we consider an intermediate level of auditing, $p = 1/2$. Here the low cost insurer audits every claim made and the high cost insurer does not audit any claims. Therefore the costs have to be $k_L \leq k^*(\alpha) \leq k_H$. Otherwise this wouldn't be an equilibrium. If fraud is very attractive, i.e. $p' \geq 1/2(1 + \bar{A})$, all insured file claims. This holds even for the most ambiguity averse insured. Consequently there is complete fraud. This is the third case of the proposition. If fraud is less appealing, some insured will abstain from filing false claims. Thus we get an level of fraud $\alpha = \frac{1}{\bar{A}} \frac{u(-P-M)+u(-P+q)-2u(-P)}{u(-P+q)-u(-P-M)} = \frac{1}{\bar{A}}(2p' - 1) < 1$. As long as fraud is not too unrewarding, i.e. $p' > 1/2(1 + \bar{A})$, there will be some fraud and $\alpha > 0$. The fourth part of the proposition describes this equilibrium. If fraud is very unrewarding, no fraud will be committed. However in this case even the low cost insurer will abstain from auditing claims, as $k^*(0) = 0$. Therefore there is no equilibrium in this case.

Assuming more auditing, $p > 1/2$, the inclination to report a false claim decreases. Thus the low cost insurer audits every claim. As the high cost insurer ex post still needs to be indifferent to find this level of auditing optimal, we solve the equation (3) of $k^*(\alpha)$ for the corresponding level of fraud α . This gives us $\alpha = \frac{\delta k_H}{(1-\delta)(q+m-k_H)}$. The intuition is as before. α is smaller than one, if and only if $k_H < (1 - \delta)(q + m)$. It holds $p_L = 1$ and $p_H > 0$. With equation (4) we can now compute the exact level of auditing in equilibrium. This gives us $p_H = \frac{1}{1-\alpha\bar{A}} \left(\frac{u(-P-M)+u(-P+q)-2u(-P)}{u(-P+q)-u(-P-M)} - \alpha\bar{A} \right)$ or $\frac{1}{1-\alpha\bar{A}}(2p' - 1 - \alpha\bar{A})$. If fraud were not attractive enough, i.e. $\frac{1}{2}(1 + \alpha\bar{A}) > p'$, there are not enough false claims in order to make the high type insurer indifferent to audit some claims.

Finally if every claim is believed to be audited, only true claims are reported. However then the best strategy of the insurer ex post is not to audit any reports. Therefore in any equilibrium that does not allow to (credibly) announce the insurer's strategy some insured will report false claims. \square

Thus in equilibrium smaller costs of auditing reduce the level of insurance fraud by increasing the level of auditing. Additionally, the level of auditing and the level of insurance fraud depend negatively on each other. We are going to discuss the effect of the ambiguity parameter \bar{A} in the next paragraph.

Comparing proposition 2 with the results of Picard (1996) we notice the following. For high costs auditing is not worthwhile and the results do not change. When the costs decrease, some claims are audited. However without ambiguity aversion, i.e. for $\bar{A} = 0$, the fourth case is not possible in equilibrium. The appeal of filing a false claim, measured by $p' = \frac{u(-P+q)-u(-P)}{u(-P+q)-u(-P-M)}$ is important for the kind of equilibrium we get. This threshold is determined by the ratio of the utility gains resp. losses of successful and unsuccessful fraud. If fraud is very attractive, there is complete fraud and the low cost type of insurer performs full auditing, while the high cost type abstains completely. In case fraud becomes less attractive the behaviour of the insurers remains the same. However there are some insured that are deterred from filing false claims. If finally fraud is not very interesting for the insured, even the low cost insurers reduce their level of auditing and the fraction of fraudulent claims depends on the costs of the low cost type. This result extends to very low costs. Yet in case of appealing fraud, both types of insurers are auditing claim reports. The low cost insurer audits to full extent and the high cost insurer to some extent. The fraction of fraudulent claims decreases with the costs of the high cost type.

Introducing ambiguity decreases the level of auditing corresponding to certain costs of auditing. This holds strictly in the second and the fifth case of the proposition and weakly in the remaining cases. This reduction is the effect discussed in section 2. To limit the amount of insurance fraud, the insured need to expect a lower level of auditing, because the uncertainty about the level of auditing has an addition deterrence effect.

The last case in the proof of proposition 2 is the point made by Picard (1996). Commitment allows to eliminate insurance fraud completely which is not possible without commitment. This result is strengthened by the fact that $k' > k^*(1)$, because of the definition of p' and the concavity of u .¹² Thus without commitment complete fraud, $\alpha = 1$, happens at lower costs of auditing than with commitment.

Next we will show, that with ambiguity aversion there are cases in which the costs of the insurer are lower without commitment than with it. This suggests that the premium might be lower without commitment, although the market equilibrium has to be determined before. In order to show this we compare the costs due to insurance fraud, $\alpha(1-\delta)(1-p)q$, and auditing, $(\delta + \alpha(1-\delta))pk$, minus the recovered fines, $m\alpha p(1-\delta)$, in the case without commitment to the cost of auditing under commitment, $\delta p'k$. Commitment implies a welfare loss, if it holds:

$$\alpha(1-\delta)(1-p)q - m\alpha p(1-\delta) + (\delta + \alpha(1-\delta))pk \leq \delta p'k$$

Proposition 3: In equilibrium insurers prefer not to commit, if:

- the costs of auditing are high, i.e. $k_L \geq k'$, in which case the insurer is indifferent between commitment and no commitment.
- fraud is attractive $\frac{1}{2} < p' < \frac{1}{2}(1 + \bar{A})$ and the costs are intermediate, $k_L \leq \min\{\frac{m\alpha(1-\delta)}{\delta(1-p')+\alpha(1-\delta)}, k^*(\alpha)\}$ and $k_H \geq \max\{\alpha k', k^*(\alpha)\}$ with $\alpha = \frac{1}{A}(2p' - 1)$.
- fraud is very attractive $p' \geq \frac{1}{2}(1 + \bar{A})$ and the costs are intermediate, $k_L \leq \min\{\frac{m(1-\delta)}{1-\delta p'}, k^*(1)\}$ and $k_H \geq k'$ with $\alpha = 1$.

Proof: For high costs of auditing, the insurers will abstain from auditing. Consequently they will be indifferent on the commitment issue, because it leads to the same outcomes. From Proposition 2 and the fact that $k' > k^*(1)$, this is the case for $k \geq k'$.

Below k' under commitment audits become worthwhile and no insurance fraud is committed. The costs for the deterring level of auditing decreases with k . Without commitment auditing is still too expensive. Therefore there is complete fraud and commitment is better.

However once the costs of auditing of one type of insurer drop below $k^*(1)$, there is auditing even without commitment. We distinguish now the following cases which are the same as in proposition 2.

Assume we are in case two or five of proposition 2. Then commitment is always preferable to no commitment, because the insurer that is made indifferent without commitment has an incentive to commit herself. The concavity of the utility function is the reason for this. The details can be found in lemma 2 in the appendix. Consequently in these cases commitment is better.

¹² To show $k' > k^*(1)$ is equivalent to $q/(\delta p') > q + m$. Consequently it is enough to proof that $q > (q + m)p'$ or $q - p'(q + m) > 0$. This is done in lemma 2 in the appendix.

Now assume we are in case four of proposition 2. Then we have $\alpha = \frac{1}{\bar{A}}(2p' - 1) < 1$ and audits of $p_H = 0$ and $p_L = 1$. The low cost type prefers not to commit, if and only if:

$$\alpha(1 - \delta)(1 - p_L)q - m\alpha p_L(1 - \delta) + (\delta + \alpha(1 - \delta))p_L k_L \leq \delta p' k_L$$

$$\text{Inserting the value of 1 for } p_L \text{ we get: } -m\alpha(1 - \delta) + (\delta + \alpha(1 - \delta))k_L \leq \delta p' k_L$$

$$\text{Rearranging the terms gives: } (\delta(1 - p') + \alpha(1 - \delta))k_L \leq m\alpha(1 - \delta) \text{ or } k_L \leq \frac{m\alpha(1 - \delta)}{\delta(1 - p') + \alpha(1 - \delta)}$$

The fraction on the right hand side is obviously positive and does not depend on k_L . Consequently for k_L small enough the low cost type of insurer does not commit. What about the high cost type?

For her no commitment is better, if and only if:

$$\alpha(1 - \delta)(1 - p_H)q - m\alpha p_H(1 - \delta) + (\delta + \alpha(1 - \delta))p_H k_H \leq \delta p' k_H$$

$$\text{Inserting the value of 0 for } p_H \text{ we get: } \alpha(1 - \delta)q \leq \delta p' k_H \text{ and this leads to: } k_H \geq \frac{\alpha(1 - \delta)q}{\delta p'} = \alpha k'$$

Thus the high cost type has no incentive to commit, if her costs are high enough. In summary, we have found an equilibrium in which the insurer choose not to commit to an auditing level, even if they had the possibility to do so credibly.

It remains to scrutinise the third case. This gives complete fraud, $\alpha = 1$, and audits of $p_H = 0$ and $p_L = 1$. We begin with the high cost type. As before she prefers commitment, if and only if $k_H \geq \frac{(1 - \delta)q}{\delta p'} = k'$ as $\alpha = 1$. Analogously the corresponding condition for the low cost insurer is $k_L \leq \frac{m(1 - \delta)}{1 - \delta p'}$. The fraction $\frac{m(1 - \delta)}{1 - \delta p'}$ is strictly positive, such that these conditions are viable. To guarantee the case distinction from proposition 2 we compare the values to $k^*(1)$. Thus there is an equilibrium with no commitment, if fraud is attractive $p' \geq \frac{1}{2}(1 + \bar{A})$ and the costs are intermediate, $k_L \leq \min\{\frac{m(1 - \delta)}{1 - \delta p'}, k^*(1)\}$ and $k_H \geq k'$. \square

If the costs of auditing of the high cost type are high enough, she prefers not to audit any claims. Consequently she has no incentive to deviate to an equilibrium with commitment. If on the other hand the cost of the low cost insurer are low enough, she chooses to do a lot of auditing, here even complete auditing. As the insured do not know which type of insurer they face, there will be some fraud. The insurer could signal her type by committing to a level of auditing p' and completely deter the insured from filing fraudulent claims. However in this case she does not get any fines awarded for finding false claim reports. If the ratio of these parts of the fines to the costs is high enough, the insurers prefer an equilibrium without commitment.

4 Extension

We now modify the model to show that ambiguity is necessary for the results we obtain. Therefore we assume there is only one type of insurer with costs of auditing k , which are commonly known. However there are two types of insured. There is a fraction $0 < \beta < 1$ of insured that are very difficult to deter from committing fraud. They might be risk loving or have their fines subsidised. Consequently the level of auditing necessary to deter them from filing fraudulent claims is too high, $p'_\beta > 1$ in our notation, and they cannot be deterred. The insurers know this fraction β from e.g.

historical observations. However the insured do not know β exactly, but only $\beta \in \{\beta_L, \beta_H\}$ with $0 \leq \beta_L < \beta_H < 1$ and $\beta_L < \frac{\delta k}{(1-\delta)(q+m-k)} < \beta_H$. As before they have a capacity v about β :

$$v(\beta_i) = (1 - \mathcal{A})\frac{1}{2}, \quad i \in \{L, H\}$$

We assume here that every insured has the same ambiguity aversion of $\bar{A} > 0$. If an insurer faces a population with β_H , she will choose to audit every claim made. Therefore the insured expected the level of auditing to be $(1 + \bar{A} + (1 - \bar{A})p_L)/2$ with p_L being the probability of auditing of the insurer having a β_L pool of insured. The auditing game is solved by the following equilibrium.

Lemma 1:

- For high costs of auditing, i.e. $k \geq k^*(1)$, the insurer with the good risk pool abstains from auditing completely and faces a level of false claims of

$$\alpha = \begin{cases} 1 & \text{if } p' \geq \frac{1}{2}(1 + \bar{A}) \\ \beta_L & \text{if } p' < \frac{1}{2}(1 + \bar{A}) \end{cases}$$

- For intermediate costs of auditing, i.e. $k^*(\beta_L) \leq k < k^*(1)$, the low type insurer has a level of fraudulent claims of $\alpha = \beta_L$ and a level of auditing of $p_L = 0$, if fraud is not very attractive $p' \leq \frac{1}{2}(1 + \bar{A})$.
- For low costs of auditing, i.e. $k < k^*(\beta_L)$, the low type insurer has a level of fraudulent claims of $\alpha = \beta_L$ and a level of auditing of $p_L = 1$.

Proof: The proof is similar as in proposition 2. □

In the case with commitment there will be no additional frauds, $\alpha = \beta_i$, and some auditing, $p = p'$, if the costs of auditing are low, i.e. $k \leq \tilde{k} = \frac{q(1-\delta)[1 - \beta_i(1-p')] + m\beta_i p'(1-\delta)}{(\delta + \beta_i(1-\delta))p'}$. If the costs are high, there will be complete fraud, $\alpha = 1$, and no auditing, $p = 0$.

This extension allows us to derive the following additional results. First, even for high costs of auditing, i.e. $k \geq \max\{\tilde{k}, k^*(1)\}$ the insurer with β_L may gain by not committing, if $p' < \frac{1}{2}(1 + \bar{A})$. Second, the average level of auditing $(1 + p_L)/2$ without commitment needed to deter all normal insured from committing fraud is $1/2$, if $p' < \frac{1}{2}(1 + \bar{A})$. The interesting point is that this average level may be lower than the level of auditing p' needed with commitment, if $p' > 1/2$. Therefore the insurance industry as a whole might be interested in not announcing fraud detection strategies. This effect is uniquely caused by the ambiguity, as it disappears with $\bar{A} = 0$.

5 Conclusion

In this article we discussed an insurance fraud model with ambiguity averse agents. We could show that ambiguity aversion reduces the inclination to commit insurance fraud. In some cases this effect is so strong that the costs caused by fraud and its deterrence are lower than under credible commitment to an auditing level. So the additional uncertainty caused by not committing might in fact be welfare enhancing. This complements the results of the previous literature which recommended such

a commitment, when possible. Additionally the results of this paper might explain, why insurance companies are rather reluctant to publish information about their fraud detection efforts, like the level of auditing.

It still remains to implement the equilibria characterised here into a market equilibrium such that the insurance premium and the coverage is determined endogenously.

Appendix

Here we give the details of the proof for proposition 3.

Lemma 2:

Assume we are in case 2 or 5 of proposition 2 and $i \in \{H, L\}$ denotes the type of insurer made indifferent by the level of false claims. Then the type i insurer will always prefer to commit to p' .

Proof: Given that $\bar{A} > 0$ the following expressions are equivalent:

The costs without commitment are higher than with commitment:

$$\alpha(1 - \delta)(1 - p_i)q - m\alpha p_i(1 - \delta) + (\delta + \alpha(1 - \delta))p_i k_i \geq \delta p' k_i$$

Collecting the p_i terms we get:

$$\alpha(1 - \delta)q - p_i[\alpha(1 - \delta)q + m\alpha(1 - \delta) - (\delta + \alpha(1 - \delta))k_i] \geq \delta p' k_i$$

Rearranging the terms in the square brackets gives:

$$\alpha(1 - \delta)q - p_i[\alpha(1 - \delta)(q + m - k_i) - \delta k_i] \geq \delta p' k_i$$

As $\alpha = \frac{\delta k_i}{(1 - \delta)(q + m - k_i)}$ the term in square brackets equals 0 and we get:

$$\alpha(1 - \delta)q \geq \delta p' k_i$$

This means that the auditing costs, $(\delta + \alpha(1 - \delta))p_i k_i$, and cost savings due to discovered frauds, i.e. indemnities not paid out, $q\alpha p(1 - \delta)$, and fines received by the insurer, $m\alpha p(1 - \delta)$, offset each other. Consequently *only* the possible losses due to falsely stated claims, $\alpha(1 - \delta)q$, matter. This effect is caused by the indifference condition (3) of the insurer in equilibrium.

By inserting α we get:

$$\frac{\delta k_i}{(1 - \delta)(q + m - k_i)}(1 - \delta)q \geq \delta p' k_i$$

Multiplying the inequality by $q + m - k_i$ leads to:

$$q \geq p'(q + m - k_i)$$

Ordering the terms for k_i and dropping p' gives us:

$$-\frac{q}{p'} + q + m \leq k_i$$

It holds $m \leq M$ and $\epsilon = [u(-P) - u(-P - M)]q - [u(-P + q) - u(-P)]M > 0$, because the utility function u is strictly concave. Therefore $q - p'(q + m) = (1 - p')q - p'm \geq (1 - p')q - p'M = \epsilon[u(-P + q) - u(-P - M)] > 0$. Consequently the left hand side of the last inequality is always negative. Yet in this case the inequalities are always satisfied and the respective insurer can make

herself better off by committing to a level of auditing p' . □

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