Closure and Growth of Fatigue Cracks at Notches

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The closure behavior of fatigue cracks growing out of notches is studied with an elastic-plastic finite element model. Crack opening stresses are shown to change significantly as the crack extends. Opening stresses are low at first and then gradually rise to stable values as the crack tip moves away from the notch field. These transient changes are not limited to the region of the original inelastic notch field. The rate of change of opening stresses with increasing crack length is a function of both nominal maximum stress and nominal stress ratio. Stable levels are reached more quickly at higher stress ratios and lower maximum stresses. These transient changes in S_{open} have been emulated with a simple model which considers only changes in S_{open} due to changes in the local stress field. The numerical results are quantitatively consistent with observed trends in experimental crack growth data, which show that accelerated crack growth can occur beyond the original notch plastic boundary. Finite element results and experimental data also both suggest that the accelerated short crack growth effect for cracks near notches is much less pronounced at higher stress ratios.

Introduction

A common site for the initiation and subsequent propagation of fatigue cracks in an engineering component is a local stress concentration, such as a hole or the root of a sharp notch. When the crack has grown sufficiently far beyond its initiation site, its growth may be easily modeled by traditional concepts of fatigue crack propagation, neglecting the hole or notch altogether except as a contributor to the total crack length. When the crack is still very short relative to characteristic notch dimensions, however, the problem is more complex. Broek [1] was one of the first to point out clearly that short cracks at notch roots tended to grow more rapidly than would be predicted by a traditional ΔK-based analysis. Others [2–15] have since confirmed this anomalous behavior.

At first glance this behavior should not be surprising. The constitutive response at the notch root of the uncracked body is likely to include some significant plastic strains, and these will both accelerate crack growth rates and invalidate K-based parameters. Even in the absence of large plastic deformations, microstructurally based small crack effects [16] and the nonlinear form of the exact K-solution for short cracks near notches [17] can influence crack growth rates. All of these issues will tend to be most significant when crack lengths are extremely short relative to notch dimensions. And yet anomalous behavior has also been reported for cracks which are longer relative to notch sizes [3, 14].

The total picture may be better organized in terms of Fig. 1, a schematic which is adapted from the ideas of Leis [8] and Hammouda et al. [18, 19]. Cracks growing from notches can be classified into three groups as a function of the crack tip location with respect to the original notch stress-strain fields in an uncracked body. Very long cracks fall in region (1), in which case the original notch has no direct influence on the local (crack-tip) stress-strain fields and hence can be neglected except as a component of total crack length. Very short cracks fall into region (3), where the crack tip lies within material which experienced plastic strains prior to the formation of any significant crack. Cracks of intermediate length are in region (2). Here there is no inelastic deformation which can be at-

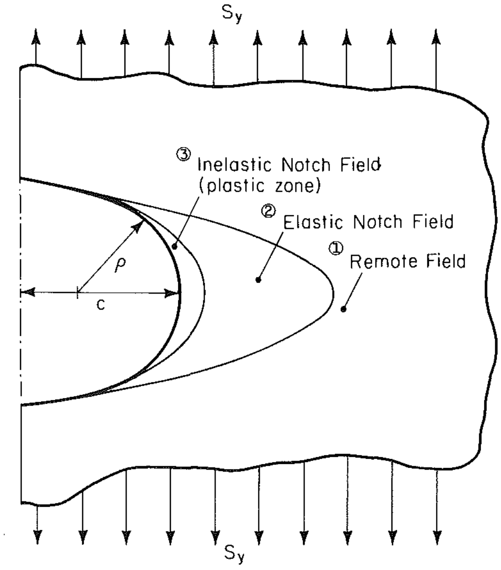

Fig. 1 Schematic representation of the stress fields around a notch in a remotely stressed body

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tributed to the notch alone, but the notch does cause perturbations in the elastic stress-strain field within which the crack tip and associated near-tip plastic zones are contained.

Many investigators have assumed that only two regions exist, Miller [20], for example, considered analytical K solutions and experimental crack growth data for cracks near notches. They suggested that the notch field was of approximate length They suggested that the notch field was of approximate length $0.13\sqrt{c\rho}$, where $2c$ is the total width of a center notch and ρ is the root radius. Beyond this distance, K was assumed to be independent of the notch, while within this distance, K was estimated as a simple function of $\sqrt{c/\rho}$. Dowling [21] separated the crack growth problem into two smaller problems: first, an edge crack growing totally in the notch field, and second, a center crack growing independent of the notch. By setting K solutions for the two problems equal to each other, he derived a transition crack length as

$$l_t = c/[(1.12K_t)^2 + 1] \quad (1)$$

where K_t is the stress concentration factor for a center notch in a wide plate (for different geometries, more general forms of Eq. (1) can be developed). Values for l_t generally range from $\rho/20$ to $\rho/4$ for moderate to sharp notches. Note that neither the Smith and Miller nor the Dowling model considers plastic deformation or any stress amplitude effects, and their "notch field" was not related to the notch plastic zone.

Hammouda and Miller [18, 19] analyzed the inelastic deformation at both cracked and uncracked notch roots. They concluded that cracks would grow at abnormally high rates within the notch plastic zone, but that this effect would die away (and crack growth rates perhaps decrease) until the crack reached the elastic-plastic boundary. Beyond this distance, normal linear elastic fracture mechanics would be sufficient to characterize crack growth. Leis [8] also concluded from an analysis of his experimental data that the transition to normal long crack behavior from anomalous short crack behavior would occur when the length of the crack approached or equaled the length of the inelastic notch field.

Other data are available, however, which demonstrate accelerated crack growth rates at crack lengths well beyond the original notch plastic zone. Sehitoglu [14] has presented such data for cracks growing from a center slot notch in a low-carbon structural steel. Some of the data of El Haddad et al. [3], exhibit a similar behavior. In both cases, cracks are also long with respect to the microstructure. Clearly, some effect other than notch plasticity is also significant.

Another possible contributor to this "short crack effect" is crack closure. Closure (or, more properly, the absence of closure) has been frequently suggested as a reason for the accelerated growth of very small cracks in unnotched members [16]. Since closure may be primarily a wake effect, a microcrack which has not yet developed a significant wake may, in theory, open at the minimum load in the cycle. Leis [8] assumed that the crack opening stress S_{open} was equal to the minimum stress S_{min} at the notch root[1]. He then allowed S_{open} to increase linearly up to the nominal, notch-independent value (dependent only on the stress ratio, R) over a distance equal to the width of the inelastic notch field. Tanaka and Nakai [9], Sehitoglu [15], Ogura et al. [10], and Shin and Smith [12, 13] all made some experimental determination of crack opening stresses for cracks growing out of notches. All determined that S_{open} did change significantly with crack length, and all were able to relate this change to variations in crack growth rates. While the greatest changes in S_{open} occurred within the inelastic notch zone, some further variations occurred beyond this region. Ogura et al.

[1]Note that in this paper, the symbol "S" will be used to represent far-field or nominal applied stresses. The more traditional symbol "σ" will be used to denote local stresses or strength properties.

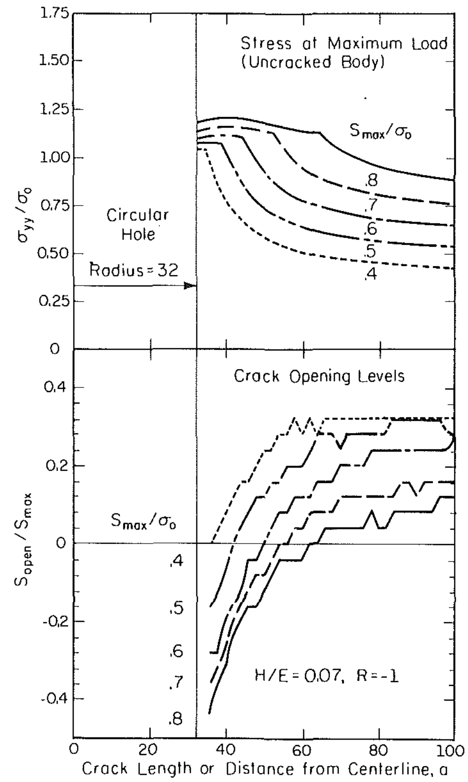

Fig. 2 Stress distributions at maximum load for a circular notched, uncracked body (top), and changes in opening stresses for cracks growing from the notch (bottom)

[22], in early work, and Lalor et al. [23–25], in more recent research related to the present article, have employed elastic-plastic finite element analyses to show that crack opening stresses change as cracks grow away from the notch root. Similar results were obtained by Newman [26] and Sehitoglu [27] using simpler analytical models based on a modified Dugdale crack.

Finite Element Model

An elastic-plastic finite element model [28–29] was used to determine crack opening stresses for fatigue cracks growing out of notch roots. The crack was allowed to "grow" on each loading cycle by releasing the crack tip node, and this growth process typically continued for twenty to fifty cycles. Stresses and displacements along the crack line behind the crack tip were continuously monitored during each cycle to check for "opening" or "closing" of the crack faces, and appropriate changes were made in the boundary conditions at each nodal location. Unlike most experimental techniques, which measure crack opening behavior at a location relatively remote from the crack tip, the finite element technique permitted monitoring of closure immediately behind the crack tip. The material model employed kinematic hardening with a von Mises yield surface translating according to Ziegler's rule. The constitutive relationship included a linear hardening model characterized by $H/E = 0.07$. Here, H is $d\bar{\sigma}/d\bar{\epsilon}_p$, the slope of the plastic line, and E is Young's modulus, the slope of the elastic line. All analyses were plane stress.

The meshes were composed of four-noded isoparametric elements and represented a rectangular plate with symmetric cracks growing from a center hole. The size of the hole ranged from $c/W = .008$ to $.032$ and the shape of the hole ranged from elliptic (aspect ratio of 3) to circular in different meshes. The crack was initiated at the notch root and permitted to grow beyond the influence of the notch. Mesh refinement along the crack line corresponded to $\Delta a/W = 0.002$.

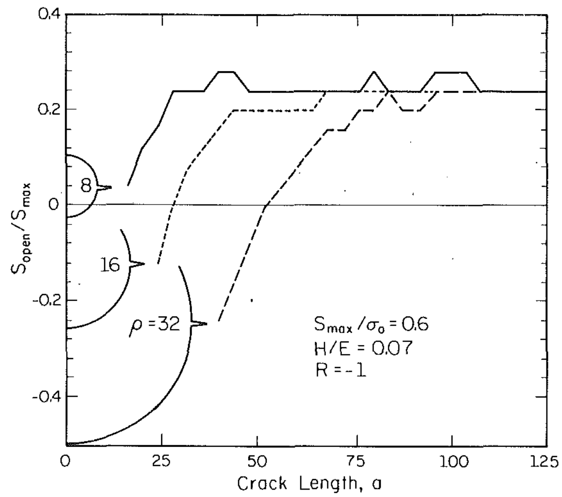

Fig. 3 Opening stresses for cracks growing out of variously sized circular holes

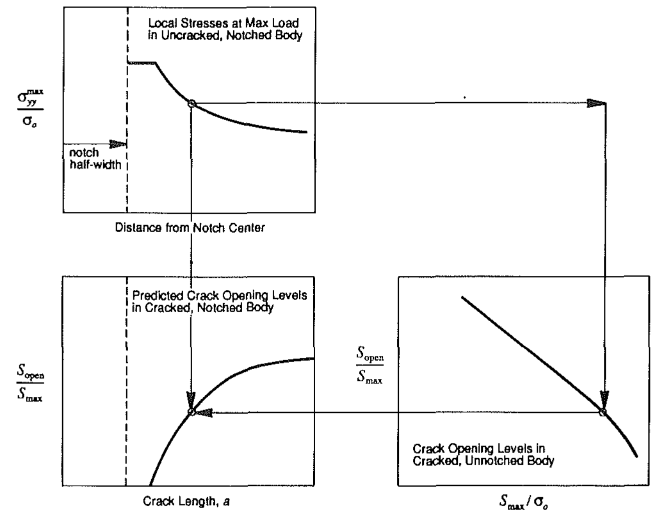

Fig. 4 Schematic outline of simple model to predict crack opening stresses for cracks growing from notches

Typical results from $R = -1$ cycling of a plate with a circular hole are summarized in Fig. 2 (bottom). Here, S_{open} is the nominal (far-field) stress at which the crack tip first becomes fully open during the load-increasing half of the loading cycle, S_{max} is the maximum nominal stress in the cycle, and σ_0 is the yield stress, the point of intersection of the elastic and plastic lines. Note that normalized opening stresses are low for very short cracks and then rise to stable levels as the crack extends. This is consistent, of course, with faster growth of the shortest cracks. Note further that the rate of increase of S_{open} and the final stable value are both lower at higher remote maximum stresses.

It is important to insure that these changes in opening stress with crack length are truly due to notch effects and are not simply artifacts of the modeling process. Previous finite element analyses of closure in *unnotched* bodies have demonstrated a similar tendency for opening stresses to start low and gradually increase to stable values [30], since some finite amount of simulated crack growth is necessary in order to develop the plastic wake associated with closure. This check was carried out by conducting analyses with identical loading parameters but three different notch sizes. The results are shown in Fig. 3. Note that the "crack length" here is measured from the centerline of the notch, and so the dimensions of the original notches (with the initial crack length used in the finite element analysis) have been sketched directly on the figure to provide a point of reference for the three corresponding curves of crack opening stress. The crack opening curves are geometrically similar with respect to the notch sizes, indicating a true notch effect. In all three cases, the crack opening stress stabilizes when the total crack length is about three times the radius of the circular notch.

It is particularly interesting to compare these curves of crack opening stress (Fig. 2, bottom) with the stress distributions in an identically notched but *uncracked* body at corresponding maximum stresses (Fig. 2, top). These are simply the stresses that would be present near the notch prior to crack initiation. Here σ_{yy} is the local stress parallel to the loading axis. Since a bilinear stress-strain curve is being used, the boundaries of the original notch plastic zones are clearly indicated by the sharp corners in the stress distributions. As the maximum stress increases, of course, the width of the plastic zone increases. The important phenomenon to note, however, is that the crack opening stresses have not yet stabilized when the crack tip reaches the location of the original inelastic notch field boundary. Further changes in S_{open} occur in the elastic field of the notch.

Why does S_{open} change as the crack grows away from the notch root? One factor is crack length. A freshly initiated crack has no previous history and no wake, and therefore has no residual displacements which induce closure. As the crack moves beyond the notch field (as defined by Dowling [21], for example), the crack tip sees a stress intensity at maximum load which is nearly identical to a notch-free center crack. Upon unloading, however, all of the reversed deformation (which reduces the plastic stretch and hence reduces closure levels) is concentrated in a very small distance between notch root and crack tip, instead of being distributed over the full crack length $2a$. Sehitoglu's simple analytical model based on the Dugdale crack captures some of these effects [27].

Another factor is stress magnitude. A crack tip near the notch effectively sees a "remote" stress which is larger than the true nominal stress, magnified by the notch stress concentration. Even though the crack tip significantly perturbs the exact form of the stress and strain distributions around the notch, the notch is still a major factor in the total stress redistribution problem. Previous finite element results for longer cracks which were not influenced by notches [28–29] have clearly shown that opening stresses are lower when nominal (far-field) stresses are higher. The logic is consistent with lower opening stresses closer to the notch root.

A simple model was previously developed [31] to confirm this functional dependence of opening level on the local stress-strain field. The model is summarized schematically in Fig. 4. First the local stress at maximum load, σ_{yy}^{max}, at the location of the crack tip in an equivalent notched but uncracked body, was estimated from companion elastic-plastic finite element analyses. Next, the normalized crack opening stress S_{open}/S_{max} at this crack tip position was estimated from the known dependence of crack opening stress on maximum applied stress in an unnotched body [28–29]. Here the local stress σ_{yy}^{max} in the notched, uncracked body was equated to the nominal stress S_{max} in the hypothetical cracked, unnotched body. The model predictions of crack opening stresses were shown to match the finite element simulations closely.

Figure 2 and the related discussion focused on crack behavior near a circular notch ($K_t = 3$). Finite element results for cracks growing from an elliptical notch ($K_t = 7$) are shown in Fig. 5. Again note the changes in opening stresses (bottom diagram) well beyond the original notch plastic zone boundary (top diagram). The predictions of the simple model [31] are also given in the bottom diagram. Note that the differences in opening behavior between the circular and elliptical notches are not great. The finite element and simple models both predict that the sharper notch will experience slightly lower opening

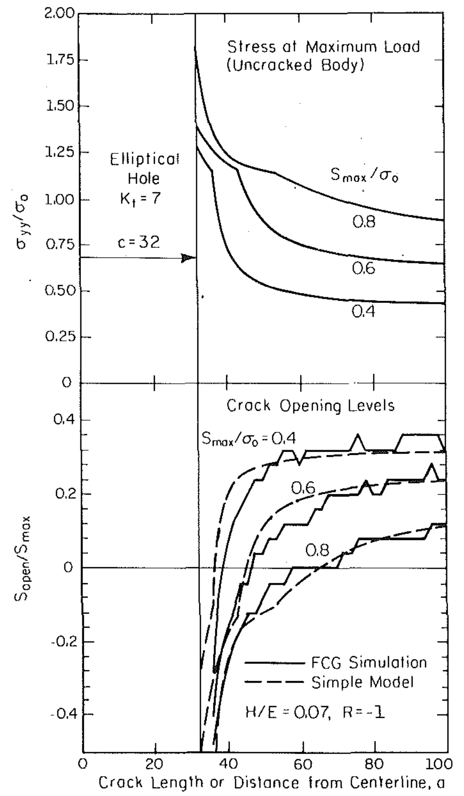

Fig. 5 Stress distributions at maximum load for an elliptically notched, uncracked body (top), and changes in opening stresses for cracks growing from the notch, as determined by a complete finite element simulation and a simpler model

stresses at very short crack lengths and slightly more rapid stabilization of opening stresses with further crack growth.

Crack Growth Data and Correlations

This information about closure is useful only if it can be related to changes in observed crack growth rates in a practical way. In order to do this, we must (1) estimate ΔK for a given notch and crack length; (2) estimate the crack opening stresses for a given crack tip location; and (3) estimate the crack growth rates corresponding to a given value of ΔK_{eff}.

It is not practical, of course, to conduct a full finite element simulation of every exact problem we wish to solve in order to determine the local crack opening levels. But the simple model referenced above was shown to provide an alternative means of estimating closure behavior near notches (step 2). A more robust formulation of this simple approach was developed by estimating the local elastic-plastic stress distribution from the analytical elastic stress distribution and the well-known Neuber relationship (instead of a separate elastic-plastic finite element analysis). The local crack opening stress in the notched body was then estimated as before in the simple model by employing archival finite element results for crack opening stresses in unnotched bodies. The stress intensity factor for a crack growing from a notch (step 1) was estimated from a semi-empirical equation which agrees closely with the benchmark Newman numerical solutions [17]. Step 3 was then carried out with the usual closure-modified Paris Law formulation:

$$\frac{da}{dN} = C(\Delta K_{eff})^m \qquad (2)$$

Complete details of this extended simple model and the K solution are given in [31].

Experimental fatigue crack growth rate data were available for notched specimens of a hot-rolled 1026 steel with a cyclic yield strength of 322 MPa. Central circular and slot-notched holes of varying size and shape were machined in specimens with a rectangular cross-section of 0.1″ × 1.5 in. (2.54 mm × 38.1 mm) and a length (between the grips) of 4 in. (101.6 mm). All tests were conducted in load control at $S_{max}/\sigma_0 = 0.54$ and $R = -1$. Crack lengths were measured by a replica technique.

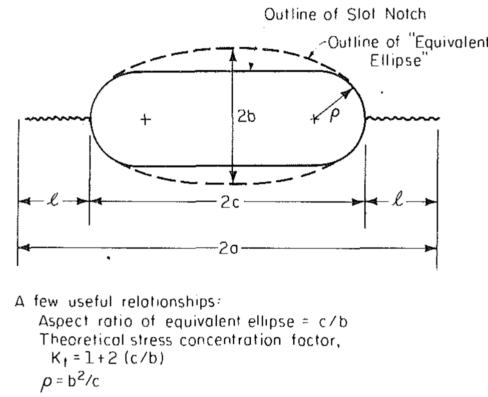

Fig. 6 Nomenclature for a cracked slot notch and relationship to its "equivalent ellipse"

The nomenclature associated with the slot notches is illustrated in Fig. 6. The stress distribution for the notch is essentially identical to that of its "equivalent ellipse" [32], an imaginary ellipse with corresponding total width $2c$ and root radius ρ. The elastic stress distribution for the ellipse, which was also required in order to calculate the stress intensity factor, was determined from the original equations of Inglis [33].

Crack growth data from four specimens are shown in Figure 7. The solid line represents the predictions of the extended simple model. The empirical constants C and m in Eq. (2) were determined from a fit of long crack data, independent of notch effects. The effective stress intensity factor range ΔK_{eff} in the simple short crack model was computed by replacing the stress range in the ΔK solution with the effective stress range, $S_{eff} = (S_{max} - S_{open})$. The dashed line represents a crack growth rate prediction based on a "long crack" model [21], which includes the notch only as a length contribution to a center crack and neglects any local stress concentration: $\Delta K_{eff} = (U_{stable})\Delta S\sqrt{\pi(l+c)}$. All data on any one plot are from a single specimen. The four different symbols used on each plot distinguish crack length measurements made on the front and back surfaces of the specimen of the cracks growing out of the left and right sides of the center notch.

The correlation of the simple model with the data provides a general confirmation that crack closure is the key issue in accelerated short crack growth near notches. Remembering that the simple model was originally designed to emulate the finite element results, the data also provide general confirmation of the numerical closure simulations.

Crack growth rates are typically underestimated slightly at the very shortest crack lengths. This is not surprising, however, since the simple model uses a linear-elastic crack tip parameter (ΔK) and does not explicitly consider any effects of yielding. At the very shortest crack lengths, the crack tip is still embedded in the original notch plastic zone. The increased scatter in the data for the circular notch ($K_t = 3$) is due to nonsymmetric growth of very small cracks, including the temporary development of corner cracks at the notch root.

It is especially important to note that accelerated crack growth is occurring well beyond the original notch plastic zone. For these stress levels and notch shapes, the boundary of the (uncracked) inelastic notch field, determined from elastic-plastic finite element analyses which account for stress redistribution, is located between $l/c = 0.25$ and $l/c = 0.30$. It is clear that an analysis of crack growth behavior which assumes "long crack" growth rates (the dashed lines) immediately beyond the

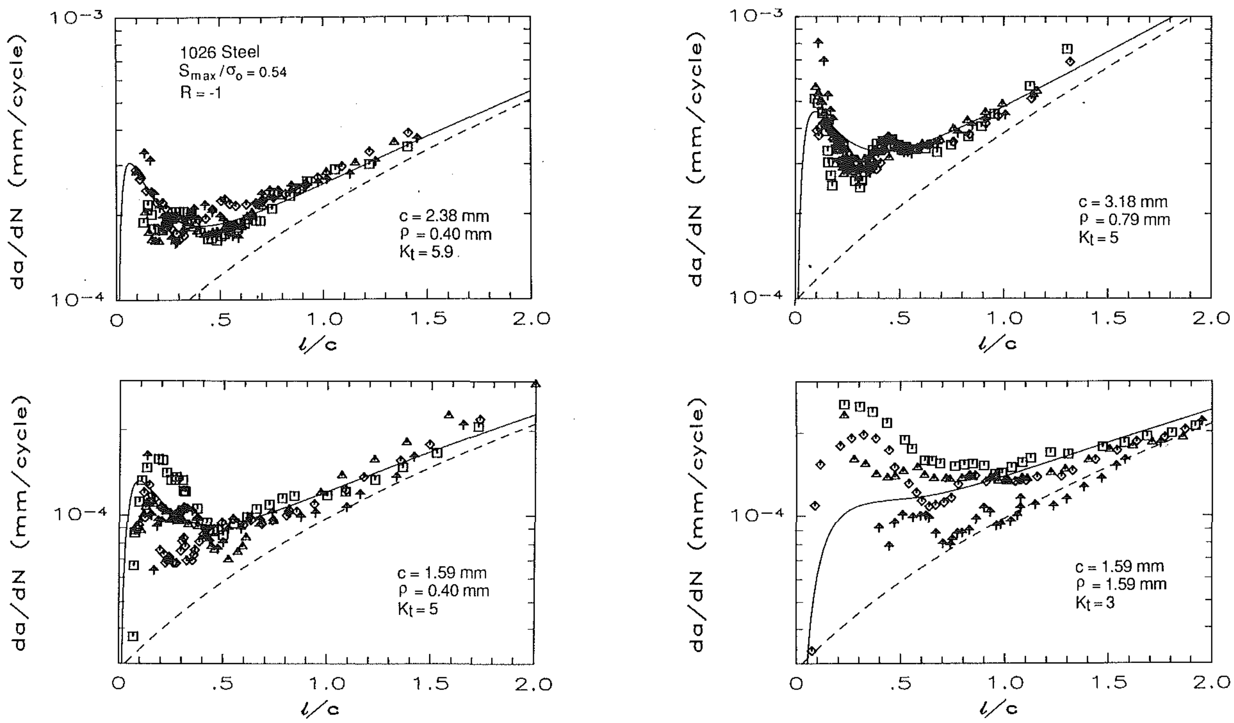

Fig. 7 Experimental data for cracks growing from notches in a 1026 steel, compared with the predictions of a simple short crack model (solid line) and a long crack model (dashed line)

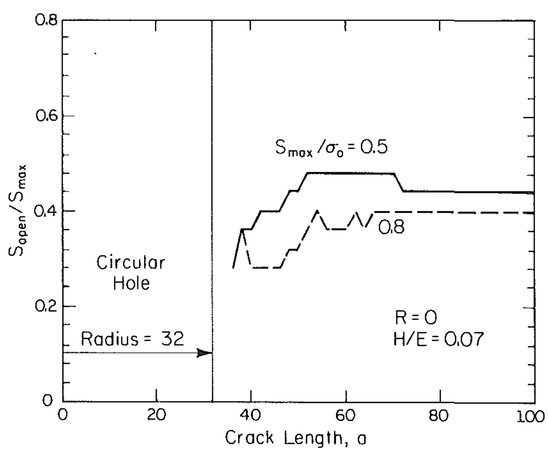

Fig. 8 Opening stresses for cracks growing out of notches at $R = 0$

original notch plastic zone boundary could seriously underestimate the actual speed of crack advance.

Discussion

Stress Ratio Effects. All of the analyses and data discussed so far have corresponded to a remote stress ratio of $R = -1$. A key factor in the simple model has been the strong dependence of S_{open}/S_{max} on S_{max} for $R = -1$. What happens when the remote stress ratio is $R = 0$? In that case the S_{open}/S_{max} values change only slightly with S_{max} at lower nominal maximum stresses [28], and therefore if the simple model is accurate, the short crack effect will be much less pronounced at $R = 0$.

This is apparently true. Finite element simulations of closure for cracks growing out of notches at $R = 0$, Fig. 8, show much less change in S_{open} with changes in crack length. Experimental data are also consistent with this trend. Broek [1], who considered only $R = 0.1$ cycling, found only a slight short crack effect at notches. Usami [11] observed a strong short crack effect at $R = -1$ but a much weaker effect at $R = 0$. Sehitoglu

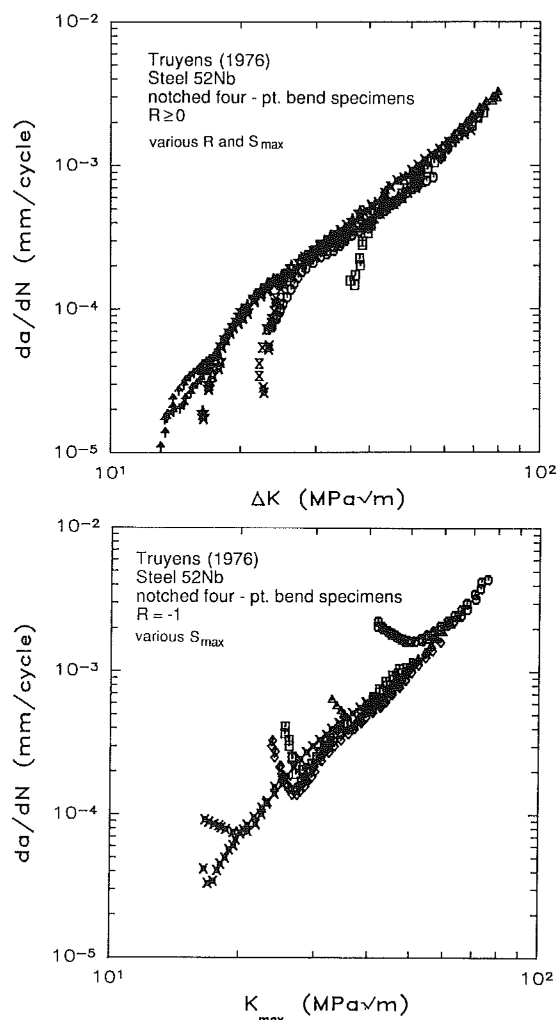

Fig. 9 Crack growth data of Truyens [2] for four-point bending of a notched beam. (top) $R \geq 0$ (bottom) $R = -1$

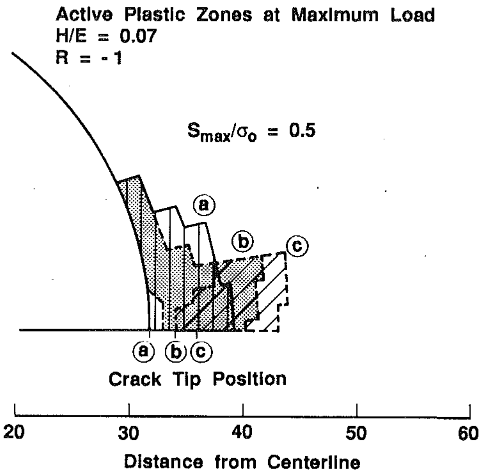

Fig. 10 Plastic zone for a short crack near a notch in comparison to the original notch plastic zone

[27] found a large effect at $R = -1$ and a small effect at $R = 0$ on tests with a 1070 steel. Some of the clearest data are those of Truyens [2], who conducted four-point bend tests on rectangular beams 75 mm deep with a saw-cut edge notch 10 mm deep. Data from his unpublished Ph.D. thesis are reproduced here as Fig. 9. Calculated K_{max} or ΔK values were based entirely on long crack relationships, ignoring any short crack effects. For various stress ratios all greater than or equal to zero, top figure, no accelerated short crack growth was observed. Very short cracks actually grew at rates slower than "long" crack trends, as would be expected from the stress intensity factor solution. For $R = -1$ cycling, bottom figure, a strong acceleration effect appeared at a wide range of maximum stresses.

Influence of the Notch Plastic Zone. The simple model referenced in this paper essentially ignored any influence of the notch plastic zone. Clearly, however, the inelastic deformation will have some impact on crack growth behavior. It is worthwhile to consider briefly these contributions.

First of all, when a crack is growing in an inelastic field, a linear elastic parameter will obviously not be an accurate representation of the crack tip fields. An elastic-plastic crack growth parameter similar to ΔJ or ΔK_ϵ should be employed instead.

On the other hand, it should be realized that notch plastic zones are typically very small. Even when remote stresses are half of the yield stress, the plastic zone at the root of a circular hole will have a width only about ten percent of the hole diameter. At lower stresses and for smaller holes, the notch plastic zone width may be on the order of the microstructure. Cracks that are this short will be influenced not only by notch plasticity but also by microstructural small crack effects. Often crack lengths in this range are simply assumed to be included in a crack "initiation" portion of the life, where crack growth is not tracked at all.

Furthermore, even when crack tips are located within the original notch plastic zone, the notch zone does not necessarily dominate the crack tip events. Hammouda and Miller [18] have previously suggested that for many short cracks, original notch plastic zones were considerably bigger than the extent of crack tip plasticity, and this concept of relatively tiny crack tip shear ears within a large zone of notch plasticity was promoted by their schematic Fig. 3 [18]. In contrast, Fig. 10 of the present paper shows finite element results for plastic zones corresponding to an uncracked body (a) and two different lengths of short cracks at the notch root (b and c). By the time the crack has grown only halfway through the original inelastic notch field, the crack tip plastic zone at maximum load (diagonally cross-hatched area) is large and has taken a form which is relatively independent of the original notch deformation fields (vertically cross-hatched area). The shape of this crack tip plastic zone is typical of a crack in an unnotched body, and the crack tip zone extends far beyond the original notch plastic zone.

In summary, the short crack effect at notches cannot be explained entirely in terms of notch root plasticity, or region (3) behavior as illustrated in Fig. 1. Finite element analyses and experimental data both show clearly that under certain conditions, crack opening stresses and crack growth rates exhibit a short crack effect in the notch elastic field, region (2) in Fig. 1.

It is also interesting to note from Figs. 2, 5, and 7 the relatively complex interactions between the notch stress distribution and the crack opening or crack growth behaviors. As the crack grows away from the notch root, the crack closure load increases at a rate which depends on the elastic-plastic notch stress distribution. For a very sharp notch, the stress ahead of the notch decays very rapidly with distance, resulting in more rapid development of crack closure compared to a blunt notch. It is possible, therefore, to observe growth rates for cracks growing from sharp notches which are similar to or even lower than growth rates for cracks growing from blunt notches, particularly at a distance away from the notch root (much closer to the notch root, however, cracks typically grow faster near sharp notches). In the limiting case, it is possible to encounter non-propagating cracks in sharply notched members. This phenomenon is related to the sharp decrease in crack driving force (or effective stress intensity). The simple model has the potential to predict these effects.

Significance of the Short Crack Effect. Demonstrating cleverness in modeling unusual crack growth behaviors may be great fun for the researcher, but the designer has a different perspective. His primary concern here is probably life prediction, and his question is whether this effect is really significant for total life.

Socie et al. [34] have shown that for a number of design problems involving the initiation and propagation of fatigue cracks at notches, it is possible to totally ignore any sort of short crack behavior, go directly from initiation to a long crack model, and still make good life predictions. This will especially be the case, for example, when notches are small and total component sizes large. On the other hand, if notches are large and component sizes smaller, a much larger percentage of the total life may be spent in the "short crack" stage. In this case, a temporary acceleration in da/dN of 4X may be a very damaging event. Furthermore, this closure behavior may also be significant at lower stresses, when fatigue thresholds and non-propagating cracks become issues.

In a larger perspective, the analysis presented in this article may have significance beyond the notch life prediction problem. The principles involved in the simple model, relating crack opening levels to local stresses in corresponding uncracked bodies, may have application to a broader class of problems involving crack propagation in nonuniform stress fields. While these simple models do not consider all effects, their very simplicity makes them attractive to the engineering analyst.

Conclusions

1 Crack opening stresses change significantly as short cracks grow from notches. Opening stresses are low at first and then gradually rise to steady-state values as the crack tip moves away from the notch field.

2. Transient changes in crack opening stresses are not limited to the region of the original inelastic notch field. Further changes occur in the elastic notch field.

3 The rate of change of opening stresses with increasing crack length is a function of both nominal maximum stress

and nominal stress ratio. Steady-state levels are reached more quickly at higher stress ratios and lower maximum stresses.

4 These transient changes in S_{open} are quantitatively consistent with observed trends in experimental crack growth data.

5 Numerical crack closure studies and experimental crack propagation data both suggest that the "short crack effect" (i.e., accelerated crack growth) for cracks at notches will be much less pronounced at higher stress ratios, such as $R = 0$, than at lower ratios, such as $R = -1$.

6 The "short crack effect" can occur beyond the original inelastic notch field boundary.

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