
A Game Theoretic Analysis of Price-QoS Market Share in Presence of Adversarial Service Providers

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Additional information is available at the end of the chapter

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1. Introduction

Recently, the selfish behavior of customers and Service Providers (SPs) in telecommunications systems has been widely analyzed using game theory with all its powerful solution concepts. It was shown in several works that customer's selfish behavior leads to a network collapse, where a typical prisoner's dilemma situation arises. Despite of the bounty of works and efforts investigated in analyzing market share game, this field is still an ideal tool to understand interaction among SPs and customers. Indeed, it is common in the literature to assume a single decision action (e.g., cost) through which an equilibrium would be computed. Yet, in order to take into account Quality of Service (QoS), it is necessary to incorporate into the model more than one decision parameter. A simple example is to include both price and some measure of QoS (e.g., delay, throughput, loss probability, etc.). Other multi-criteria models may incorporate, for example, delay and reliability, the latter representing the QoS, price or delay and jitter, etc.

The competition in terms of prices and QoS among SPs entails the formation of non-cooperative games. We consider multiple SPs (players of the game), where each one seeks to maximize its own revenue, whereby the whole system of SPs would have no incentive to deviate from the Nash equilibrium¹ point, i.e., the vector of equilibrium strategies. Yet, such equilibrium point should first mathematically exist. In this chapter, we present a general model for computing a bi-criteria Nash equilibrium for multiple SPs. We shall then analyze the interactions between SPs who won't attract more clients and maximize their respective profits. We address the important problem of Nash Equilibrium characterization with two-component action, when the two components of each provider are the service price and a measure of QoS. Our model is mainly inspired from, [6], where the authors studied a

¹ A Nash equilibrium is a strategy profile where no player has sensitive to deviate unilaterally from its current strategy.

non-cooperative game for pricing problem considering QoS as an extra decision parameter. The authors build a Markovian model to derive the behavior of customers depending on the strategic actions of the SPs. In contrast to this chapter, we base our study on the concepts of demand for the services of a given SP (defined by linear function that depends on the vectors of prices and QoSs), which is a commonly used function in research related to competitive network and equilibrium models, [9], [5], to calculate the reputation of an SP in the market.

We focus our studies on the non-cooperative games in terms of stable solutions, which are the pure strategy Nash equilibria of the game. We do not consider mixed strategy equilibria, because our environment requires a concrete strategy rather than a randomized strategy, which would be the result of a mixed strategy. Hence, when using the term Nash equilibrium we mean pure strategy exact Nash equilibria unless mentioned otherwise.

We note that the most fundamental assumption in relative works of game theory is rationality. It implies that every player is motivated by increasing his own payoff, i.e. every player is looking to maximize his own utility. John V. Neumann and Morgenstern justified the idea of maximizing the expected payoff in their work in [23]. In this context, all information concerning the game is known to all players, i.e., there is complete information. So, we consider that all players are said to be rational and intelligent. A rational person is one who acts in such a way as to maximize his or her expected payoff or utility as economists would say. An intelligent person is one who can deduce what his or her opponent will do when acting rationally. In fact, humans use a propositional calculus in reasoning, the propositional calculus concerns truth functions of propositions, which are logical truths (statements that are true in virtue of their form). For this reason, the assumption of rational behaviour of players in telecommunications systems is more justified, as the players are usually devices programmed to operate in certain ways. However, there are previous studies that have shown that humans do not always act rationally [10].

Related Works :

Applying game theory in telecommunications problems is an active research area, in which game-theoretic models have been developed and studied in the last decades, [1, 2, 6, 7, 9, 16, 18]. These models are interested in pricing issues, they proposed non-cooperative game formulations to analyze behaviours of players that selfishly decide their strategies to maximize their respective profits. Other works consider the criteria of price as an implicit parameter, which is determined as a function of the degree of saturation on the network. Typically in these approaches, the price is a shadow price. For more details on those approaches see, [14, 15, 24]. Nonetheless, the price of anarchy has been studied in a large and diverse number of games, e.g., in areas like wireless ad-hoc networks [8, 13], routing and congestion [4, 19], network creation [3], or facility location [22]. In our model, we do not take into account network topology, but rather the effective service proposed by each SP as a single entity. In other words, the price and QoS proposed by an SP will not depend on the source or destination, distance, etc. that underlies the request of each user. After we have proved existence of Nash equilibrium, we propose a joint price and QoS algorithm which allows to learn the equilibrium price and QoS strategies decided by SPs. This is a simple algorithm implementation with lower computational complexity.

Organization :

The rest of the chapter is organized as follows : in Section 2 we describe the system model and introduce a new demand and utility functions. In Section 3 we formulate the joint price and QoS problem as a non-cooperative game, and investigate existence and uniqueness of a Nash equilibrium solution. Then, we present numerical results obtained from simulations that exploits our joint price and QoS algorithm in Section 4 . Conclusions and future guidelines are drawn in Section 5.

2. Problem modeling

In this chapter, we formulate the interaction among service providers (SPs) as a non-cooperative game. Each SP chooses the Quality of Service to guarantee (it depends on the amount of requested bandwidth) and the corresponding price.

We consider a system with N service providers. Let p_i and q_i be, respectively, the tariff/pricing policy and the QoS guaranteed by SP- i . Now, each customer seeks to subscribe to the operator which allows him to meet a QoS sufficient to satisfy his/her needs, at suitable price. We consider that behaviors of customer's has been handled by a simple function so called demand functions, see equation (1). This later depends on the price and QoS strategies of all SPs. From a tagged SP's point of view, the question is to set the best pricing strategy and the best QoS (amount of bandwidth to request from the network owner). SPs are supposed to know the effect of their policy on the customer's subscription policy. Whereas from customer's point of view, the question is to find the SP that has the best price-QoS tradeoff conditions.

2.1. Demand model

For simplicity, we consider that the demand function D_i for services of the tagged SP- i is linear with respect to the set price p_i and the promised QoS q_i , see, [9]. This demand function depends also on prices \mathbf{p}_{-i} and QoS \mathbf{q}_{-i} set by the competitors. Namely, the demand function of SP- i depends on $\mathbf{p} = [p_1, \dots, p_N]$ and $\mathbf{q} = [q_1, \dots, q_N]$. Eventually, D_i is decreasing w.r.t. p_i and increasing w.r.t. p_j , $j \neq i$. Whereas it is increasing w.r.t q_i and decreasing w.r.t. q_j , $j \neq i$. Then, the demand functions w.r.t services of SP- i can be written as follows:

$$D_i(\mathbf{p}, \mathbf{q}) = D_i^0 - \alpha_i^i p_i + \beta_i^i q_i + \sum_{j, j \neq i} [\alpha_i^j p_j - \beta_i^j q_j], \quad \forall i \in \{1, \dots, N\}. \quad (1)$$

where D_i^0 is a positive constant used to insure non-negative demands over the feasible region. While α_i^j and β_i^j are positive constants representing respectively the sensitivity of service provider i to price and QoS of service provider j .

2.2. Utility model

The total revenue of SP- i is $D_i(\mathbf{p}, \mathbf{q})p_i$. We assume that we have a single network owner, this latter charges each SP- i a cost ϑ_i per unit of requested bandwidth. In order to insure the customers loyalty, the amount of bandwidth μ_i required by SP- i should depend on $D_i(\cdot)$ and

on the QoS q_i it wishes to offer to its customers. Therefore, the net profit of SP- i is simply the difference between the total revenue and the fee paid to the network owner:

$$U_i(\mathbf{p}, \mathbf{q}) = D_i(\mathbf{p}, \mathbf{q})p_i - F_i(q_i, D_i), \quad \forall i \in \{1, \dots, N\}.$$

where $F_i(q_i, D_i)$ is the fee paid by SP- i (investment of SP- i):

$$F_i = \vartheta_i \mu_i (q_i, D_i)$$

where μ_i is the amount of bandwidth required by SP- i , such that ϑ_i is a cost per unit of requested bandwidth. We assume that the QoS corresponds to the expected delay, also we consider the Kleinrock delay which is a common delay used in Networking Games, so :

$$q_i = \frac{1}{\sqrt{\text{Delay}_i}} = \sqrt{\mu_i - D_i}$$

that mean that:

$$\mu_i = q_i^2 + D_i$$

While, the utility function of the SP- i is given by the following formula:

$$U_i(\mathbf{p}, \mathbf{q}) = D_i(\mathbf{p}, \mathbf{q}) (p_i - \vartheta_i) - \vartheta_i q_i^2, \quad \forall i \in \{1, \dots, N\}. \quad (2)$$

3. A non-cooperative game formulation

For a precise formulation of a non-cooperative game, we have to specify (i) the number of players, (ii) the possible actions available to each player, and any constraints that may be imposed on them, (iii) the objective function of each player which she attempts to optimize. Here we will consider formulation of games where items (i)-(iii) above are relevant.

Let $\mathbf{G} = [\mathcal{N}, \{P_i, Q_i\}, \{U_i(\cdot)\}]$ denote the non-cooperative price and QoS game (NPQG), where $\mathcal{N} = \{1, \dots, N\}$ is the index set identifying the SPs, P_i is the price strategy set of SP- i , Q_i is the QoS strategy set of SP- i , and $U_i(\cdot)$ is the utility function. Each SP- i selects a price $p_i \in P_i$ and a QoS measure $q_i \in Q_i$. Let the price vector $\mathbf{p} = (p_1, \dots, p_N)^T \in P^N = P_1 \times P_2 \times \dots \times P_N$, QoS vector $\mathbf{q} = (q_1, \dots, q_N)^T \in Q^N = Q_1 \times Q_2 \times \dots \times Q_N$ (where T represents the transpose operator). The utility of SP- i when it decides the strategy price p_i to allocate the QoS q_i is given in equation (2). We assume that the strategy spaces P_i and Q_i of each SP are compact and convex sets with maximum and minimum constraints, For any given user i we consider strategy spaces the closed intervals $P_i = [p_i, \bar{p}_i]$ and $Q_i = [q_i, \bar{q}_i]$.

In order to maximize their utilities, each SP- i decides a price p_i and QoS q_i . Formally, the NPQG problem can be expressed as:

$$\max_{p_i \in P_i, q_i \in Q_i} U_i(\mathbf{p}, \mathbf{q}), \quad \forall i \in \mathcal{N}. \quad (3)$$

3.1. The Nash equilibrium

Considering rationality of service providers, the Nash equilibrium concept is the natural concept solution of the NPQG game. We first will investigate the Nash equilibrium solution for the induced game as defined in the previous section. We will show that a Nash equilibrium solution exists and is unique by using the theory of concave games, [20]. We recall that a non-cooperative game \mathbf{G} is called concave if all players' utility functions are strictly concave with respect to their corresponding strategies, [20].

According to, [20], a Nash equilibrium exists in a concave game if the joint strategy space is compact and convex, and the utility function that any given player seeks to maximize is concave in its own strategy and continuous at every point in the product strategy space. Formally, if the weighted sum of the utility functions with nonnegative weights:

$$\varphi = \sum_{i=1} x_i U_i, \quad x_i > 0 \quad \forall i. \quad (4)$$

is diagonally strictly concave, this implies that the Nash equilibrium point is unique. The notion of diagonal strict concavity means that an individual user has more control over its utility function than the other users have on it, and is proven using the pseudo-gradient of the weighted sum of utility functions, [20].

Fixed-Price Game : Considering some fixed price policy, a Nash equilibrium in QoS is formally defined as:

Definition 1. A QoS vector $\mathbf{q}^* = (q_1^*, \dots, q_N^*)$ is a Nash equilibrium of the NPQG : $\mathbf{G} = [\mathcal{N}, \{P_i, Q_i\}, \{U_i(\cdot)\}]$ if, for every $i \in \mathcal{N}$, $U_i(q_i^*, \mathbf{q}_{-i}^*) \geq U_i(q_i', \mathbf{q}_{-i}^*)$ for all $q_i' \in Q_i$.

Theorem 1. A Nash equilibrium in terms of QoS for game $\mathbf{G} = [\mathcal{N}, \{P_i, Q_i\}, \{U_i(\cdot)\}]$ exists and is unique.

Proof. To prove existence, we note that each SP's strategy space Q_i is defined by all QoSs in the closed interval bounded by the minimum and maximum QoSs. Thus, the joint strategy space Q is a nonempty, convex, and compact subset of the Euclidean space \mathbb{R}^N . In addition, the utility functions are concave with respect to QoSs as can be seen from the second derivative test:

$$\frac{\partial^2 U_i(\mathbf{p}, \mathbf{q})}{\partial q_i^2} = -2\theta_i < 0, \quad \forall i \in \mathcal{N}, \quad (5)$$

which ensures existence of a Nash equilibrium.

In order to prove uniqueness, we follow, [20], and define the weighted sum of user utility functions.

$$\varphi(\mathbf{q}, \mathbf{x}) = \sum_{i=1}^N x_i U_i(q_i, \mathbf{q}_{-i}), \quad (6)$$

The pseudo-gradient of (6) is given by :

$$g(\mathbf{q}, \mathbf{x}) = [x_1 \nabla U_1(q_1, \mathbf{q}_{-1}), \dots, x_N \nabla U_N(q_N, \mathbf{q}_{-N})]^T \quad (7)$$

The Jacobian matrix \mathbf{J} of the pseudo-gradient (w.r.t. \mathbf{q}) is written

$$\begin{aligned} \mathbf{J} &= \begin{pmatrix} x_1 \frac{\partial^2 U_1}{\partial q_1^2} & x_1 \frac{\partial^2 U_1}{\partial q_1 \partial q_2} & \cdots & x_1 \frac{\partial^2 U_1}{\partial q_1 \partial q_N} \\ x_2 \frac{\partial^2 U_2}{\partial q_2 \partial q_1} & x_2 \frac{\partial^2 U_2}{\partial q_2^2} & \cdots & x_2 \frac{\partial^2 U_2}{\partial q_2 \partial q_N} \\ \vdots & \vdots & \ddots & \vdots \\ x_N \frac{\partial^2 U_N}{\partial q_N \partial q_1} & x_N \frac{\partial^2 U_N}{\partial q_N \partial q_2} & \cdots & x_N \frac{\partial^2 U_N}{\partial q_N^2} \end{pmatrix} \\ &= \begin{pmatrix} -2x_1 \theta_1 & 0 & \cdots & 0 \\ 0 & -2x_2 \theta_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -2x_N \theta_N \end{pmatrix}. \end{aligned}$$

□

Thus, \mathbf{J} is a diagonal matrix with negative diagonal elements. This implies that \mathbf{J} is negative definite. Henceforth $[\mathbf{J} + \mathbf{J}^T]$ is also negative definite, and according to Theorem (6) in, [20], the weighted sum of the utility functions $\varphi(\mathbf{q}, \mathbf{x})$ is diagonally strictly concave. Thus the fixed-price Nash equilibrium point

$$q_i^* \in \operatorname{argmax}_{q_i \in Q_i} U_i(q_i, \mathbf{q}_{-i}^*), \quad \forall i \in \mathcal{N}. \quad (8)$$

is unique.

Fixed-QoS Game : When fixing the QoS, a Nash equilibrium in terms of price is formally defined as :

Definition 2. A price vector $\mathbf{p}^* = (p_1^*, \dots, p_N^*)$ is a Nash equilibrium of the NPQG : $\mathbf{G} = [\mathcal{N}, \{P_i, Q_i\}, \{U_i(\cdot)\}]$ if, for every $i \in \mathcal{N}$, $U_i(p_i^*, \mathbf{p}_{-i}^*) \geq U_i(p_i', \mathbf{p}_{-i}^*)$ for all $p_i' \in P_i$.

Theorem 2. A Nash equilibrium in terms of price for the game $\mathbf{G} = [\mathcal{N}, \{P_i, Q_i\}, \{U_i(\cdot)\}]$ exists and is unique.

Proof. To prove existence, we note that each SP's strategy space P_i is defined by all prices in the closed interval bounded by the minimum and maximum prices. Thus, the joint strategy space P is a nonempty, convex, and compact subset of the Euclidean space \mathbb{R}^N . In addition, the utility functions are concave with respect to prices as can be seen from the second derivative test:

$$\frac{\partial^2 U_i(\mathbf{p}, \mathbf{q})}{\partial p_i^2} = -2\alpha_i^i < 0, \quad \forall i \in \mathcal{N}, \quad (9)$$

which ensures existence of a Nash equilibrium.

To prove uniqueness we define now the weighted sum of user utility functions

$$\phi(\mathbf{p}, \mathbf{x}) = \sum_{i=1}^N x_i U_i(p_i, \mathbf{p}_{-i}), \quad (10)$$

the pseudo-gradient of this later is given by

$$g(\mathbf{p}, \mathbf{x}) = [x_1 \nabla U_1(p_1, \mathbf{p}_{-1}), \dots, x_N \nabla U_N(p_N, \mathbf{p}_{-N})]^T. \quad (11)$$

In order to show that $\phi(\mathbf{p}, \mathbf{x})$ is diagonally strictly concave in this case we use the following lemma proved in, [11].

Lemma 1. *If each $U_i(\mathbf{p})$ is a strictly concave function in p_i , each $U_i(\mathbf{p})$ is convex in p_{-i} and there is some $\mathbf{x} > 0$ such that $\phi(\mathbf{p}, \mathbf{x})$ is concave in \mathbf{p} , then $[\mathbf{J}(\mathbf{p}, \mathbf{x}) + \mathbf{J}^T(\mathbf{p}, \mathbf{x})]$ is negative definite, where $\mathbf{J}(\mathbf{p}, \mathbf{x})$ is the Jacobian of $g(\mathbf{p}, \mathbf{x})$.*

From equation (9), we know that $U_i(\mathbf{p})$ is strictly concave in p_i . Further

$$\frac{\partial^2 U_i}{\partial p_j^2} = 0, \quad \forall i \neq j,$$

which implies that $U_i(\mathbf{p})$ is convex in p_{-i} as well. Also, we have that

$$\begin{aligned} \frac{\partial^2 \phi(\mathbf{p}, \mathbf{x})}{\partial p_i^2} &= x_i \frac{\partial^2 U_i(p_i, \mathbf{p}_{-i})}{\partial p_i^2} + \sum_{j \neq i}^N x_j \frac{\partial^2 U_j(p_i, \mathbf{p}_{-i})}{\partial p_i^2} \\ &= -2x_i \alpha_i^i < 0, \quad \forall i, \end{aligned}$$

then $\phi(\mathbf{p}, \mathbf{x})$ is concave in p_i and from Lemma 1 we have that $[\mathbf{J}(\mathbf{p}, \mathbf{x}) + \mathbf{J}^T(\mathbf{p}, \mathbf{x})]$ is negative definite. Thus the weighted sum of utility functions $\phi(\mathbf{p}, \mathbf{x})$ is diagonally strictly concave. The fixed-QoS Nash equilibrium point is then unique and is given by

$$p_i^* \in \operatorname{argmax}_{p_i \in P_i} U_i(p_i, \mathbf{p}_{-i}^*), \quad \forall i \in \mathcal{N}. \quad (12)$$

□

3.2. The joint price and QoS game

As shown in equations (5) and (9), the utility functions $U_i(\mathbf{p}, \mathbf{q})$, $\forall i \in \mathcal{N}$, are concave respectively w.r.t. q_i and p_i . So, for all, $i \in \mathcal{N}$, the QoS and price conditions which maximizes the utility given in equation (2) are respectively :

$$\begin{cases} \frac{\partial U_i(\mathbf{p}, \mathbf{q})}{\partial q_i} = 0 \\ \frac{\partial U_i(\mathbf{p}, \mathbf{q})}{\partial p_i} = 0 \end{cases}$$

Thus, the computation of Nash Equilibrium can be performed by solving latter system.

Now, we turn to develop a fully distributed algorithm to learn the two-parameter equilibrium. Designing distributed algorithms that converge quickly to equilibrium is one of the foremost research goals in algorithmic game theory, and convex programs have played a crucial role in the design of algorithms for markets. Assuming that Providers are selfish and choose dynamically each one the best price and QoS that maximize his profiles, the distributed algorithms can be thought of as protocols that players are programmed to follow. The design and analysis of distributed algorithms converging to equilibria in the context of games has also received considerable attention, most commonly convergence of best response dynamics.

Solutions of equations induced by vanishing the partial derivatives correspond respectively to the best response in terms of QoS $BR_q^i(\cdot)$, and best response Price $BR_p^i(\cdot)$, of each SP- i as a function of the strategies of its opponents. Since Nash equilibrium point is unique, then a best response-based dynamics would converge to the joint Price-QoS NE. The two-parameters best response dynamics is detailed in Algorithm 1.

Algorithm 1 Best response dynamics

- 1: Initialize price and QoS vectors \mathbf{p} and \mathbf{q} randomly;
 - 2: For each service provider $i \in \mathcal{N}$ at iteration t :
 - a) $p_i^{t+1} = BR_p^i(\mathbf{p}^t, \mathbf{q}^t)$;
 - b) $q_i^{t+1} = BR_q^i(\mathbf{p}^t, \mathbf{q}^t)$.
-

3.3. Social welfare and price of anarchy

The concept of social welfare [17] or total surplus [21], is defined as the sum of the utilities of all agents in the systems (i.e. Providers). It is well known in game theory that agent selfishness, such as in a Nash equilibrium, does not lead in general to a socially efficient situation. As a measure of the loss of efficiency due to the divergence of user interests, we use the Price of Anarchy (PoA) [19], this latter is a measure of the loss of efficiency due to actors' selfishness. This loss has been defined in [19] as the worst-case ratio comparing the global efficiency measure (that has to be chosen) at an outcome of the noncooperative game played among actors, to the optimal value of that efficiency measure. A PoA close to 1 indicates that the equilibrium is approximately socially optimal, and thus the consequences of selfish behavior are relatively benign. The term Price of Anarchy was first used by Koutsoupias and Papadimitriou [19] but the idea of measuring inefficiency of equilibrium is older. The concept in its current form was designed to be the analogue of the "approximation ratio" in Approximation Algorithms or the "competitive ratio" in Online Algorithms. As in [12], we measure the loss of efficiency due to actors' selfishness as the quotient between the social welfare obtained at the Nash equilibrium and the maximum value of the social welfare:

$$PoA = \frac{\min_{\mathbf{p}, \mathbf{q}} W_{NE}(\mathbf{p}, \mathbf{q})}{\max_{\mathbf{p}, \mathbf{q}} W(\mathbf{p}, \mathbf{q})} \quad (13)$$

where $W(\mathbf{p}, \mathbf{q}) = \sum_{i=1}^N U_i(\mathbf{p}, \mathbf{q})$ is a welfare function and $W_{NE}(\mathbf{p}, \mathbf{q}) = \sum_{i=1}^N U_i(\mathbf{p}^*, \mathbf{q}^*)$ is a sum of utilities of all actors at Nash Equilibrium.

4. Numerical investigations

To clarify and show how to take advantage from our theoretical study, we suggest to study numerically the market share game while considering the best response dynamics and expressions of demand as well as utility functions of SPs. Hence, we consider a system with two SPs seeking to maximize their respective revenues. Table 1 represents the system parameter values considered in this numerical study.

$\alpha_1^1 = \alpha_2^2$	$\alpha_2^1 = \alpha_1^2$	$\beta_1^1 = \beta_2^2$	$\beta_2^1 = \beta_1^2$	D_0^1	D_0^2
0.7	0.3	0.7	0.3	300	250
$\theta_1 = \theta_2$	$p_1 = p_2$	\bar{p}_1, \bar{p}_2	$q_1 = q_2$	\bar{q}_1, \bar{q}_2	
20	100	1000	0	10	

Table 1. System parameters used for numerical examples.

Figures 1 and 2 present respectively curves of the convergence to Nash Equilibrium Price and to Nash Equilibrium QoS. It is clear that the best response dynamics converges to the unique Nash equilibrium price and QoS. We also remark that the speed of convergence is relatively high (around 9 rounds are enough to converge to the joint price and QoS equilibrium).

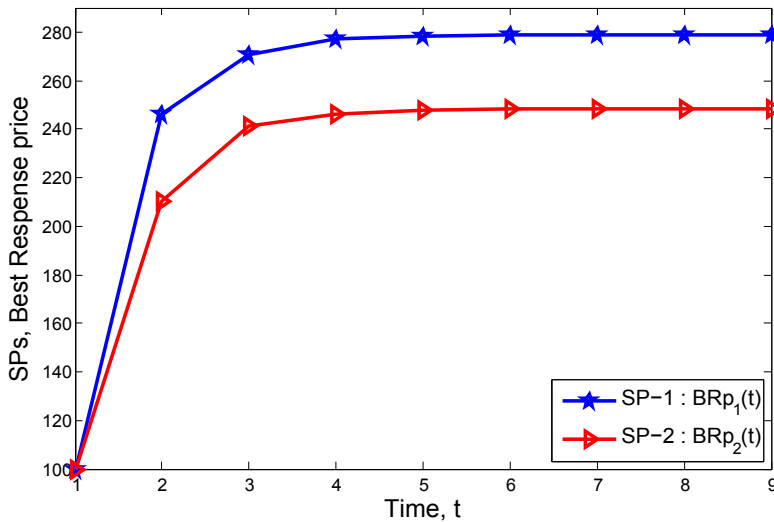


Figure 1. Price game : Convergence to the Price Nash equilibrium.

Next we plot in figures 3 and 4, respectively, the interplay of bandwidth cost ($\theta_i, i \in \{1, 2\}$) on the price and QoS at Nash equilibrium, for both SPs that we consider in this example. On one hand, we note that the equilibrium price for both SPs is increasing with respect to the

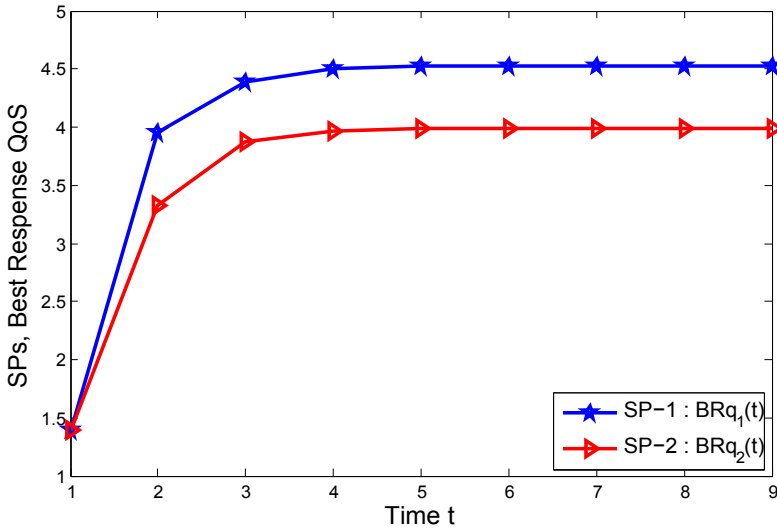


Figure 2. QoS game : Convergence to the QoS Nash equilibrium.

bandwidth cost. On the other hand, we note that the equilibrium QoS for all SPs is decreasing with the bandwidth cost. When the cost of bandwidth decided by the network owner is cheaper, the SPs invest for more bandwidth, so as to offer better QoS and an attractive price.

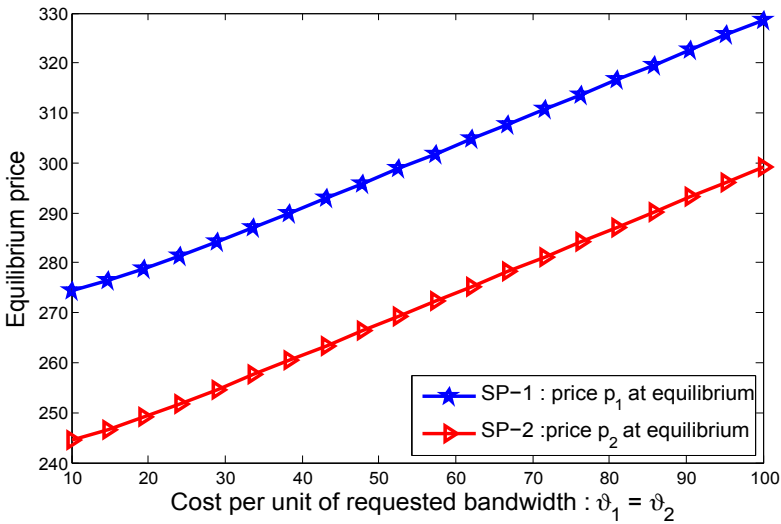


Figure 3. Equilibrium Prices w.r.t cost per unit of requested bandwidth ϑ_i .

In the following, we discuss the impact of the system parameters on the system efficiency in terms of Price of anarchy:

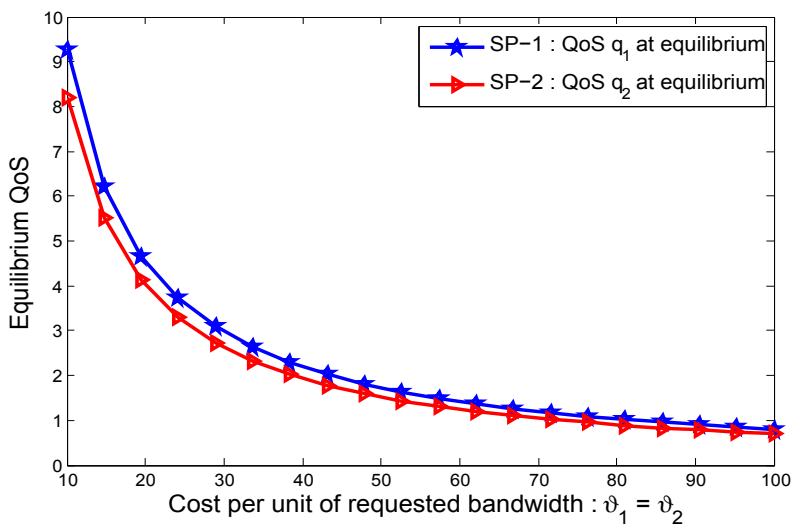


Figure 4. Equilibrium QoSs w.r.t cost per unit of requested bandwidth ϑ_i .

Influence of ϑ_i (cost per unit of requested bandwidth): Figure 5 shows the PoA variation curve as a function of the providers’ bandwidth cost ϑ_i . Without loss of generality, we assume that $\vartheta_1 = \vartheta_2$. A special feature is that the Nash equilibrium performs well and the loss of efficiency is only around 8%. This result indicates that the Nash equilibrium of this game is fair and socially efficient. Henceforth, selfish players would not need the help of a third-part regulator (who recommends the players the best strategy profile to achieve their respective best outcomes) to get attracted by the optimum social welfare. However, the network owner can use the value of the bandwidth cost to control the selfishness/aggressiveness of the service providers, which will improve the whole network performance.

Influence of α (Sensitivity of SP- i to his price p_i): Figure 6 plots the variation curve of price of anarchy with respect to α which represents the sensitivity of SP- i to his price p_i . In that figure, we first notice that the price of anarchy increases when α increases, the fact that the price of anarchy increases with α finds the simple intuition that increasing the sensitivity of SPs to their prices gives more and more freedom to SPs for optimizing the Nash equilibrium. On the other hand, when $\alpha = \alpha_1^1 = \alpha_2^2 = 1$, in the other word, when the sensitivity of an SP to the price of its competitor is zero ($\alpha_1^2 = \alpha_2^1 = 0$), price of anarchy converges to 1 and so the equilibrium is approximately socially optimal.

Influence of β (Sensitivity of SP- i to his QoS q_i): Figure 7 illustrates variations of PoA as a function of, β , which is the sensitivity of SPs to their respective own QoS. We first notice that the loss of efficiency is around 8%. Moreover the curve of PoA is concave, this latter mean that there are some, $\beta^* < 1$, which optimizes the equilibrium, ($\beta^* = \beta_1^1 = \beta_2^2 = 0.76$, $PoA^* = 0.925$). Surprisingly, the price of anarchy varies slightly (variation of almost 0.001). To explain this behaviour, Figures 8 and 9 depict, respectively, the curves of equilibrium Price and QoS of SP-1 and SP-2. We find that the induced variation of the price is much higher compared to

that of QoS, and subsequently, β (Sensitivity of SPs to their QoS) has a smaller impact on the system.

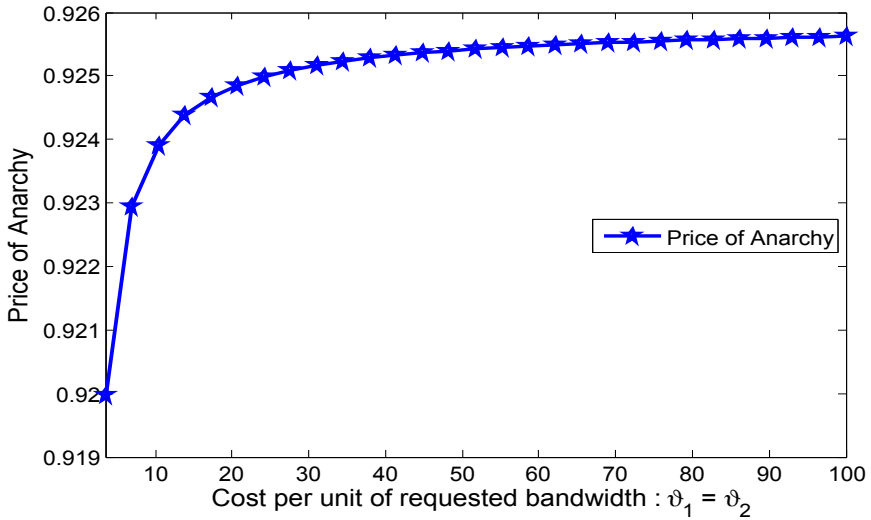


Figure 5. Price of Anarchy as a function of cost per unit of requested bandwidth ϑ_i .

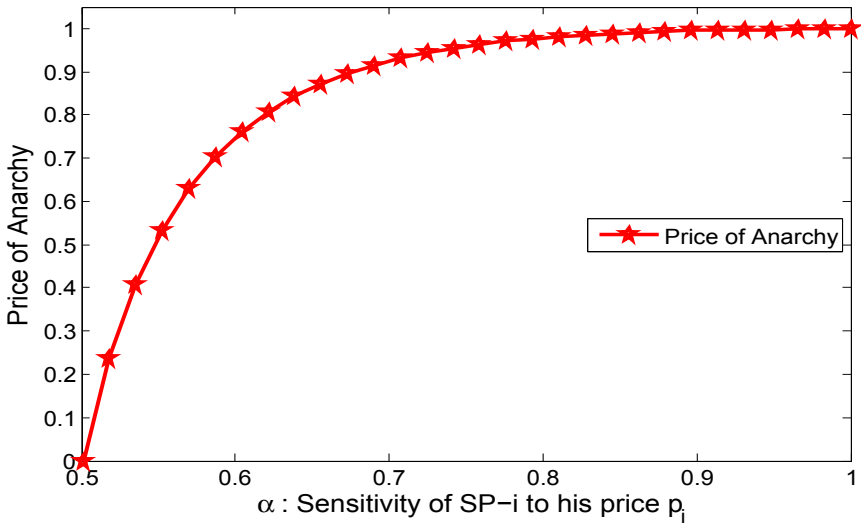


Figure 6. Price of Anarchy as a function of $\alpha = \alpha_1^1 = \alpha_2^2$ (Sensitivity of SP-i to his price p_i)

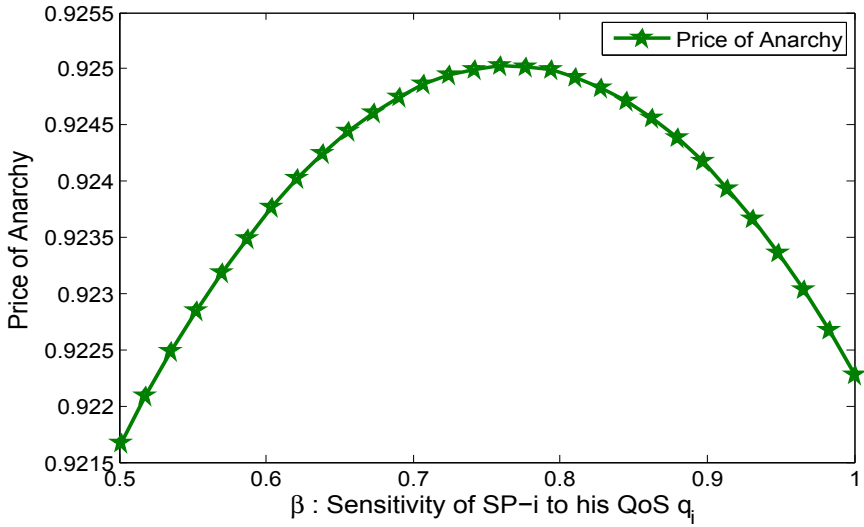


Figure 7. Price of Anarchy as a function of $\beta = \beta_1^1 = \beta_2^2$ (Sensitivity of SP-*i* to his QoS q_i)

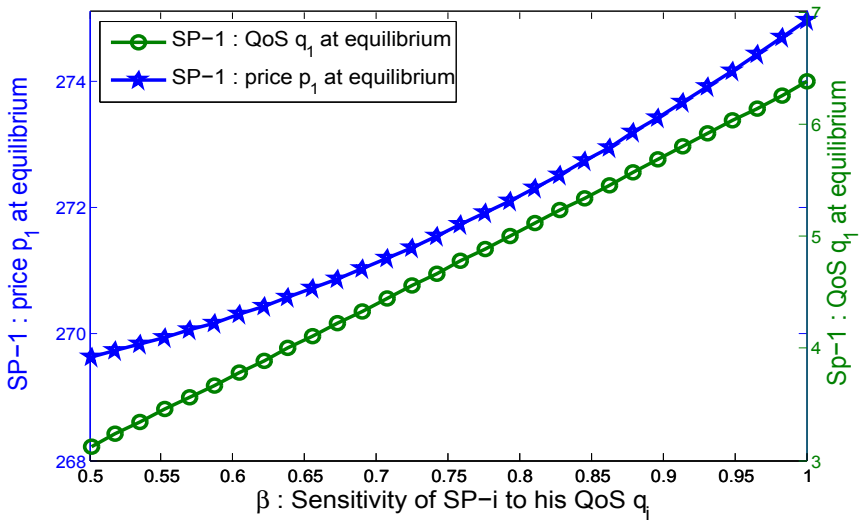


Figure 8. equilibrium Price and QoS of SP-1 as a function of $\beta = \beta_1^1 = \beta_2^2$ (Sensitivity of SP-*i* to his QoS q_i)

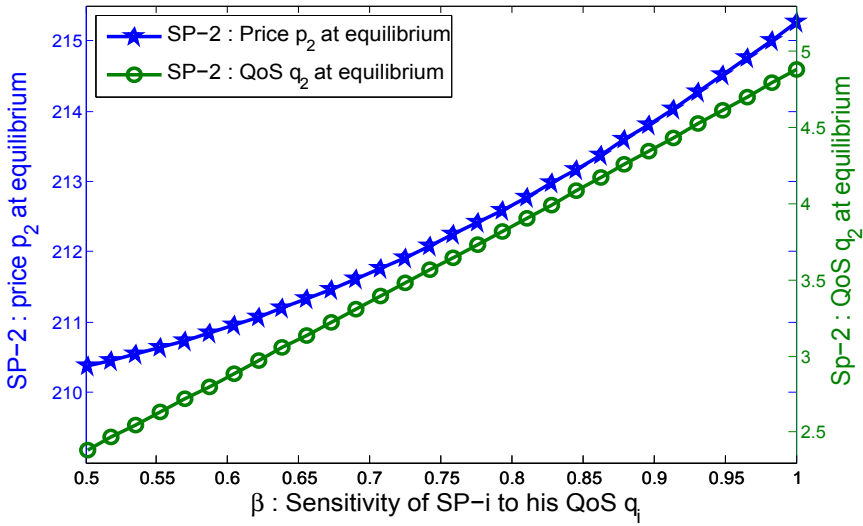


Figure 9. equilibrium Price and QoS of SP-2 as a function of $\beta = \beta_1^1 = \beta_2^2$ (Sensitivity of SP- i to his QoS q_i)

5. Conclusion

In this work, we presented and analyzed a framework to model the complex interactions among SPs as players through a class of two parameter Nash equilibrium models. The model is based on a simple linear demand functions which describe customer behaviour, take into account not only the characteristics of a current SP, (SP- i), but also of all other SPs, (SP- j , $j \neq i$), the presence of two parameters describing each SP's service price and QoS level. We established uniqueness of a Nash equilibrium point and developed a distributed algorithm to learn it. Then, our proposed algorithm finds very fast the equilibrium price and the equilibrium QoS to be chosen by each provider. Our scheme is different from previous approaches since it involves two varying parameters in a simple implementation and low complexity. Yet, we have obtained some insightful results such as the interplay of bandwidth cost. Results found in this work can be further extended to general network considerations, in particular under non-neutrality perspective or non-linear demand.

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