# Predicting Converter Utilization Convergence under Various Dynamic Loading Conditions of Paralleled Converters 

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#### Abstract

In this paper, converter utilization equalization under various dynamic loading conditions experienced by parallel buck converters were observed. Equalization of converter utilization is ensured by using fuzzy logic control while output voltage regulation and current sharing between converters is maintained by employing sliding mode control. This research work has also contributed the derivation of the working equation in determining the converter's utilization point of equalization even in dynamic loading. Simulation results have confirmed that the value solved from the working equations are correct and equal to the value of the utilizations each of the converters converged under a given dynamic load.


Index Terms-dynamic loading conditions, converter utilization, fuzzy logic, sliding mode control

## I. Introduction

Paralleling DC-DC converters has many desirable advantages including reliability due to redundancy, flexibility due to modularity, and stability due to reduced thermal stress since each module handles a lower power level [1]. Paralleled power supplies have also increased load power capacity through equal contribution from each converter in the paralleled system. In conventional parallel systems, each of the power supplies equally share the load, irregardless of how high or low the load current is. For example, if a certain parallel system with four converters installed can handle up to 10 A and sees a 6 A load current, each converter will supply 1.5 A each even though it is possible that only three of the four converters are turned ON to supply 2 A each while the other remaining converter is turned OFF. Such scenario has been studied in [2] applying an elementary and simplified control of turning converters ON or OFF. The latter case reduces the usage time of a certain converter which is turned OFF during a given load condition. Low load condition is characterized when the number of turned ON

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converters needed to meet the load requirement is less than the total number of converters in parallel.

The study of using fuzzy logic to address equal converter utilization under static and dynamic loading has already shown remarkable results, whether in low or in high load conditions [3] and was first mentioned in [4]. In static loading, it was observed that during low load conditions, converter turns ON and OFF and after a certain amount of time, achieves equal utilization, i.e., converter usage time is the same for all converters. In dynamic loading, system stability was checked. During a change in load current, the parallel system was able to regulate at the desired output voltage. It is to be noted that classical compensation and average current control was used to achieve equal current sharing and output voltage regulation. This research work is the extension of an earlier work [3] primarily focusing on two contributions: 1) the use of sliding mode control to achieve equal current sharing and voltage regulation and 2) derivation of the working equation to determine the point of convergence, $U_{\text {conv }}$, of each of the converter in the parallel system during dynamic loading conditions.

This paper is organized as follows. Section II provides the Simulink hierarchical model of the parallel converters, derivation of the sliding mode control coefficients to be used to regulate the parallel buck converters and fuzzy control rule base used to determine the probability of the converter turning ON. Section III discusses the derivation of the converter utilization value during dynamic loading conditions while section IV presents the simulation results of such system and empirical verification of the derived formula of the parallel converters during various dynamic loading conditions.

## II. Parallel Buck Converters Simulink Model

The buck or step-down DC-DC converter provides an output that is less than its input voltage, $V_{\text {out }}<V_{i n}$. The steady state voltage transfer function is defined by the ratio $\frac{V_{\text {out }}}{V_{\text {in }}}=D$, where $D$ is the switch duty cycle. The

Matlab Simulink buck converter model used is derived from the works from Colorado University [4]. The Simulink hierarchical model of the parallel converters is shown in Fig. 1. Sliding mode control which is a type of non-linear controller developed by V. Utkin primarily
intended for variable structure system was used to regulate the output voltage and current sharing of the parallel converters [5]. This control has a guaranteed stability and robustness against parameter changes independent of the variation of load and line changes.


Figure 1. Hierarchical model simulink model of four parallel DC-DC converters

Sliding mode control (SMC) is a very attractive method for switched mode power supplies (SMPS) due to their inherent variable state structure (VSS). However, traditional SMC requires infinite switching frequency which is not optimum for (SMPS). In an earlier work by Tan et al. the principles for a fixed frequency buck converter has been laid [6]. However, this was only used for a single converter. An earlier work by one of the authors has shown effectiveness of using sliding mode control in parallel converters and the effectiveness of the control in delivering equal load current to the converters [7]. This paper used the principles laid out in the use of sliding mode control and extended the study to four buck converters in parallel. The specifications of the buck converter are shown in Table I.

The chosen state variables for the sliding mode control are stated below in (1) [7]

$$
\begin{align*}
& x_{1}=i_{\text {ref }}-\delta i_{L} \\
& x_{2}=V_{\text {ref }}-\beta V_{\text {out }}  \tag{1}\\
& x_{3}=\int\left(i_{\text {ref }}-\delta i_{L}\right) d t+\int\left(V_{\text {ref }}-\beta V_{\text {out }}\right) d t
\end{align*}
$$

where:
$i_{L}$ - load current
$\delta$-inductor current scaling factor
$i_{\text {ref }}$ - reference load current and is given by
$V_{\text {ref }}$ - output volage reference
$V_{\text {out }}$ - output volage
$\beta$ - voltage divider gain

TABLE I. Buck Converter Specifications

| Specifications | Value |
| :---: | :---: |
| Input Voltage | 20 V |
| Output Voltage | 12 V |
| Output Regulation Band | $\pm 0.2 \mathrm{~V}$ |
| Maximum Load Current | 2.5 Amps |
| Minimum Load Current | 0 Amp |
| Switching frequency | 100 kHz |
| Number of Buck Converters in <br> Parallel | 4 |

Since average current sharing method is used as the sharing method, the current reference is defined in (2).

$$
\begin{equation*}
i_{r e f}=\frac{1}{n} \sum_{i=1}^{i=n} i_{L i} \tag{2}
\end{equation*}
$$

where:
$n$ - number of VRM modules connected in parallel
$i_{L}-$ load current
The sliding surface is then given as [6]:

$$
\begin{align*}
S= & \alpha_{1}\left(i_{\text {ref }}-\delta i_{L}\right)+\alpha_{2}\left(V_{\text {ref }}-\beta V_{\text {out }}\right)+ \\
& \alpha_{3}\left[\int\left(i_{\text {ref }}-\delta i_{L}\right) d t+\int\left(V_{\text {ref }}-\beta V_{\text {out }}\right) d t\right] \tag{3}
\end{align*}
$$

The solutions of the trajectories for the sliding surface are shown in Fig. 2. Since the buck converter changes its
structure depending on the position of its power switch, a closer inspection of the surface plot is in order.


Figure 2. Trajectory paths for the different solutions of the SMC differential equation

Notice that there are two ellipses in the surface plot, with the smaller ellipse representing the state of the power switch as "off" and the larger ellipse representing the state of the power witch as "on". Redrawing the surface plot reveals that the state trajectories are only present in the x 1 x 2 dot plane. Thus it can be redrawn in a 2-dimensional graph as seen in Fig. 3. Since the objective is to have a path going to equilibrium $(0,0)$ it was assumed that the sliding surface is a line passing through the origin. The state of the switch being "off" is indicated above by the red line. It intersected a solution with the trajectory of the small ellipse, thus, this path is followed as it intersected the sliding surface as it moved towards the origin.


Figure 3. Surface plot with switch at "OFF" position
The derivation of the equivalent control gives the duty function in terms of the circuit parameters [6]:

$$
\begin{equation*}
u_{\text {eq }}=\frac{\delta V_{\text {out }}}{\delta V_{\text {in }}}-\frac{K_{1} i_{c}}{\delta V_{\text {in }}}+\frac{K_{2}}{\delta V_{\text {in }}}\left[\left(i_{\text {ref }}-\delta i_{L}\right)+\left(V_{\text {ref }}-\beta V_{\text {out }}\right)\right] \tag{4}
\end{equation*}
$$

where:

$$
\begin{equation*}
K_{1}=\frac{\alpha_{2}}{\alpha_{1}} \frac{\beta L}{C} \text { and } K_{2}=\frac{\alpha_{3}}{\alpha_{1}} L \tag{5}
\end{equation*}
$$

The same derived duty function was utilized in this study for the control of the four buck converters in parallel. Equation (4) is easily modelled in Matlab Simulink using its gain and operation blocks. Equation (5) on the other hand determines the possible choices of coefficients to achieve equal current sharing and output voltage regulation. The control of which converter(s) will turn ON and OFF] using fuzzy logic as the control scheme has already been presented [3]. The fuzzy control takes in the input voltage and converter time of usage as its inputs and outputs the converter probability of turning ON. The rule base in the fuzzy logic controller can be represented as:

## If \{Utilization (\%)\} and \{Input/supply voltage (V)\},

Then \{Probability (P)\}
The fuzzy logic controller's rule base has been retuned using the same inputs and output represented by triangular membership functions, as presented in Table II below.

TABLE II. FuzZy Rule Base

| Vin | L | L-M | M | M-H | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| V-L | HP | P | LP | VLP | VVLP |
| UV | VHP | HP | P | LP | VLP |
| OK | VVHP | VHP | HP | HP | LP |
| OV | VHP | HP | P | LP | VLP |
| V-H | HP | P | LP | VLP | VVLP |

## III. Derivation of Converter Utilization CONVERGENCE DURING DYNAMIC LOADING Conditions

It was also shown that the fuzzy control scheme during has equalized all converter usage times while providing the regulated voltage and appropriate needed load current both for static and dynamic loading conditions [3]. For static conditions, it was easily determined that the utilization of the four converters converges at the values given in Table III below.
table iII. Converter Utilization Values at Various Static
LOADING CONDITIONS

| Load Current (Iout) | Utilization Convergence Value $\left(\mathrm{U}_{\text {conv }}\right)$ |
| :---: | :---: |
| $0<$ Iout $\leq 2.5$ | $25 \%$ |
| $2.5<$ Iout $\leq 5$ | $50 \%$ |
| $5<$ Iout $\leq 7.5$ | $75 \%$ |
| $7.5<$ Iout $\leq 10$ | $100 \%$ |

For dynamic loading conditions, the utilization convergence values are not yet provided, so it is in this paper that such formula be derived and verified by empirical simulation experiments. A dynamic loading condition such as that shown in Fig. 4 was used to determine stability of the converter's stability during a sudden change in load current. Dynamic loading can be treated as the superposition of various static loading conditions.

It was already shown that sliding mode controlled buck converters provide good transient results when compared
to classically compensated DC-DC converters, e.g. rise time and overshoot [7].


Figure 4. Dynamic loading condition
In trying to equalize the converter utilization throughout the dynamic loading condition, in each part of the loading condition, $I_{n}$ at time $t_{O N n}$, the converter's utilization, $U_{\text {conv }}$, must converge, thus, we have:

$$
\begin{equation*}
U_{\text {conv }}=\frac{1}{N} \text { ceiling }\left(\frac{I_{x}}{I o_{\max }}\right) \times 100 \% \tag{6}
\end{equation*}
$$

where:
$N=$ number of converters in parallel
$I_{x}=$ load current $x$
$I o_{\max }=$ maximum outputcurrent of a single converter
If $\mathrm{N}=4$ and the load currents in Table III are substituted in Equation (6), then the utilization convergence values found also in Table III are established. This equation is then extended to account for dynamic loading. Since Equation (6) is only applicable for a time interval in a given dynamic loading condition, it must be multiplied by the percentage of how long it was turned ON, mathematically, this is shown by:

$$
\begin{equation*}
U_{\text {conv }}=\frac{1}{N}\left(\frac{t_{O N x}}{t} \operatorname{ceiling}\left(\frac{I_{x}}{I o_{\max }}\right)\right) \times 100 \% \tag{7}
\end{equation*}
$$

Finally expanding Equation (7) to accommodate the full dynamic loading condition, the final equation is given by Equation (8).

$$
\begin{equation*}
U_{\text {conv }}=\frac{1}{N} \sum_{x=1}^{n} \frac{t_{O N x}}{t} \text { ceiling }\left(\frac{I_{x}}{I o_{\max }}\right) \times 100 \% \tag{8}
\end{equation*}
$$

where:
$N=$ number of converters in parallel
$x=\mathrm{i}$ th current load in the dy namic loading condition
$t_{\text {ON }}=$ duration of a load current $x$ in dy namic loading
$I_{x}=$ load current $x$ in dy namic loading
$I o_{\text {max }}=$ maximum outputcurrent of a single converter
$t=$ period of dynamic loading

Note that if there is a static load, $t_{O N x}=t$, thus Equation (8) is the same as Equation (6).

## IV. Experimental Results and Discussion

In verifying Equation (8) above, Table IV below summarizes the different dynamic loading tests. It must be emphasized that $t_{O N x}, x=1,2,3$ and 4 are all equal to 0.0125 sec for all test conditions. This is also the refresh rate of the fuzzy controller. For the sliding mode controller, the constants are: $\delta=0.974, \beta=0.2083, \mathrm{~V}_{\text {ref }}=$ $2.5 \mathrm{~V}, \mathrm{~K} 1=-5$ and $\mathrm{K} 2=50$. As $\delta$ approaches unity, the output voltage approaches the desired voltage of regulation. Increasing K1, which should always be negative, improves converter transient response while on the other hand, increasing K2 improves steady state response.

TABLE IV. Dynamic Loading Test Conditions

| Test | Uconv | I1 | I2 | I3 | I4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 68.75 | 1.00 | 5.10 | 9.00 | 7.00 |
| 2 | 62.5 | 10.00 | 2.00 | 6.00 | 3.00 |
| 3 | 50 | 4.00 | 2.00 | 4.00 | 7.00 |

Fig. 5 shows the parallel system's response to an 8-2-6-3 dynamic loading condition. It is evidently seen that SMC is able to perform voltage regulation and equal current sharing the same as that of classical compensation done previously [3].


Figure 5. Corresponding output voltage and current sharing outputs, Ix under dynamic loading of 8-2-6-3A

Fig. 6-8 shows the simulation results for Tests 1, 2 and 3 respectively. It is noticeable that the predicted utilization convergence of each converter even in dynamic loading conditions has been met and first occurs at $\mathrm{t}=0.2$ seconds. Equation (8) may also be extended such that varying $t_{O N x}$ may also be implemented. However, it is advised that the minimum value of $t_{O N x}$ must be also the refresh rate of the fuzzy controller to fully implement converter utilization. Using a dynamic loading condition of 3-8-4-5 having the same $t_{O N x}$, the converter utilization $U_{\text {conv }}$ is also equal to $50 \%$ similar to test 3. Therefore, Equation (8) provides us an idea of the limited values of the converter utilization even under various types of dynamic loading conditions.

It must also be emphasized that though the fuzzy controller provides the probability of a converter of turning ON, the Matlab function "Enable Switch" selects which converter will turn ON in case there are converters with the same probability. Such function chooses sequentially and in ascending order when converters have the same probability, e.g. if $P(C 2)=P(C 4)=0.9$, then choose converter 2 (C2).


Figure 6. Converter utilization under dynamic loading of 1-5.1-9-7A


Figure 7. Converter utilization under dynamic loading of 10-2-6-3A


Figure 8. Converter utilization under dynamic loading of 4-2-4-7A

## V. CONCLUSION

This paper has successfully derived and verified the converter's utilization convergence under various dynamic loading conditions. It has been proven that even in such cases, stability is ensured, regulation and converter utilization equalization have been met. It has also been shown that sliding mode control is an effective control method for current sharing and output voltage regulation. It is in the list of future directives of this work that simulation results be verified by actual experimentation using fuzzy logic and sliding mode control.

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