



Spatial competition in networks under delivered pricing*

Pablo Dorta-González, Dolores R. Santos-Peñate, Rafael Suárez-Vega

Departamento de Métodos Cuantitativos en Economía y Gestión, Universidad de Las Palmas de Gran Canaria, Facultad de Ciencias Económicas y Empresariales, Campus de Tafira, 35017 Las Palmas de Gran Canaria, Spain (e-mail address: pdorta@dmc.ulpgc.es)

Received: 3 April 2003 / Accepted: 6 October 2004

Abstract. We consider a two-stage non-cooperative Bertrand game with location choice involving r firms. There are n spatially separated markets located at the vertices of a network. Each firm first selects the location of a facility and then selects the delivered price in the markets in order to maximise its profit. The article extends the duopolistic model with completely inelastic demand (Lederer and Thisse 1990) to the oligopolistic scenario. Under moderate assumptions, a pure strategy equilibrium, which minimises social costs, exists. Furthermore, an equilibrium location can be obtained by finite steps and consists of vertices only.

JEL classification: L13, R32, D43

Key words: Bertrand oligopoly, networks, location, pricing, equilibrium

1 Introduction

The spatial competition literature, beginning with Hotelling (1929), has focused on the use of mill (or f.o.b.) pricing by competing firms. Hotelling considered two firms competing in a bounded linear market in which consumers with inelastic demand are uniformly distributed. The firms compete in price and location.

Different spatial pricing policies exist. In delivered pricing policy the firm provides and pays for the transportation costs. A particular case is the spatial

* Partially financed by FEDER and Ministerio de Ciencia y Tecnología, grant BFM2002-04525-C02-01.

discriminatory pricing, in which the firm charges for the product, depending on its delivery location. Hoover (1936) analysed spatial discriminatory pricing for firms with fixed locations and concluded that a firm serving a particular market would be constrained in its local price by the delivery cost of the other firms serving in that market. In situations where demand elasticity is “not too high”, market price is equal to the delivery cost of the firm with the next lowest delivery cost. This result was later extended to a spatial duopoly by Lederer and Hurter (1986) in a compact subset of the plane, and Lederer and Thisse (1990) in a network. We find a similar property in our oligopolistic model illustrated here.

Two different approaches are usually considered when firms choose location and price. Some authors assume a simultaneous choice of price and location. However, most models in this context follow Hotelling’s formulation and use a refinement of the Nash equilibrium. More precisely, firms are supposed to choose location and price, one at a time in a two-stage process, with the aim of maximising their own profits. The division into two stages is motivated by the fact that choice of location is usually prior to decision on price. In the first stage firms simultaneously choose their location. Given any outcome of the first stage, firms then simultaneously choose their price in the second stage. The corresponding two-stage solution is called a subgame perfect Nash equilibrium, which captures the idea that when firms select their location, they all anticipate the consequences of their choice on price competition.

The literature on spatial oligopoly with strategic choice location on networks is relatively small. Labbé and Hakimi (1991) present a model where firms take location and quantity decisions along a network of connected but spatially separated markets. In a duopoly with linear demand, Labbé and Hakimi (1991) show that under reasonable assumptions, a subgame perfect Nash equilibrium exists at the vertices. Sarkar et al. (1997) extend the previous results in the following ways: there is an arbitrary number of Cournot oligopolists; firms may set up multiple facilities along the network; and the demand functions may be non-linear. Dorta-González et al. (2004) in a similar model study the equilibrium of Stackelberg. Lederer and Thisse (1990) present a competitive model between two profit-maximising firms that produce and sell a single homogeneous product to customers located on a network with completely inelastic demand. The firms compete through their decisions concerning plant location, production technology and delivered pricing. They prove the existence of a Nash equilibrium in location, production technology and delivered pricing for any network configuration, and a vertex optimality property.

Gupta et al. (1994) use delivered pricing Bertrand competition with inelastic demand and discuss a vertical relationship. Spatial heterogeneity in the distribution of consumers in delivered pricing oligopoly is studied by Gupta et al. (1997).

In this article we analyse a Bertrand oligopoly where firms choose their locations on a network of connected but spatially separated markets. An example of such markets is found in large urban centres connected by highways. The firms play a two-stage game in which they take a location decision in the first stage and choose the delivered pricing in the second stage. We restrict the study to pure strategy equilibria. Delivered pricing is a common pricing policy for firms and

has been extensively discussed by Philips (1983). We assume that firms produce at a constant marginal production cost; this implies that markets can be treated independently when the locations of the firms are fixed.

Our work extends the article by Lederer and Thisse (1990) to the oligopolistic scenario. We prove the existence of a subgame perfect Nash equilibrium at the vertices of the network when transportation costs are concave with respect to distance. The concavity assumption is realistic under delivered pricing policies in certain situations, such as those where the product is transported by air (Brander and Zhang 1990). The vertex optimality property implies that an equilibrium solution to the network problem can be found by solving a discrete problem, which means that it allows us to determine an equilibrium by investigating a finite set of candidate sites. Moreover, we see that the locations of the firms are in equilibrium if each firm minimises the social cost (i.e., total cost to firms of supplying markets with goods it demands is minimised) with respect to the competitors' fixed location.

The remainder of the article is organised as follows. In Sect. 2 the notation and model are introduced, followed by the equilibrium analysis discussion in Sect. 3, and concluding remarks.

2 The model

Let $N = (V, E)$ be a undirected connected network with a finite vertex set $V = \{v_1, v_2, \dots, v_n\}$ and edge set, E . At each vertex, $v_k \in V$, a market is located where a given product is sold at unit price, p^k .

Each edge $[v_i, v_j] \in E$ has a positive length. The distance between two points x and y along the edges is denoted by $\delta(x, y)$ and is the length of the shortest path joining them.

A number of r firms located on the network at points $\mathbf{X} = (x_1, \dots, x_r)$, manufacture the product and ship it to markets $v_k \in V$. The following assumptions are made regarding the marginal production cost and the transportation cost function.

Assumption 1. *Unit transportation cost between firm i 's location x , $1 \leq i \leq r$, and market $v_k \in V$ is denoted by $t_i^k(x) = t_i(\delta(x, v_k))$ and is assumed to be positive, concave and increasing with respect to distance.*

Assumption 1 might appear artificial for those familiar with mill pricing location-price models, but who are unfamiliar with delivered pricing policies. For delivered pricing, concave transportation costs are assumed by many authors (Labbé and Hakimi 1991; Sarkar et al. 1997; Lederer and Thisse 1990, among others). This assumption is realistic in certain situations. Consider, for example, transport by air. As Brander and Zhang (1990) point out, the transportation cost is far from proportional to the distance. Large costs are incurred at take off and landing while the actual time in the air requires low costs, which yield concave transportation cost functions. Another example is the following situation. When the distance is small (large) the firm uses a truck (air cargo). Marginal cost for truck transport is larger than that for air transport, thus the transportation cost becomes concave.

Assumption 2. *Marginal production cost is assumed to be independent of the quantity produced and is denoted by $C_i(x)$, $1 \leq i \leq r$. Furthermore, it is a positive, concave function as x moves along any edge of the network.*

For the motivation of assumption 2, in terms of the objective to minimise the total cost of inputs, the reader is referred to Labbé and Hakimi (1991) who consider the following situation. Firm i , $i = 1, \dots, r$ produces the product by using J inputs $j = 1, \dots, J$. There are H_j possible sources of input j in N , denoted by y_{jh} , $h = 1, \dots, H_j$. The price of input j , at y_{jh} , is given and denoted by p_{jh} . The cost of transporting one unit of input j from y_{jh} to x_i is $t_j(\delta(y_{jh}, x_i))$, which is assumed to be increasing and concave with distance $\delta(y_{jh}, x_i)$. Let a_{ji} be the amount of input j used by firm i to produce one unit of the product. The marginal production cost at x_i is then given by:

$$C(x_i) = \sum_{j=1}^J \left[\min_{h=1, \dots, H_j} \{p_{jh} + t_j(\delta(y_{jh}, x_i))\} \right] a_{ji} \tag{1}$$

which, as a weighted sum of minima of concave functions, is a concave function of the distances $\delta(y_{jh}, x_i)$. Next, since each distance $\delta(y_{jh}, x_i)$ is a concave function of x_i as x_i moves along an edge, $C(x_i)$ is also concave.

Notice that the marginal production cost depends only on the location of the firm and the markets can therefore be treated independently of one another.

The marginal delivery cost of firm i at market v_k is given by:

$$c_i^k(x) = C_i(x) + t_i^k(x). \tag{2}$$

The price offered by firm i , $1 \leq i \leq r$, at market $v_k \in V$ is denoted by p_i^k and $\mathbf{P} = (p_i^k)_{\substack{1 \leq k \leq n \\ 1 \leq i \leq r}}$.

3 Equilibrium analysis

The equilibrium problem is solved in two steps, assuming that firms first decide location and then pricing. Suppose that at each market, v_k , customers have a fixed demand for the product of $\lambda_k > 0$. The second stage pricing problem is solved in the next subsection.

3.1 The second stage pricing problem

Let $\mathbf{X} = (x_1, \dots, x_r)$ be a vector of fixed locations for the firms. The second stage problem is a non-cooperative game in which firms determine the pricing that maximises their profit. We investigate the Nash equilibrium in prices.

We assume that customers purchase from the cheapest source. Thus, if $p_i^k < p_j^k, \forall j \neq i$, customers located at v_k will buy from firm i at price p_i^k . Define:

$$K_i = \{k : p_i^k < p_j^k; j \neq i\}, \tag{3}$$

$$M_i = \left\{k : p_i^k = \min_{j \neq i} p_j^k\right\}. \tag{4}$$

Then K_i is the set of markets controlled by firm i and M_i is the shared market by firm i .

The profit for firm i can be written as:

$$\pi_i(\mathbf{X}, \mathbf{P}) = \sum_{k \in K_i} \lambda_k (p_i^k - c_i^k) + \sum_{k \in M_i} \frac{\lambda_k}{r_i^k} (p_i^k - c_i^k), \tag{5}$$

where r_i^k is the number of firms that share the market v_k with firm i , that is:

$$r_i^k = \left| \left\{l : p_l^k = p_i^k = \min_{j \neq i} p_j^k\right\} \right|. \tag{6}$$

In our formulation we assume that firm i will not price below its marginal cost. We do not consider predatory pricing because a strategy of this type is difficult to justify in the context of a one-period model. For any pricing policy p_j^k , $j \neq i$, the optimal pricing policy for firm i is then:

$$p_{ie}^k = \max \left\{ c_i^k, \min_{j \neq i} \{c_j^k\} - \varepsilon \right\}, 1 \leq k \leq n, \tag{7}$$

where $\varepsilon > 0$ is arbitrarily small.

For small ε we see that a customer will be served by the firm with the lowest marginal delivery cost at a price slightly less than the next lowest competitors, marginal delivery cost. At its limit,

$$\bar{p}_i^k = \lim_{\varepsilon \rightarrow 0} p_{ie}^k = \max \left\{ c_i^k, \min_{j \neq i} c_j^k \right\} \tag{8}$$

and the resulting market price at which consumers will buy the homogeneous product is the second lowest marginal delivery cost of serving that market, that is:

$$\bar{p}^k = \min_{i \neq i^*} c_i^k, 1 \leq k \leq n, \tag{9}$$

with i^* such that $c_{i^*}^k = \min_{1 \leq i \leq r} c_i^k$.

If $c_{(1)}^k \leq c_{(2)}^k \leq \dots \leq c_{(r)}^k$ is a ranking of the marginal delivery costs in market v_k , then the market price at v_k is:

$$\bar{p}^k = c_{(2)}^k. \tag{10}$$

Under the rule used to define the profit function, at least two firms have to split the market demand, since they charge identical prices. Note, however, that

for any positive ε , demand goes to the lower cost firm. This discontinuity in the demand relationship may be eliminated by redefining the equal price rule, so that, if two firms have identical customer prices, the firm with the lowest marginal delivery cost will serve the customer. Such a rule is reasonable and intuitive because the firm with the cost advantage can serve the customer just by cutting its price by an arbitrarily small amount. Redefining the profit functions using this rule will result in profits being continuous for $\varepsilon = 0$, and the following pricing policy:

$$\bar{p}_i^k = \begin{cases} \min_{j \neq i} \{c_j^k\} & \text{if } c_i^k < c_j^k, \forall j \neq i \\ c_i^k & \text{otherwise} \end{cases} \tag{11}$$

is a Nash equilibrium.

Having characterised equilibrium pricing policies for fixed locations, we next analyse the existence of equilibrium locations.

3.2 The first stage location problem

The objective of the first stage game is to find an r -vector of locations such that no firm unilaterally finds it profitable to relocate. A vector of locations $\mathbf{X} = (x_1, \dots, x_r)$ is a Nash equilibrium if location x_i maximises the profit of firm i given the competitor locations $x_j, j \neq i$.

The profit for firm i can be written:

$$\pi_i(\mathbf{X}) = \sum_{k=1}^n \pi_i^k(\mathbf{X}) \tag{12}$$

and

$$\pi_i^k(\mathbf{X}) = \begin{cases} \lambda_k \left(\min_{j \neq i} \{c_j^k(x_j)\} - c_i^k(x_i) \right) & \text{if } c_i^k(x_i) < c_j^k(x_j), \forall j \neq i \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

Next we consider the equilibrium locations of the firms under the Nash equilibrium prices in the second stage. Using the concept of social cost defined by Lederer and Hurter (1986), we will illustrate that there is a relationship between social cost and a firm’s profit.

Definition (Lederer and Hurter 1986) *The social cost is the total cost incurred by the firms to supply demand to consumers in the market space in a cooperative cost minimising manner. If the firms are located at \mathbf{X} , then when the firms are cooperating to supply demand in a cost minimising manner, social cost is:*

$$SC(\mathbf{X}) = \sum_{k=1}^n \lambda_k \min_{1 \leq j \leq r} \{c_j^k(x_j)\}. \tag{14}$$

There is a relationship between social cost and a firm's profit under equilibrium pricing policies.

Proposition 1. *Under assumption 2, the profit function in the first stage can be written:*

$$\pi_i(\mathbf{X}) = \sum_{k=1}^n \lambda_k \min_{j \neq i} \{c_j^k(x_j)\} - SC(\mathbf{X}), \quad 1 \leq i \leq r. \quad (15)$$

Proof. Directly from definition of π_i^k and SC . □

Thus, the profit to firm i is the total cost that would be experienced by its rivals if they were serving the entire market in a cooperative manner, minus the social cost. This means that a firm will strive to minimise social cost, not its own delivered cost, in order to maximise its profit.

The relationship stated in proposition 1 leads to the following results:

Proposition 2. *The following results are obtained:*

- (i) *Under assumption 2, a vector of locations $\bar{\mathbf{X}} = (\bar{x}_1, \dots, \bar{x}_r) \in N^r$ is a Nash equilibrium if and only if each location \bar{x}_i minimises the social cost SC , given the location of each competitor $\bar{x}_j, j \neq i$. Moreover, a socially optimal location decision is a Nash equilibrium of the first stage game.*
- (ii) *Under assumptions 1 and 2, $SC(\mathbf{X})$ is a concave function in x_i when x_i moves along any edge of the network and x_j is unchanged, $\forall j \neq i$.*

Proof.

- (i) From proposition 1, assuming x_j fixed, $\forall j \neq i$, the first element in the profit function is constant. The profit function is then maximised in that location which minimises the social cost. Therefore, $\bar{x}_1, \dots, \bar{x}_r$ is a Nash equilibrium if and only if:

$$SC(\bar{x}_1, \dots, \bar{x}_i, \dots, \bar{x}_r) \leq SC(\bar{x}_1, \dots, x_i, \dots, \bar{x}_r), \quad \forall x_i \in N, 1 \leq i \leq r. \quad (16)$$

Moreover, if $\bar{\mathbf{X}} = (\bar{x}_1, \dots, \bar{x}_r)$ is a global minimum of $SC(\mathbf{X})$, then:

$$SC(\bar{\mathbf{X}}) \leq SC(\mathbf{X}), \quad \forall \mathbf{X} \in N^r, \quad (17)$$

and therefore,

$$SC(\bar{x}_1, \dots, \bar{x}_i, \dots, \bar{x}_r) \leq SC(\bar{x}_1, \dots, x_i, \dots, \bar{x}_r), \quad \forall x_i \in N, 1 \leq i \leq r. \quad (18)$$

- (ii) For each vertex $v_k \in V$, the distance $\delta_{x_i v_k} = \delta(x_i, v_k)$ is a concave function with respect to x_i when x_i moves along any edge. Since t_i is a concave increasing function of the distance, then $t_i^k = t_i \circ \delta_{x_i v_k}$ is a concave function with respect to x_i when x_i moves along any edge. Since $C_i(x)$ is a concave function in x when x moves along any edge of the network, then $c_i^k(x) = C_i(x) + t_i^k(x)$ is a concave function of x when x moves

along any edge of the network. Furthermore, $SC(\mathbf{X})$ is the addition of concave functions, therefore $SC(\mathbf{X})$ is a concave function of x_i when x_i moves along any edge of the network and x_j is unchanged. \square

This proposition shows that the locations of the firms are in Nash equilibrium if for each firm, its location minimises the social cost given the competitor locations. Moreover, an equilibrium may be found by minimising social cost. This means that if a firm anticipates that equilibrium prices will be employed by the other firms which are already located, the firm will strive to minimise social cost in order to maximise its profit.

In general, uniqueness is not satisfied because $SC(x_1, \dots, x_r)$ may have several global minimisers and, more importantly, equilibrium strategies need not globally minimise social cost SC . The equation:

$$SC(\bar{x}_1, \dots, \bar{x}_i, \dots, \bar{x}_r) \leq SC(\bar{x}_1, \dots, x_i, \dots, \bar{x}_r), \forall x_i \in N, 1 \leq i \leq r. \tag{19}$$

only requires minimisation in each individual component of SC . Therefore, socially optimal equilibria may not exist.

An example with several global minimisers is portrayed in the following: Consider $r = 2$ and the segment line $[1, 3]$ with nodes at points 1, 2, 3, and $\lambda_1 = 2, \lambda_2 = 1, \lambda_3 = 1$. The marginal cost at any point x on the network is $C_i(x) = 1$, and $t_i(d(x, v_k)) = d(x, v_k)$, for $i = 1, 2, 3$. We can check that the social cost is minimised when one firm is located at v_1 and the other at v_2 or at v_3 . Then (v_1, v_2) and (v_1, v_3) are Nash equilibria in the network and the equilibrium is therefore not unique.

The concavity of the social cost function guarantees the optimisation at the vertices, as shown in the following result:

Proposition 3. *Under assumptions 1 and 2, each firm minimises the social cost at some vertex of N , regardless of where its competitors are located.*

Proof. This is a direct consequence of proposition 2(ii). \square

Thus, we must limit ourselves to the vertices of the network to find equilibrium locations. This corresponds to a Hakimi-type property, where firms may limit themselves to vertices on the network, established here in the context of an oligopolistic spatial competition model.

Using proposition 3 we can now establish the existence of a subgame perfect Nash equilibrium consisting of vertices only in the following result:

Proposition 4. *Under assumptions 1 and 2, equilibrium locations always exist in the first stage of the game. Furthermore, there exists an equilibrium consisting of vertices only.*

Proof. From proposition 3 we can restrict the set of candidate sites to the set V of vertices. Given any starting location for a firm, we show that if each firm in turn responds by relocating at a vertex that minimises the social cost, then this process converges, in a finite number of iterations, to an equilibrium. By the

reduction-to-absurd method, assume that r sets $\{v_1^1, \dots, v_i^s\}$, $1 \leq i \leq r$, of vertices exist, which represent the choice sequence of locations by firms $1, 2, \dots, r$, respectively, with firm 1 being the first player, firm 2 being the second player, and so on, such that:

$$\begin{aligned} SC &= (v_1^2, \dots, v_{i-1}^2, v_i^2, v_{i+1}^1, \dots, v_r^1) \leq SC(v_1^2, \dots, v_{i-1}^2, v_i^1, v_{i+1}^1, \dots, v_r^1), 1 \leq i \leq r, \\ SC &= (v_1^3, \dots, v_{i-1}^3, v_i^3, v_{i+1}^2, \dots, v_r^2) \leq SC(v_1^3, \dots, v_{i-1}^3, v_i^2, v_{i+1}^2, \dots, v_r^2), 1 \leq i \leq r, \\ &\vdots \end{aligned} \tag{20}$$

$SC(v_1^1, \dots, v_{i-1}^1, v_i^1, v_{i+1}^s, \dots, v_r^s) \leq SC(v_1^1, \dots, v_{i-1}^1, v_i^s, v_{i+1}^s, \dots, v_r^s)$, $1 \leq i \leq r$, i.e., the process cycles. Since at least one of these expressions is a strict inequality, when we add up all these expressions, we obtain:

$$\sum_{j=1}^s \sum_{i=1}^r SC(v_1^{j+1}, \dots, v_i^{j+1}, v_{i+1}^j, \dots, v_r^j) < \sum_{j=1}^s \sum_{i=1}^r SC(v_1^{j+1}, \dots, v_i^j, v_{i+1}^j, \dots, v_r^j), \tag{21}$$

where $v_i^{s+1} = v_i^1$, $1 \leq i \leq r$. Putting all the terms on the left-hand side, we get:

$$\sum_{j=1}^s \sum_{i=1}^r [SC(v_1^{j+1}, \dots, v_i^{j+1}, v_{i+1}^j, \dots, v_r^j) - SC(v_1^{j+1}, \dots, v_i^j, v_{i+1}^j, \dots, v_r^j)] < 0. \tag{22}$$

Finally, the fact that $v_i^{s+1} = v_i^1$, $1 \leq i \leq r$, implies that all the terms on the left-hand side cancel and a contradiction occurs. \square

Thus, there exists a set of equilibrium locations consisting of vertices only and the search for an equilibrium can be limited to vertices that minimise social cost.

Proposition 4 provides an easy method to find a subgame perfect Nash equilibrium for networks satisfying assumptions 1 and 2. The following algorithm can be used to obtain a subgame perfect Nash equilibrium.

Algorithm

Step 0. Take $h = 0$ and let $\{v_1^h, \dots, v_r^h\}$ be a set of initial locations in V . Let $SC^h = SC(v_1^h, \dots, v_r^h)$.

Step 1. For $1 \leq i \leq r$, find v_i^{h+1} such that $SC(v_1^{h+1}, \dots, v_{i-1}^{h+1}, v_i^{h+1}, v_{i+1}^h, \dots, v_r^h) = \min_{v \in V} SC(v_1^{h+1}, \dots, v_{i-1}^{h+1}, v, v_{i+1}^h, \dots, v_r^h)$ and take $SC^{h+1} = SC(v_1^{h+1}, \dots, v_r^{h+1})$

Step 2. If $SC^{h+1} = SC^h$ then stop. The equilibrium is (v_1^h, \dots, v_r^h) . Otherwise, set $h = h + 1$ and go to step 1.

From proposition 4 this process converges, in a finite number of iterations, to an equilibrium.

4 Conclusion

We have analysed the Bertrand game involving r firms that first locate their facilities on a network connecting n spatially separated markets and then determine

the pricing in order to maximise profit. Under moderate assumptions a pure strategy equilibrium exists. An equilibrium location consists of vertices only and can be obtained by finite steps. Each firm will strive to minimise social cost in order to maximise its profit. In general, uniqueness is not satisfied because social cost may have several global minimisers and, more importantly, equilibrium strategies need not globally minimise social cost.

References

- Brander JA, Zhang A (1990) Market conduct in the airline industry: An empirical investigation. *Rand Journal of Economics* 21: 567–583
- Dorta-González P, Santos-Peñate DR, Suárez-Vega R (2004) Cournot oligopolistic competition in spatially separated markets: The Stackelberg equilibrium. *The Annals of Regional Science* 38: 499–511
- Gupta B, Kats A, Pal D (1994) Upstream monopoly, downstream competition and spatial price discrimination. *Regional Science and Urban Economics* 24: 529–542
- Gupta B, Pal D, Sarkar J (1997) Spatial Cournot competition and agglomeration in a model of location choice. *Regional Science and Urban Economics* 27: 261–282
- Hoover EM (1936) Spatial price discrimination. *Review of Economic Studies* 4: 182–191
- Hottelling H (1929) Stability in competition. *Economic Journal* 39: 41–57
- Labbé M, Hakimi SL (1991) Market and location equilibrium for two competitors. *Operations Research* 39: 749–756
- Lederer PJ, Hurter AP (1986) Competition of firms: Discriminatory pricing and location. *Econometrica* 54: 623–640
- Lederer PJ, Thisse JF (1990) Competitive location on networks under delivered pricing. *Operations Research Letters* 9: 147–153
- Philips L (1983) *The economics of price discrimination*. Cambridge University Press, Cambridge
- Sarkar J, Gupta B, Pal D (1997) Location equilibrium for Cournot oligopoly in spatially separated markets. *Journal of Regional Science* 2: 195–212